DEA Models for Parallel Systems: 
Game-Theoretic Approaches

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In many settings, systems are composed of a group of independent sub-units. Each sub-unit produces the same set of outputs by consuming the same set of inputs. Conventional data envelopment analysis (DEA) views such a system as a “black-box”, and uses the sum of the respective inputs and outputs of all relevant component units to calculate the system efficiency. Various DEA-based models have been developed for decomposing the overall efficiency. This paper further investigates this kind of structure by using

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the cooperative (or centralized) and non-cooperative (Stackelberg or leader–follower) game theory concepts. We show that the existing DEA approaches can be viewed as a centralized model that optimizes the efficiency scores of all sub-units jointly. The proposed leader–follower model will be useful when the priority sequence is available for sub-units. Consider, for example, the evaluation of relative efficiencies of a set of manufacturing facilities where multiple work shifts are operating. Management may wish to determine not only the overall plant efficiency, but as well, the performance of each shift in some priority sequence. The relationship between the system efficiency and component efficiencies is also explored. Our approaches are demonstrated with an example whose data set involves the national forests of Taiwan.

**Keywords**: Data envelopment analysis (DEA); parallel systems; efficiency; centralized model; leader–follower model.

1. Introduction

Data envelopment analysis (DEA) is an effective approach for measuring the relative efficiency of peer decision making units (DMUs) that produce the same set of outputs by consuming the same set of inputs. In conventional DEA models, DMUs are seen as black-boxes in the sense that the internal structure of DMUs is ignored. In recent years, a number of studies have looked at DMUs with network structures. For example, Seiford and Zhu (1999) use a series-connected two-stage process to measure the profitability and marketability of US commercial banks. To further address potential conflicts caused by intermediate measures existing between the two stages, Kao and Hwang (2008), and Liang et al. (2008) propose various DEA-based methods to measure efficiency for the overall process and for each individual stage. Färe and Grosskopf (2000) develop a network model to evaluate the overall network DMU efficiency.

One particular type of network structure is a parallel system where a production system or a DMU can be composed of a set of independent sub-units, with each consuming the same set of inputs to produce the same set of outputs as is true of the entire system (Kao, 2009). A typical example provided by Kao (2009) is a firm with several independently-operating plants. Each of the firm’s inputs and outputs can be obtained by summing up those of all its plants respectively. In modeling this kind of parallel structure, Beasley (1995), Kao (2009), Kao and Hwang (2010), and Castelli et al. (2010) propose their own versions of DEA models. All these models are essentially equivalent.

Beasley (1995) develops a joint DEA maximization model for determining teaching and research efficiencies for university departments devoted to the same discipline. Moreover, special situations are considered where certain resources (general and equipment expenditure) are shared by sub-units (teaching and research activities).

Kao (2009) further investigates the relationship between the inefficiency of component units and the inefficiency of the entire parallel system, and proposes a parallel DEA model to calculate the overall and component efficiencies. Instead of maximizing efficiency, Kao’s (2009) parallel model minimizes the inefficiency slack of a DMU, and decomposes the inefficiency slack into its production sub-units.
Castelli et al. (2004) investigate single-level and two-level hierarchical structures where each DMU is composed of consecutive stages of parallel sub-units. For the two-level situation, they introduce two kinds of balancing constraints (virtual weight balancing constraints and flow balancing constraints), and accordingly set up two different DEA models. In particular, they prove that the maximum relative efficiency of a DMU is obtained when it is compared with all the existing sub-units. However, in their work, the operation of each component unit is treated independently, and the relationship among these components is not considered. Castelli et al. (2010) proposed an elementary DEA model which is further extended into shared flow, multi-level, and network models.

In order to address both series and parallel structures in network systems, Kao and Hwang (2010) develop a relational model, based on which the system and component efficiencies can be directly determined. In terms of inefficiency, they show that the system inefficiency is a weighted average of process/component inefficiencies.

However, the DEA-based models in the above literature, including the joint maximization model (Beasley, 1995), parallel model (Kao, 2009), parallel relational model (Kao and Hwang, 2010), and elementary model (Castelli et al., 2010), may have alternative optimal solutions, thus calculating process efficiencies directly from its optimality can lead to non-unique efficiency decompositions. Thus an approach for testing for unique efficiency decomposition is required.

In the current paper, the parallel structure is further studied and extended using the game-theoretic perspectives presented in Liang et al. (2008). In Liang et al. (2008), two game approaches are proposed for series-connected two-stage network processes. One is a centralized model based upon the concept of a cooperative game where the overall efficiency is maximized in an effort to optimize both stages’ efficiency scores jointly. The overall efficiency is defined as the weighted arithmetic average or geometric average of stage efficiencies. The other is a non-cooperative game model which assumes that one stage is viewed as the leader with action priority to optimize its efficiency first, and then the efficiency of the follower stage is calculated subject to maintaining the first-derived leader’s efficiency.

Based upon the Liang et al. (2008) concept, we show that the existing DEA approaches can be viewed as centralized models that optimize the overall efficiency of a DMU subject to the unity restrictions on each component sub-unit and on the entire system. We then propose a leader–follower approach that assumes that one first decides on the relative importance of the sub-units, and then the optimal relative efficiencies of the sub-units are derived in sequential and priority order. Such a concept of evaluating both overall and sub-unit efficiencies can be readily applicable in many industrial and service sector settings. Consider, for example, the problem of evaluating the relative efficiencies of a set of manufacturing facilities where multiple work shifts are operating. Management may wish to determine not only the overall efficiency of the plant, but may, as well, wish to determine the performance of each of the multiple shifts, and in some priority sequence.
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In Kao’s (2009) approach, the overall efficiency of a DMU under evaluation is measured in terms of inefficiency in forms of “slacks”. As a result, the “overall slack” is a sum of sub-unit slacks, and implies a form of additive efficiency decomposition. We should point out, however, that these slacks in Kao’s (2009) approach are neither the standard DEA slacks in the second stage calculation of the envelopment DEA model, nor the slacks in slacks-based models (Tone, 2001). We point out that when efficiency is measured in terms of ratio efficiency as in the standard DEA model, such additive decomposition does not arise and thus is not inherent in the newly developed models.

The remainder of the paper is organized as follows. The centralized and leader–follower approaches for measuring system and process efficiencies are developed in Secs. 2 and 3, respectively. The mathematical relationship between the system and component efficiencies is also investigated. In Sec. 4, a real-world application in forest production in Taiwan is used to illustrate both approaches and compare results. Section 5 presents concluding remarks.

2. The Centralized Approach

Suppose that there are \( n \) DMUs, with unit \( o \) is denoted by DMU\(_o\) \((o = 1, \ldots, n)\), and with the \( i \)th input and \( r \)th output of DMU\(_o\) denoted by \( x_{io} \) \((i = 1, \ldots, m)\) and \( y_{ro} \) \((r = 1, \ldots, s)\), respectively. The conventional efficiency score, \( E_o \) for DMU\(_o\) is calculated by solving the following CCR model (1) (Charnes et al., 1978):

\[
E_o = \text{Max} \sum_{r=1}^{s} \frac{u_r y_{ro}}{v_i x_{io}},
\]

\[
\text{s.t.} \quad \sum_{r=1}^{s} \frac{u_r y_{ro}}{v_i x_{io}} \leq 1, \quad j = 1, \ldots, n
\]

\[
u_r, v_i \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.
\]

Now, suppose that each DMU has a parallel internal structure as shown in Fig. 1. For each DMU\(_o\), there are \( k \) sub-units operating independently, denoted by DMU\(_p^o\) \((p = 1, \ldots, k)\), and each of them utilizes the same \( m \) inputs to produce the same \( s \) outputs (in varying amounts), denoted by \( x_{io}^p \) \((i = 1, \ldots, m)\) and \( y_{ro}^p \) \((r = 1, \ldots, s)\), respectively. The sum \( x_{io} \) of all \( x_{io}^p \) over \( p \) is the amount of input \( i \) available to DMU\(_o\), and \( y_{ro} \), the sum of all \( y_{ro}^p \) over \( p \) is the amount of output \( r \) available to DMU\(_o\).

We first view this parallel system from a centralized and cooperative perspective, and determine a common set of optimal weights to maximize the overall efficiency for each DMU. In a cooperative sense, all component sub-units are supposed to agree on the absolute importance of the entire system performance. They cooperate first to achieve the optimal overall efficiency, after which the component efficiency is obtained for each sub-unit at the premise of maintaining the optimal system efficiency.
Therefore, for DMU_o, we first optimize its overall efficiency subject to the usual unity constraints in the form of unity restrictions not only on the DMUs, but also on each sub-unit in each DMU. Because of the centralized assumption, DMU_o has the central control over its sub-units. Thus the weight attached to each input/output is assumed to be unified in both system and component levels. The centralized model is given as follows:

\[
\theta^*_o = \max \left( \sum_{r=1}^{s} u_r y_{ro} \right),
\]

s.t. \( \sum_{r=1}^{s} u_r y_{rj} \leq 1, \quad j = 1, \ldots, n \) \hspace{2cm} (2)

\[
\sum_{r=1}^{s} u_r y_{prj} \leq 1, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n
\]

\[
u_r, \nu_i \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.
\]

Since \( x_{ij} = \sum_{p=1}^{k} x_{ij}^p, \quad y_{rj} = \sum_{p=1}^{k} y_{prj}^p \), it is obvious that the first set of constraints in model (2) is redundant. Using the Charnes–Cooper transformation (Charnes and Cooper, 1962), we get the following linear program (3) equivalent to fractional program (2):

\[
\theta^*_o = \max \left( \sum_{r=1}^{s} \mu_r y_{ro} \right)
\]

s.t. \( \sum_{r=1}^{s} \mu_r y_{rj}^p - \sum_{i=1}^{m} \omega_i x_{ij}^p \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n \) \hspace{2cm} (3)

\[
\sum_{i=1}^{m} \omega_i x_{io} = 1
\]

\[
\mu_r, \omega_i \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.
\]
In model (3), let $\mu^*_r, r = 1, \ldots, s$ and $\omega^*_i, i = 1, \ldots, m$ represent an optimal solution for $\mu_r$ and $\omega_i$. Then the overall efficiency score for the whole system is calculated as $\theta^*_o = \sum_{r=1}^{s} \mu^*_r y^*_r$, and the efficiency score for its sub-unit $p$, denoted by $\theta^*_p, p = 1, \ldots, k$, can be computed as

$$\theta^*_p = \frac{\sum_{r=1}^{s} \mu^*_r y^*_p}{\sum_{i=1}^{m} \omega^*_i x^*_p}, \quad p = 1, \ldots, k.$$

One can find that this centralized model (3) is the same with the joint maximization model in Beasley (1995), the parallel relational model in Kao and Hwang (2010), and the elementary model in Castelli et al. (2010). If we measure the efficiency in (3) in terms of inefficiency as in Kao (2009), namely letting $s^e_p \geq 0 (p = 1, \ldots, k)$ represent the slacks for DMU$_o$’s inequality constraints in model (3), then the centralized model (3) is also equivalent to the parallel model proposed by Kao (2009). However, although the overall system efficiency is obtained via similar models from existing literature, we determine the efficiency decomposition among all sub-units in a very different way as follows.

Note that our centralized model (3), or other similar models from existing studies, may have alternative optimal solutions, which can lead to non-unique efficiency decompositions for sub-units. This can be demonstrated by a simple numerical example presented in Table 1.

Suppose that there are two DMUs consuming two inputs to produce two outputs. Each DMU has two parallel sub-units with the same types of inputs and outputs. We evaluate DMU 1 via centralized model (3) and obtain two optimal solutions, which lead to two different efficiency decompositions for sub-units 1 and 2. For one optimal solution, the overall efficiency for DMU 1 is $\theta^*_1 = 0.5$, and the efficiency scores for sub-units 1 and 2 are $\theta^*_1 = 1$ and $\theta^*_2 = 0.3333$, respectively. The corresponding results from the other optimal solution are $\theta^*_1 = 0.5, \theta^*_1 = 0.6667$, and $\theta^*_2 = 0.4444$, respectively.

The above results indicate that the standard approach of calculating sub-unit efficiencies directly from the optimal solutions to related models, may lead to different component efficiency combinations. Therefore, to test for uniqueness, we follow Kao and Hwang’s (2008), or Liang et al. (2008) approach to find a set of multipliers which satisfies the following conditions: (i) produce the largest score for sub-unit with the first priority, while maintaining the overall efficiency score $\theta^*_o$.
calculated from model (3); (ii) produce the largest score for the sub-unit with the $q$th ($q = 2, \ldots, k$) priority while maintaining the optimal efficiencies for the overall system and for sub-units in the first $(q - 1)$ priority positions. Through the above process, we obtain a set of efficiency scores for all sub-units.

For example, assume that sub-unit $p_1$ is given pre-emptive priority. The following model (4) determines its efficiency score while maintaining the overall efficiency score at $\theta^*_o$.

$$\theta^{p_1*}_o = \text{Max} \sum_{r=1}^{s} \mu^{p_1}_r y^{p_1}_{ro}$$

s.t. $\sum_{r=1}^{s} \mu^{p_1}_r y^{p_1}_{rj} - \sum_{i=1}^{m} \omega^{p_1}_i x^{p_1}_{io} \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n$

$$\sum_{i=1}^{m} \omega^{p_1}_i x^{p_1}_{io} = 1$$

$$\sum_{r=1}^{s} \mu^{p_1}_r y^{p_1}_{ro} = \theta^*_o\sum_{i=1}^{m} \omega^{p_1}_i x^{p_1}_{io} = 0$$

$$\mu^{p_1}_r, \omega^{p_1}_i \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.$$  \hfill (4)

For sub-unit with the $q$th ($q = 2, \ldots, k$) priority, the following model (5) optimizes its efficiency score while maintaining the optimal efficiencies for the entire system and sub-units with the first to the $(q - 1)$th priority.

$$\theta^{p_q*}_o = \text{Max} \sum_{r=1}^{s} \mu^{p_q}_r y^{p_q}_{ro}$$

s.t. $\sum_{r=1}^{s} \mu^{p_q}_r y^{p_q}_{rj} - \sum_{i=1}^{m} \omega^{p_q}_i x^{p_q}_{io} \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n$

$$\sum_{i=1}^{m} \omega^{p_q}_i x^{p_q}_{io} = 1$$

$$\sum_{r=1}^{s} \mu^{p_q}_r y^{p_q}_{ro} - \theta^*_o\sum_{i=1}^{m} \omega^{p_q}_i x^{p_q}_{io} = 0$$

$$\sum_{r=1}^{s} \mu^{p_q}_r y^{p_q}_{ro} - \theta^{p_q*}_o\sum_{i=1}^{m} \omega^{p_q}_i x^{p_q}_{io} = 0, \quad \ell = 1, \ldots, q - 1$$

$$\mu^{p_q}_r, \omega^{p_q}_i \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.$$  \hfill (5)

Therefore, an efficiency decomposition is obtained for all component units of DMU$_o$ as $(\theta^{1*}_o, \theta^{2*}_o, \ldots, \theta^{k*}_o)$. It is very likely that when $k$ is greater than a certain number, there is only one unique optimal solution to model (5). Let $\theta^{p*}_o$ represent the optimal efficiency score for sub-unit $p$ calculated from model (4) when sub-unit $p$ itself is given the first priority, $p = 1, \ldots, k$. If $\theta^{q*}_o = \theta^{p*}_o$ for all $p = 1, \ldots, k$, then the
Table 2. Efficiency results from centralized approach.

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Efficiency</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sub-unit 1</td>
<td>Sub-unit 2</td>
</tr>
<tr>
<td></td>
<td>takes priority</td>
<td>takes priority</td>
</tr>
<tr>
<td>DMU 1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Sub-unit 1</td>
<td>1</td>
<td>0.6667</td>
</tr>
<tr>
<td>Sub-unit 2</td>
<td>0.3333</td>
<td>0.4444</td>
</tr>
<tr>
<td>DMU 2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sub-unit 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sub-unit 2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

efficiency score for each sub-unit is uniquely determined by centralized model (3), indicating that a unique efficiency decomposition for $DMU_o$ is obtained. Here, $\theta^*_o$ is defined above and computed directly from one optimal solution of model (3).

To demonstrate the above idea, we revisit the numerical example in Table 1. The efficiency results are reported in Table 2. The second column lists efficiency scores for DMU 1, 2 and their two sub-units when sub-unit 1 is given the first priority. The third column lists the corresponding results when sub-unit 2 is given the first priority to realize its optimal efficiency. From the results in Table 2, we note that a unique efficiency decomposition for DMU 2 can be directly determined by centralized model (3). However, for DMU 1, the efficiency decomposition for sub-units varies when different sub-units are given first priority.

Note that in Kao’s (2009) parallel model, the overall “slack” in the objective function is a sum of sub-units’ slacks in their constraints. As a result, Kao (2009) obtains a form of additive efficiency decomposition in terms of the “slacks”. If we use ratio efficiency measures as in model (2) or (3), such decomposition is not available. We point out, however, that the slacks in Kao’s (2009) model are neither the standard DEA slacks in the second stage calculation of the envelopment DEA model, nor the slacks in slacks-based models (Tone, 2001). As a result, these slacks do not completely indicate the levels of performance inefficiency. One still needs to convert such slacks into ratio efficiency measures to get the magnitude of inefficiency.

Next we explore the mathematical relationship between the overall efficiency $\theta^*_o$ and component efficiency scores $\theta^*_p$ ($p = 1, \ldots, k$) calculated from models (4) and (5).

For a sub-unit with the $k$th priority, let \{\$\theta^*_o, \mu^*_k, \omega^*_i, r = 1, \ldots, s, i = 1, \ldots, m\} represent an optimal solution to model (5); then it is true that

$$\sum_{r=1}^s \mu^*_k y_{ro} = \theta^*_o \sum_{i=1}^m \omega^*_i x_{io}$$  \hspace{1cm} (6)

and

$$\sum_{r=1}^s \mu^*_k y_{pt} = \theta^*_p \sum_{i=1}^m \omega^*_i x_{pt}, \hspace{0.5cm} t = 1, \ldots, k.$$  \hspace{1cm} (7)
Summing over \( t \) on both sides of Eq. (7), we have

\[
\sum_{r=1}^{s} \mu_{p_r}^* y_{r0} = \sum_{i=1}^{k} \theta_{0}^* \sum_{p=1}^{m} \omega_{i}^{p_k} p_{r}^{*} x_{i0},
\]

\[
\Rightarrow \theta_{0}^* \sum_{i=1}^{m} \omega_{i}^{p_k} x_{i0} = \sum_{t=1}^{k} \theta_{0}^* \sum_{p=1}^{m} \omega_{i}^{p_k} p_{t}^{*} x_{i0},
\]

\[
\Rightarrow \theta_{0}^* = \sum_{t=1}^{k} \left( \sum_{i=1}^{m} \omega_{i}^{p_k} x_{i0} \right) \frac{\theta_{0}^*}{\sum_{i=1}^{m} \omega_{i}^{p_k} x_{i0}},
\]

\[
\Rightarrow \theta_{0}^* = \sum_{t=1}^{k} w_{p_t} \theta_{0}^*{p_t}^*, \quad \text{where } w_{p_t} = \frac{\sum_{i=1}^{m} \omega_{i}^{p_k} x_{i0}}{\sum_{i=1}^{m} \omega_{i}^{p_k} x_{i0}}.
\]

Since \( 0 < w_{p_t} < 1 \) and \( \sum_{t=1}^{k} w_{p_t} = 1 \), Eq. (11) demonstrates that the overall efficiency is a weighted average of all component efficiencies. The weight attached to each component efficiency is the proportion of total aggregated resources devoted to each sub-unit, reflecting the relative size of a sub-unit, and furthermore the relative importance or contribution of each sub-unit’s performance compared to the system’s overall performance.

It is also easily derived from Eq. (11) that \( \min_{p=1,...,k} \{ \theta_{0}^{*} \} \leq \theta_{0}^{*} \leq \max_{p=1,...,k} \{ \theta_{0}^{*} \} \).

This implies that in our centralized approach, although the overall efficiency is optimized first, it still lies within the efficiency range of the sub-units.

3. The Leader–Follower Approach

In this section, we view the parallel system shown in Fig. 1 from a non-cooperative game perspective, and suppose that each sub-unit intends to make its efficiency score as high as possible, given the current input and output levels. Thus, we adopt the idea from the Stackelberg game concept where the leader firm/party moves first, and then the process continues for each follower down through the hierarchical structure. In that sense, the Stackelberg model is also referred to as leader–follower model.

In a parallel structure, if we assume that the action sequence for all sub-units under DMU \( o \) is \((p_1, p_2, \ldots, p_k)\), then sub-unit \( p_1 \) is the leader and moves first, sub-unit \( p_2 \) is follower 1 and moves after sub-unit \( p_1 \)’s action, sub-unit \( p_3 \) is follower 2 and moves after sub-unit \( p_1 \) and \( p_2 \)’s action. This process continues until sub-unit \( p_k \) (follower \((k - 1)\)) moves after sub-unit \( p_{k-1} \) through \( p_{k-1} \)’s action. In other words, each subordinate player executes his policies after, and with the full knowledge of, his superior players. No matter what tactics will be taken by the followers, the best strategy for leader \( p_1 \) is to optimize its efficiency, and the best strategy for follower \((t - 1)\), \( p_t \), \( t = 1, \ldots, k \) is to optimize its efficiency subject to the requirement that the efficiencies for leader and follower 1 through \((t - 2)\) all remain unchanged. It implies that the decision of any one player will impact its subsequent players set of feasible choices.
The above procedure can be realized in a mathematical manner as follows:

(1) For leader sub-unit \( p_1 \), its efficiency score is determined via the following model (12), the result of which is the conventional CCR efficiency when all sub-units are treated as independent DMUs and compared together.

\[
e_{p_1}^* = \text{Max} \sum_{r=1}^{s} \mu_{r}^{p_1} y_{r_0}^{p_1}
\]

s.t. \[
\sum_{r=1}^{s} \mu_{r}^{p_1} y_{r_1}^{p_1} - \sum_{i=1}^{m} \omega_{i}^{p_1} x_{ij}^{p_1} \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n
\]

\[
m \sum_{i=1}^{m} \omega_{i}^{p_1} x_{io}^{p_1} = 1
\]

\[
\mu_{r}^{p_1}, \omega_{i}^{p_1} \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.
\]

(12)

(2) Based upon the leader’s efficiency result, the efficiency for follower 1 (sub-unit \( p_2 \)) is derived by solving the following linear model (13).

\[
e_{p_2}^* = \text{Max} \sum_{r=1}^{s} \mu_{r}^{p_2} y_{r_0}^{p_2}
\]

s.t. \[
\sum_{r=1}^{s} \mu_{r}^{p_2} y_{r_1}^{p_2} - \sum_{i=1}^{m} \omega_{i}^{p_2} x_{ij}^{p_2} \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n
\]

\[
m \sum_{i=1}^{m} \omega_{i}^{p_2} x_{io}^{p_2} = 1
\]

\[
\sum_{r=1}^{s} \mu_{r}^{p_2} y_{r_1}^{p_1} - e_{p_1}^* \sum_{i=1}^{m} \omega_{i}^{p_2} x_{ij}^{p_1} = 0
\]

\[
\mu_{r}^{p_2}, \omega_{i}^{p_2} \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.
\]

(13)

(3) Based upon leader’s and all the previous followers’ efficiency results, the efficiency for follower \((k-1)\) (sub-unit \( p_k \)) is derived from the solution to:

\[
e_{p_k}^* = \text{Max} \sum_{r=1}^{s} \mu_{r}^{p_k} y_{r_0}^{p_k}
\]

s.t. \[
\sum_{r=1}^{s} \mu_{r}^{p_k} y_{r_1}^{p_k} - \sum_{i=1}^{m} \omega_{i}^{p_k} x_{ij}^{p_k} \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n
\]

\[
m \sum_{i=1}^{m} \omega_{i}^{p_k} x_{io}^{p_k} = 1
\]

\[
\sum_{r=1}^{s} \mu_{r}^{p_k} y_{r_1}^{p_t} - e_{p_t}^* \sum_{i=1}^{m} \omega_{i}^{p_k} x_{ij}^{p_t} = 0, \quad t = 1, \ldots, k - 1
\]

\[
\mu_{r}^{p_k}, \omega_{i}^{p_k} \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.
\]

(14)
(4) After the efficiency scores for all sub-units are obtained as $e^*_p, p = 1, \ldots, k$, we calculate the overall efficiency for DMU$_o$ as

$$e^*_o = \text{Max}_{r=1}^{s} \sum_{r=1}^{s} \mu_r y_{ro}$$

subject to

$$\sum_{p=1}^{k} \mu_r y_{rj} - \sum_{i=1}^{m} \omega_i x_{ij}^p \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n$$

$$\sum_{i=1}^{m} \omega_i x_{io} = 1$$

$$\sum_{p=1}^{k} \mu_r y_{rj} - e^*_o \sum_{i=1}^{m} \omega_i x_{io}^p = 0, \quad p = 1, \ldots, k$$

$$\mu_r, \omega_i \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.$$  \hfill (15)

Note that in the non-cooperative (leader–follower) model proposed by Liang et al. (2008), the overall efficiency score is directly determined as the product of two individual stages’ scores. This is due to the special relationship among inputs, intermediate measures and outputs from their series-connected two-stage structures. However, in our parallel case, there is no similar relationship between the system and component efficiencies. Therefore, once all component efficiency scores are determined from (1) to (3) in the above procedure, we need to further solve an additional model (15) in order to evaluate the overall efficiency for the parallel system. The last set of constraints requires that the calculated efficiency scores for sub-units remain unchanged.

It is worth noting that in the case of leader–follower approach, although the overall efficiency is optimized last, after all sub-units obtain their respective optimal scores in a sequential manner, it does not necessarily mean that the entire system will be surpassed by any sub-unit in terms of efficiency measurement. Rather, the overall efficiency $e^*_o$ lies between the lowest and highest of sub-unit efficiency scores.

Also, if we let \{ $e^*_o, \mu^*_r, \omega^*_i, r = 1, \ldots, s, i = 1, \ldots, m$ \} be an optimal solution to model (15), we have

\[
e^*_o = \sum_{r=1}^{s} \mu^*_r y_{ro},
\]

\[
= \sum_{p=1}^{k} e^*_o \sum_{i=1}^{m} \omega^*_i x_{io}^p,
\]

\[
= \sum_{p=1}^{k} \left( \sum_{i=1}^{m} \omega^*_i x_{io}^p \right) e^*_o.
\]  \hfill (16)
Equation (16) indicates that the overall efficiency is a weighted average of all component efficiencies. The weight attached to sub-unit \( p \)'s efficiency is its aggregated inputs, reflecting to some extent the relative importance or contribution of sub-unit \( p \)'s performance.

To demonstrate our leader–follower approach, we consider the numerical example in Table 1. We first assume that sub-unit 1 is the leader, and sub-unit 2 is the follower. The corresponding efficiency results are reported in the second column of Table 3. The third column shows the efficiency results when sub-unit 2 is assumed to be leader. Comparing the two groups of efficiency scores, we notice that DMU 1 has different system and sub-unit efficiencies when different sub-units act as leader. However, for DMU 2, its overall and component efficiencies remain the same no matter which sub-unit is the leader.

Finally, we can also develop the leader–follower approach based upon inefficiency or slacks.

(1) For leader sub-unit \( p_1 \), its efficiency score is determined by model (12) which can be converted into the following linear program:

\[
\begin{align*}
\frac{e^{p_1}}{\sigma} &= 1 - \frac{s^{p_1}}{\sigma} = 1 - \min s^{p_1} \\
\text{s.t.} & \sum_{r=1}^{s} \mu^{p_1} r y^{p_1} - \sum_{i=1}^{m} \omega^{r} \sum_{r=1}^{s} \mu^{p_1} y^{p_1} = 0 \\
& \sum_{r=1}^{s} \mu^{p_1} r y^{p_1} - \sum_{i=1}^{m} \omega^{r} \sum_{r=1}^{s} \mu^{p_1} x^{p_1} \leq 0, \quad p = 1, \ldots, k, \ j = 1, \ldots, n \\
& \sum_{i=1}^{m} \omega^{p_1} x^{p_1} = 1 \\
& \mu^{p_1}, \omega^{p_1} \geq \varepsilon, \quad r = 1, \ldots, s, \ i = 1, \ldots, m \\
& s^{p_1} \geq 0.
\end{align*}
\]

Table 3. Efficiency results from leader–follower model.

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sub-unit 1 is leader</td>
</tr>
<tr>
<td>DMU 1</td>
<td>0.5</td>
</tr>
<tr>
<td>Sub-unit 1</td>
<td>1</td>
</tr>
<tr>
<td>Sub-unit 2</td>
<td>0.3333</td>
</tr>
<tr>
<td>DMU 2</td>
<td>1</td>
</tr>
<tr>
<td>Sub-unit 1</td>
<td>1</td>
</tr>
<tr>
<td>Sub-unit 2</td>
<td>1</td>
</tr>
</tbody>
</table>

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(2) Based upon leader’s efficiency result, the efficiency for follower 1 (sub-unit $p_2$) is calculated by solving the following linear model (18).

\[ e_{o}^{P^{2}} = 1 - s_{o}^{P^{2}} = 1 - \text{Min} s_{o}^{P^{2}} \]

\[
\text{s.t.} \quad \sum_{r=1}^{s} x_{iio}^{P^{2}r} - \sum_{i=1}^{m} \omega_{i}^{P^{2}} x_{iio}^{P^{2}r} + s_{o}^{P^{2}} = 0 \\
\sum_{r=1}^{s} x_{rj}^{P^{2}r} - \sum_{i=1}^{m} \omega_{i}^{P^{2}} x_{rj}^{P^{2}r} \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n \\
\sum_{i=1}^{m} \omega_{i}^{P^{2}} x_{iio}^{P^{2}} = 1 \tag{18}
\]

\[
\sum_{r=1}^{s} x_{rj}^{P^{2}r} - (1 - s_{o}^{P^{2}}) \sum_{i=1}^{m} \omega_{i}^{P^{2}} x_{rj}^{P^{2}r} = 0 \\
\mu_{i}^{P^{2}}, \omega_{i}^{P^{2}} \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m \\
s_{o}^{P^{2}} \geq 0.
\]

(3) Based upon the leader’s and all the previous followers’ efficiency results, the efficiency for follower $(k-1)$ (sub-unit $p_k$) is calculated by

\[ e_{o}^{P^{k}} = 1 - s_{o}^{P^{k}} = 1 - \text{Min} s_{o}^{P^{k}} \]

\[
\text{s.t.} \quad \sum_{r=1}^{s} x_{iio}^{P^{k}r} - \sum_{i=1}^{m} \omega_{i}^{P^{k}} x_{iio}^{P^{k}r} + s_{o}^{P^{k}} = 0 \\
\sum_{r=1}^{s} x_{rj}^{P^{k}r} - \sum_{i=1}^{m} \omega_{i}^{P^{k}} x_{rj}^{P^{k}r} \leq 0, \quad p = 1, \ldots, k, \quad j = 1, \ldots, n \\
\sum_{i=1}^{m} \omega_{i}^{P^{k}} x_{iio}^{P^{k}} = 1 \tag{19}
\]

\[
\sum_{r=1}^{s} x_{rj}^{P^{k}r} - (1 - s_{o}^{P^{k}}) \sum_{i=1}^{m} \omega_{i}^{P^{k}} x_{rj}^{P^{k}r} = 0, \quad t = 1, \ldots, k-1 \\
\mu_{i}^{P^{k}}, \omega_{i}^{P^{k}} \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m \\
s_{o}^{P^{k}} \geq 0.
\]

(4) After the optimal slacks for all sub-units are obtained as $s_{o}^{P^{p}}, p = 1, \ldots, k$, we calculate the overall efficiency for DMU$_{o}$ as

\[ e_{o}^{*} = 1 - s_{o}^{*} = 1 - \text{Min} s_{o} \]

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The mathematical relationship between the overall optimal slack \( s^*_o \) and component optimal slacks \( s^*_p (p = 1, \ldots, k) \) is expressed as \( s^*_p = \sum_{p=1}^{k} \omega^*_p x^p_{io} \), where \( \omega^*_i \) is the optimal value for \( \omega_i (i = 1, \ldots, m) \) in model (20). This implies that similar to efficiency, the overall inefficiency is also a weighted average of all component inefficiencies, which was previously mentioned in Kao and Hwang (2010).

4. Forest Production in Taiwan

We here revisit the forest production example in Kao (2009) to illustrate the cooperative and non-cooperative game approaches proposed in this paper, and further compare our efficiency results with Kao’s (2009) results.

As pointed out by Kao (2009), the forest production system is a typical parallel production system, where each district has several sub-districts, referred to as working circles (WCs), operating independently. In Taiwan, the forestlands are divided into eight districts, and each is further divided into four or five WCs. A WC is the basic unit in forest management, but it is not regarded as a so-called independent unit because it does not possess an administrator. Only a district is viewed as an independent unit in this forest production system.

The data set is presented in Table 4, which was previously used by Kao (1998, 2000, 2009). There are four inputs, including land in thousands of hectares, labor in persons, expenditures each year in ten-thousand New Taiwan dollars, and initial stocks before the evaluation period in 10,000 cubic meters. Three outputs are taken into account, including timber production each year in cubic meters, soil conservation in 10,000 cubic meters, and recreation each year in thousands of visits. For a detailed explanation regarding the forest production system and the data set, refer to Kao (1998, 2000, 2009).
Table 4. Data for Taiwan forest system.

<table>
<thead>
<tr>
<th>Working circles</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Land</td>
<td>Labor</td>
</tr>
<tr>
<td></td>
<td>(1000 hectares)</td>
<td>(persons)</td>
</tr>
<tr>
<td><strong>Lotung District</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Taipei</td>
<td>175.73</td>
<td>248.33</td>
</tr>
<tr>
<td>(2) Tai-ping-shan</td>
<td>18.23</td>
<td>45.33</td>
</tr>
<tr>
<td>(3) Chao-chi</td>
<td>31.44</td>
<td>51.00</td>
</tr>
<tr>
<td>(4) Nan-ao</td>
<td>28.94</td>
<td>27.33</td>
</tr>
<tr>
<td>(5) Ho-pei</td>
<td>41.63</td>
<td>26.67</td>
</tr>
<tr>
<td><strong>Hsinchu District</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6) Guay-shan</td>
<td>162.81</td>
<td>316.67</td>
</tr>
<tr>
<td>(7) Ta-chi</td>
<td>29.72</td>
<td>58.00</td>
</tr>
<tr>
<td>(8) Chu-tung</td>
<td>59.28</td>
<td>77.67</td>
</tr>
<tr>
<td>(9) Ta-hu</td>
<td>32.33</td>
<td>94.67</td>
</tr>
<tr>
<td><strong>Tungsh District</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10) Shan-chi</td>
<td>138.42</td>
<td>310.34</td>
</tr>
<tr>
<td>(11) An-ma-shan</td>
<td>10.40</td>
<td>50.67</td>
</tr>
<tr>
<td>(12) Li-yaang</td>
<td>33.64</td>
<td>111.33</td>
</tr>
<tr>
<td>(13) Li-shan</td>
<td>38.01</td>
<td>97.67</td>
</tr>
<tr>
<td><strong>Nantou District</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(14) Tai-chung</td>
<td>56.37</td>
<td>50.67</td>
</tr>
<tr>
<td>(15) Tan-ta</td>
<td>211.82</td>
<td>287.32</td>
</tr>
<tr>
<td>(16) Pu-li</td>
<td>10.57</td>
<td>64.33</td>
</tr>
<tr>
<td>(17) Shui-li</td>
<td>52.69</td>
<td>49.00</td>
</tr>
<tr>
<td>(18) Chu-shan</td>
<td>77.22</td>
<td>68.33</td>
</tr>
<tr>
<td>Working circles</td>
<td>Land (1000 hectares)</td>
<td>Labor (persons)</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Chiayi District</td>
<td>139.65</td>
<td>203.00</td>
</tr>
<tr>
<td>(19) A-li-shan</td>
<td>42.81</td>
<td>69.33</td>
</tr>
<tr>
<td>(20) Fan-chi-hu</td>
<td>19.28</td>
<td>35.33</td>
</tr>
<tr>
<td>(21) Ta-pu</td>
<td>32.86</td>
<td>44.67</td>
</tr>
<tr>
<td>(22) Tai-nan</td>
<td>44.70</td>
<td>53.67</td>
</tr>
<tr>
<td>Pingtung District</td>
<td>196.06</td>
<td>250.33</td>
</tr>
<tr>
<td>(23) Chih-shan</td>
<td>35.64</td>
<td>61.33</td>
</tr>
<tr>
<td>(24) Chao-chou</td>
<td>70.19</td>
<td>62.00</td>
</tr>
<tr>
<td>(25) Liu-guay</td>
<td>70.96</td>
<td>55.67</td>
</tr>
<tr>
<td>(26) Heng-chun</td>
<td>19.27</td>
<td>71.33</td>
</tr>
<tr>
<td>Taichung District</td>
<td>226.54</td>
<td>141.67</td>
</tr>
<tr>
<td>(27) Kuan-shan</td>
<td>113.42</td>
<td>54.67</td>
</tr>
<tr>
<td>(28) Chi-chen</td>
<td>44.54</td>
<td>41.00</td>
</tr>
<tr>
<td>(29) Ta-wu</td>
<td>44.03</td>
<td>20.33</td>
</tr>
<tr>
<td>(30) Chao-hong</td>
<td>24.55</td>
<td>25.67</td>
</tr>
<tr>
<td>Hualien District</td>
<td>320.43</td>
<td>284.00</td>
</tr>
<tr>
<td>(31) Shin-chan</td>
<td>85.95</td>
<td>64.00</td>
</tr>
<tr>
<td>(32) Nan-hua</td>
<td>51.60</td>
<td>76.00</td>
</tr>
<tr>
<td>(33) Wan-yang</td>
<td>59.53</td>
<td>74.00</td>
</tr>
<tr>
<td>(34) Yu-li</td>
<td>123.35</td>
<td>70.00</td>
</tr>
</tbody>
</table>
Table 5 reports the results for districts and WC obtained from our centralized approach and leader–follower approach in columns 2 and 3, respectively. Within each district, without any additional information available to us, the priority order for both approaches is assumed to be set according to the manner in which they
are numbered, that is from the WC with the lowest number to the one with the highest number.

The results from Kao’s (2009) parallel model are shown in column 4. As expected, the efficiency results from our centralized approach are exactly the same as those from the parallel model. Furthermore, to test the uniqueness for each district, we assume the pre-emptive priority and solve model (4) for each WC. All of the resulting optimal efficiency scores are the same with our centralized approach. This implies that a unique efficiency decomposition is obtained for all WC under each district. Also, note that for both of our approaches, the overall efficiency for each district lies within the efficiency range of its subordinate WCs, and can be demonstrated to be a weighted average of related WC efficiency scores.

If we treat the 34 WCs as independent DMUs, we can calculate their efficiency scores by solving the conventional CCR model (1). The results are listed in the fifth column of Table 5, and generally speaking, they are quite consistent with those calculated from the centralized approach (or Kao’s (2009) parallel model), and with those from the leader–follower approach. Since the CCR WC efficiency is the best score that each WC can possibly achieve, these CCR efficiencies are greater than or equal to the corresponding efficiency results obtained from our centralized or leader–follower approach. This is especially true for our leader–follower approach, where the first WC under each district is given the first priority in efficiency optimization. Thus, its leader–follower efficiency result is equal to its CCR WC efficiency.

Comparing the efficiency results solved from the centralized model with the CCR WC efficiency scores, we find that there are 9 out of 34 WCs whose CCR efficiencies are greater than their centralized efficiencies by more than 0.1. The largest difference occurs at the 14th WC Tai-chung under Nantou district, where the efficiency score is 1 versus 0.5701. This indicates that sacrificing the efficiency score for Tai-chung by 0.4299 (= 1 - 0.5701), could realize a higher efficiency for Nantou, the district it belongs to. If WC Tai-chung is allowed to be efficient with the score 1, then the highest possible efficiency score for Nantou district becomes 0.5577, a drop by 0.2157 (= 0.7734 - 0.5577) compared with its original centralized efficiency. We also notice that 12 out of 34 WCs have equal centralized efficiencies to their respective CCR efficiencies, 2/3 of which have an efficient score of unity. Similarly, we compare the efficiency results obtained from the leader–follower model with the CCR WC efficiencies, and find that 12 out of 34 WCs with a CCR efficiency greater than the leader–follower efficiency by more than 0.1. The largest difference occurs at the 26th WC Heng-chun under Pingtung district, which is 1 versus 0.0853. This is because Heng-chun is the last follower and is given the least priority in efficiency optimization among all four WCs belonging to Pingtung district. In other words, its efficiency is sacrificed to realize higher efficiency scores for the remaining three WCs from the same district, all of which have priority over Heng-chun in optimizing efficiency. Also, note that 11 out of 34 WCs have leader–follower efficiencies equal to their respective CCR efficiencies, 8 of which do so because they are the leader in
the action sequence within each DMU/district, and have the priority to receive the best possible efficiency scores.

The last column of Table 5 shows the conventional CCR efficiency scores for eight districts without considering any WC. As pointed out by Kao (2009), ignoring the requirement that each sub-unit should have an aggregated output that is smaller than its aggregated input will lead to a higher efficiency measure for every district. The efficiency results obtained either from the centralized or from the leader–follower approach, show that none of the eight districts is efficient. However, in terms of conventional CCR efficiency, only two districts (Lotung and Nantou) are inefficient, and the remaining six are efficient. This implies that both of our centralized approach and leader–follower approach have a stronger discrimination power in performance evaluation than the conventional DEA model. Also comparing the constraints from leader–follower model (15) with those from centralized model (3), we find that the constraints of the leader–follower overall-efficiency model are much stronger than those of the centralized model. Therefore, the overall efficiency scores calculated from the former are smaller than those calculated from the latter.

We point out again that our application of the leader–follower model in this particular problem setting is illustrative only, and acknowledge that in the absence of additional information, one may not be able to easily decide upon which sub-unit or WC in a particular district should be treated as the leader or follower. Specifically, if none of the WCs can legitimately be treated as leader, then the leader–follower model may not be appropriate for capturing sub-unit level efficiency.

5. Conclusions

In this paper, we first examine the existing DEA approaches on DMUs that have a parallel internal network structures with independently-operating sub-units. We show that the existing DEA approaches can be viewed as a DEA model adopting the concept of cooperative (or centralized) game theory. Next we further develop a DEA approach based upon non-cooperative (Stackelberg/leader–follower) game theory. The two approaches study the same problem from different perspectives. The centralized model supposes all component sub-units agree on the absolute importance of the overall efficiency for the entire system. The overall efficiency is optimized first, after which an efficiency-decomposition is obtained for each sub-unit. On the contrary, in the leader–follower approach, a priority is placed on the sub-units. Each sub-unit determines its best possible score with the restriction that it must follow those leader sub-units that come before it in the optimizing sequence. The overall efficiency for the DMU is calculated last subject to that all sub-units maintain their respective efficiency scores.

The decision on which of the two approaches is more preferable depends on the specific real-world application. Additional information from specific empirical study supports decision-makers to make the choice on models and to decide the priority
order for component units. If additional information indicates that one component is of vital importance during a production process, then the leader–follower approach may be more appropriate for performance evaluation. But if decision-makers pay much more attention to the overall system rather than any individual sub-unit, or none of the sub-units can legitimately be treated as leader, then the centralized approach is a better choice for efficiency analysis.

We emphasize that the key assumption made in this paper and in the work of Kao (2009), is that the parallel sub-units all produce exactly the same outputs using the same inputs (albeit in differing amounts). In many sub-unit situations, however, this property of identical output/input factors may not hold. Consider, for example, the case of an organization where different business units operate within the DMU — different wards in hospitals, service versus sales components in banks, different production lines in a factory, and so on. In future work, the authors propose to extend the development herein to accommodate settings where non-homogenous parallel sub-units operate.

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DEA Models for Parallel Systems: Game-Theoretic Approaches


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