ON PIECEWISE LOGLINEAR FRONTIERS AND LOG EFFICIENCY MEASURES

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Scope and Purpose—There are several data envelopment analysis (DEA) models that employ loglinear production functions to measure the efficiency of a set of decision making units (DMUs). Some models are units invariant to changes in the units of the inputs and outputs, and some are not. Chang and Guh (1995) provide a distance efficiency measure for logarithmic data and claim that their measure is units invariant and can serve as an alternative efficiency measure. However, we show that because their model is limited to the constant output case, the resulting loglinear frontier is the same as that obtained from the units invariant loglinear efficiency measure. It is this assumption of constant output, rather than their actual efficiency measure, which produces the units invariance of the resulting loglinear efficient frontier. Thus, their distance efficiency measure is developed and works under an implicit condition of units invariance. We examine the nature of invariance in logarithmic DEA models and correct erroneous statements and claims in Chang and Guh’s (1995) flawed paper.

Abstract—Chang and Guh (1995) provide a so-called units invariant distance efficiency measure and argue that Sueyoshi and Chang’s (1989) remedy for Charnes et al. (1982, 1983) is not units invariant. However, we show that (i) Chang and Guh (1995) misunderstand Sueyoshi and Chang’s (1989) remedy, and (ii) the loglinear frontier determined by the Chang and Guh’s distance efficiency measure, which is for the case of constant output and a linearly homogeneous production technology, is the same as obtained from Charnes et al. (1982, 1983). Chang and Guh (1995) ignore the fact that it is the constant output case, rather than the distance efficiency measure, that produces units invariance. We show that (i) their constant output assumption implies units invariance of the resulting loglinear efficient frontier, and (ii) their distance efficiency measure is developed and works under an implied condition of units invariance. We examine invariance in loglinear DEA models and correct erroneous calculations and claims in Chang and Guh’s (1995) paper. © 1998 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

Charnes et al. (CCR) [1] develop a mathematical programming formulation for assessing the relative efficiencies and inefficiencies of decision making units (DMUs). They term this methodology Data Envelopment Analysis (DEA). As discussed in Seiford [2], the evolution of DEA has been rapid and widespread resulting in a host of publications. One important branch of DEA models is based upon loglinear efficiency measures. Charnes et al. [3] develop a variant multiplicative (log) DEA model which is not invariant under change of units in the inputs or outputs. In [4], Charnes et al. provide an invariant multiplicative measure by introducing a convexity constraint. Finally, Sueyoshi and Chang [5] give a new units invariant objective function so that CCR-type DEA models are units invariant.

Chang and Guh ([6], p. 1031) suggest taking logarithms on inputs and outputs of DMUs, employing the DEA approach to estimate loglinear frontiers; then using the frontiers to construct the distance efficiency measure. However, they do not recognize that (i) their distance efficiency measure first requires that the resulting frontier is invariant to the changes in inputs and outputs; and (ii) their constant output assumption implies a convexity constraint in the DEA model which preserves units invariance in the loglinear efficiency measures. In fact, the resulting loglinear frontier determined by the DEA model used for their distance efficiency measure in [6] is the same as that obtained from the units invariant loglinear efficiency measure of [4] when one (as Chang and Guh [6] implicitly do) assumes constant output. It is

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this assumption of constant output, rather than the actual distance efficiency measure, which causes units invariance of the resulting loglinear efficient frontier in [6]. In other words, not only does their distance efficiency measure itself not ensure units invariance, but also it is actually developed under this implied condition of units invariance. The purpose of the current paper is to: (i) reveal the nature of invariance in loglinear DEA models; (ii) show the equivalence between Chang and Guh's [6] CCR loglinear model and units invariant loglinear efficiency model [4]; and (iii) point out the errors and problems in Chang and Guh's [6] paper.

The remainder of the paper is organized as follows. Section 2 discusses invariance in loglinear DEA models. Section 3 explores the false claims, errors and problems in [6]. It is shown that same efficient frontier will be obtained from the invariant multiplicative model of Charnes et al. [4] and the CCR loglinear model of Chang and Guh [6] for the case of constant output. Concluding remarks are given in Section 4.

2. INVARINCE IN DEA

To begin our discussion, we provide the following three definitions related to invariance in DEA.

**Definition 1. Units Invariance.** Solutions to DEA models are independent of the units of measurement employed.

**Definition 2. Translation Invariance.** Solutions to DEA models are independent of the displacement of measurement employed.

**Definition 3. Classification Invariance.** The classification of efficient and inefficient DMUs is independent of the data changes. Moreover, if the data changes are of unit changes, then we call units classification invariance, and if the data changes are of translations of the inputs or outputs, then we call translation classification invariance.

The Charnes et al. [4] Invariant Multiplicative Model can be written as

\[
\begin{align*}
\min \quad & -\delta \sum_{r=1}^{s} s_r^* - \delta \sum_{i=1}^{m} s_i^- \\
\text{s.t.} \quad & \sum_{j=1}^{k} \lambda_j \tilde{y}_j - s_r^* = \tilde{y}_r, \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{k} \lambda_j \tilde{x}_j + s_i^- = \tilde{x}_i, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{k} \lambda_j = 1, \\
& \lambda_j s_r^*, s_i^- \geq 0, \quad \forall j, r, i,
\end{align*}
\]

(1)

where \( \tilde{x}_j = \log x_j \) and \( \tilde{y}_j = \log y_j \), and \( x_j \) and \( y_j \) are, respectively, the \( i \)th input and \( r \)th output of DMU\(_j\) (\( j = 1, \ldots, k, \ldots, n \)). The parameter \( \delta \) in (1) is a positive scalar which also represents the bounds on the multipliers in the dual linear programming problem to (1) (see Ali et al. [7]).

**Theorem 1.** (i) Model (1) is units invariant to the original inputs and outputs; (ii) Model (1) is translation invariant to the logarithmic inputs and outputs.

**Proof.** (i) Let \( \tilde{x}_j = \alpha_j x_j \) and \( \tilde{y}_j = \beta_j y_j \). Using logarithms yields

\[
\begin{align*}
\hat{x}_j &= \log \tilde{x}_j = \log \alpha_j + \log x_j = \hat{\alpha}_j + \hat{x}_j, \\
\hat{y}_j &= \log \tilde{y}_j = \log \beta_j + \log y_j = \hat{\beta}_j + \hat{y}_j,
\end{align*}
\]

Substituting \( \hat{x}_j \) and \( \hat{y}_j \) into model (1) yields

\[
\begin{align*}
\sum_{j=1}^{k} \lambda_j \hat{y}_j - s_r^* &= (\sum_{j=1}^{k} \lambda_j \hat{y}_j - s_r^* + \beta_r \sum_{r=1}^{s} \lambda_r) = \hat{y}_r, \quad r = 1, \ldots, s, \\
\sum_{j=1}^{k} \lambda_j \hat{x}_j + s_i^- &= (\sum_{j=1}^{k} \lambda_j \hat{x}_j + s_i^- + \hat{\alpha}_i \sum_{i=1}^{m} \lambda_i) = \hat{x}_i, \quad i = 1, \ldots, m, \\
\sum_{j=1}^{k} \lambda_j &= 1.
\end{align*}
\]

Since the convexity constraint of \( \sum_{j=1}^{k} \lambda_j = 1 \), we immediately have the same constraints in (1), therefore
(1) is units invariant to the original inputs and outputs.
(ii) By the above proof and \( \sum_{j=1}^{m} \lambda_j = 1 \), we immediately have the result. \( \square \)

It can be seen that units invariance for the original inputs and outputs implies translation invariance for the logarithmic inputs and outputs, and vice versa. However, model (1) is not units invariant to the logarithmic inputs and outputs. In order to obtain this property, one may replace the objective function in (1) by (Sueyoshi and Chang [5])

\[
- \sum_{r=1}^{s} s_r^+ \gamma_{r}, \quad \sum_{i=1}^{m} s_i^- \delta_{r}.
\]

(2)

**Theorem 2.** Model (1) with new objective function (2) is units invariant to the logarithmic inputs and outputs.

**Proof.** Suppose that the logarithmic inputs and outputs are changed to \( a \tilde{x}_r \) and \( b \tilde{y}_r \). Then the new objective function is

\[
- \sum_{r=1}^{s} \frac{s_r^+}{b \tilde{y}_r} \gamma_{r}, \quad \sum_{i=1}^{m} \frac{s_i^-}{a \tilde{x}_r} \delta_{r},
\]

in which \( s_r^+/b_r \) and \( s_i^-/a_i \) are respectively the new output and input slacks in the constraints of (1). Therefore, model (1) with new objective function (2) is units invariant to the logarithmic inputs and outputs. \( \square \)

If we remove the constraint of \( \sum_{j=1}^{m} \lambda_j = 1 \) from (1), then we obtain the Variant Multiplicative Model of Charnes et al. [3]

\[
\begin{align*}
\text{min} & \quad - \delta \sum_{r=1}^{s} s_r^+ - \delta \sum_{i=1}^{m} s_i^- \\
\text{s.t.} & \quad \sum_{j=1}^{s} \lambda_j \tilde{y}_{rj} - s_r^+ = -\tilde{y}_r, \quad r=1, \ldots, s, \\
& \quad \sum_{i=1}^{m} \lambda_i \tilde{x}_i + s_i^- = \tilde{x}_i, \quad i=1, \ldots, m, \\
& \quad \lambda_j s_r^+, s_i^- \geq 0, \quad \forall j, r, i.
\end{align*}
\]

(3)

The property of units invariance to the original inputs and outputs no longer holds for (3), but the property of units invariance to the logarithmic inputs and outputs holds for (3) with the objective function (2).

Finally, the invariance property in DEA usually holds for the primal (envelopment) models, not the dual (multiplier) models. The invariance property also indicates that the ranking of DMUs is independent of changes in inputs and outputs, and that the resulting loglinear efficient frontier is invariant to changes in inputs and outputs.

3. ERRORS AND PROBLEMS IN CHANG AND GUH’S PAPER

It is well known that Charnes et al. [3] Variant Multiplicative Model [model (3)] is not units invariant to the original inputs and outputs. Therefore, Charnes et al. [4] provided an Invariant Multiplicative Model, i.e., model (1). Consider the four DMUs with two inputs and a single output as given in Chang and Guh [6]: \( P_1(10^3, 10^4, 10^5), P_2(10^4, 10^1, 10^2), P_3(10^3, 10^4, 10^5) \) and \( P_4(10^7, 10^4, 10^2) \). Model (1) will classify all of them as efficient, and model (3) will classify the first two as efficient. By Theorem 1, we know that under (1), these four DMUs remain efficient when the unit of the first input is changed from one to ten. Chang and Guh [6] write the invariant multiplicative model and state that the variant multiplicative model has the same formulation except that some variables in the invariant multiplicative model are set to zero and one. However, they do not point out that the major difference between (1) and (3) is that the latter does not contain the convexity constraint which preserves units invariance (of original inputs and outputs).

Chang and Guh ([6], p. 1033) claim that the dual of (1)\( ^\dagger \) will not give meaningful results if \( \delta \) is an infinitesimal. It is true that if \( \delta \) is set to a small positive number, the aggregated objective function value in (1) will approximately be equal to zero for all DMUs. However, such an approach is incorrect and

\( ^\dagger \) The dual of (1) in our paper is model (2) in Chang and Guh’s paper ([6], p. 1032).
indicates a lack of understanding of the definition and purpose of an infinitesimal. Note that the purpose of (1) is to detect nonzero slack values and the size of \( \delta \) does not affect this. Note also that the solution provided by Chang and Gubh ([6], p. 1033) is not an optimal solution, since the objective function value in the dual to (1) is not equal to zero for efficient DMUs. In fact, all four DMUs \( (P_1, P_2, P_3, \text{and } P_4) \) are efficient under (1) no matter what value \( \delta \) takes on. Therefore, their statement about Charnes et al. [4] is wrong and their calculations are also erroneous. Moreover, the infinitesimal is used to implement the two-stage DEA process. Therefore, their comments on Charnes et al. [4], Banker and Maindiratta [8] and Ali and Seiford [9] are all incorrect. Ali and Seiford [10] provide a detailed discussion of the treatment of the infinitesimal in DEA, and show that arbitrarily choosing a value for the infinitesimal \( \delta \) and solving the obtained linear program may result in a considerable loss of accuracy and misclassification of efficient/inefficient units. The implementation of the infinitesimal \( \delta \) is dependent upon the range of inputs and outputs, and therefore, \( \delta \) is not correctly implemented by Chang and Gubh [6] in their discussion of the models from [4,8,9].

Further, Chang and Gubh [6] misunderstand the results in Sueyoshi and Chang ([5], p. 206) which are established for the additive DEA model [model (3) with original inputs and outputs]. When applied to multiplicative measurement, i.e., (2), Sueyoshi and Chang's [5] result should be read as Theorem 2.

Finally, Chang and Gubh [6] provide a so-called distance efficiency measure under the assumption that all DMUs produce the same level of a single output, through the following CCR model\(^\dagger\)

\[
\begin{align*}
\max \quad & \sum_{i=1}^{n} u_i y_i^e \\
\text{s.t.} \quad & \sum_{i=1}^{n} u_i y_i^e - \sum_{i=1}^{n} v_i x_i^e \leq 0, \\
& \sum_{i=1}^{n} v_i x_i^e = 1, \\
& u_i, v_i \geq 0, \quad \forall r, i, \text{ and } j.
\end{align*}
\]

The only difference between the basic CCR model and (4) is that (4) uses logarithms of inputs and outputs. The dual to (4) can be written as

\[
\begin{align*}
\min \quad & \delta \\
\text{s.t.} \quad & \sum_{r=1}^{s} \lambda_r y_r^e - s^e = \hat{y}_e, \quad r = 1, \ldots, s, \\
& \sum_{r=1}^{s} \lambda^s_i x_i^e + s_i^e = \delta \hat{x}_e, \quad i = 1, \ldots, m, \\
& \lambda_r s_r^e, s_i^e \geq 0, \quad \forall j, r, i.
\end{align*}
\]

However, we show that under the case of a constant single output, model (5) implies a convexity constraint of \( \sum_{j=1}^{m} \lambda_j = 1 \). As a result, the new model is classification invariant and the resulting frontier is units invariant. Thus, we have

**Theorem 3.** In the case of a single constant output, model (5) is equivalent to the following model

\[
\begin{align*}
\min \quad & \delta \\
\text{s.t.} \quad & \sum_{j=1}^{m} \lambda_j x_j^e + s_i^e = \delta x_e, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{m} \lambda_j = 1, \\
& \lambda_j s_i^e \geq 0, \quad \forall j, i.
\end{align*}
\]

*Proof:* In the constant single output case, the first constraint of (5) reduces to \( \sum_{j=1}^{m} \lambda_j y \geq y \) and (5) becomes

\[
\begin{align*}
\min \quad & \delta \\
\text{s.t.} \quad & \sum_{j=1}^{m} \lambda_j x_j^e + s_i^e = \delta x_e, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{m} \lambda_j = 1, \\
& \lambda_j s_i^e \geq 0, \quad \forall j, i.
\end{align*}
\]

\(^\dagger\) Model (4) here is model (8) in Chang and Gubh's paper ([6], p. 1034).
Suppose $\lambda^*_j$, $\hat{\theta}^*$ are optimal solutions to (5') such that $\Sigma_{j=1}^n \lambda^*_j > 1$.

We define

$$
\lambda_j = \frac{\lambda_j^*}{\Sigma_{j=1}^n \lambda_j^*}, \quad \hat{\theta} = \frac{\hat{\theta}^*}{\Sigma_{j=1}^n \lambda_j^*}
$$

and divide all terms in the $m$ constraints of (5') by $\Sigma_{j=1}^n \lambda_j^*$. Then we obtain

$$
\Sigma_{j=1}^n \lambda_j x_{ij} + \frac{s_{ir}}{\Sigma_{j=1}^n \lambda_j^*} = \hat{\theta}^* x_{ir} - h_i
$$

Since $\hat{\theta}^* < \hat{\theta}^*$, the optimality of $\hat{\theta}^*$ is violated. Thus, there is always an optimal solution in which $\Sigma_{j=1}^n \lambda_j = 1$. Therefore (5') is equivalent to (6). ■

**Theorem 4. In the case of a constant single output, model (5) is classification invariant.**

**Proof:** By Theorem 3, (5) is equivalent to (6). Note that for an efficient DMU, $\hat{\theta}^* = 1$. Therefore, by Theorem 1, all efficient DMUs under (6) will remain efficient after units of inputs and/or outputs are changed, since $\Sigma_{j=1}^n \lambda_j = 1$; i.e., a change of units does not alter the efficient frontier and the classification of DMUs as inefficient or efficient is invariant to a change of units. ■

**Remark.** In Theorem 4, classification invariance means units classification invariance with respect to the original inputs and outputs, and translation classification invariance with respect to the logarithmic inputs and outputs.

Chang and Guh's [6] **distance efficiency measure** calculates the distance from a DMU to a related efficient facet. Therefore, their method requires that the efficient facet is invariant to unit changes of inputs and output in order for the related distance efficiency measure to be units invariant. Theorems 3 and 4 demonstrate that the constant output assumption implies a constraint of $\Sigma_{j=1}^n \lambda_j = 1$, and, as a result, implies units invariance of the efficient facet. Thus, they do not address the units variance problem of (3), but simply imply and implicitly require units invariance of the efficient facet. The distance efficiency measure itself does not have the property of units invariance. It is their constant output assumption, rather than the distance efficiency measure itself, that yields the property of units invariance. That the efficient frontier is units invariant is the necessary condition for the validity of the distance efficiency measure, i.e., the (implied) condition of $\Sigma_{j=1}^n \lambda_j = 1$ resulting from the constant output assumption ensures the units invariance. Therefore, their distance efficiency measure is at most a variation of existing loglinear efficiency measures [3,4], but limited to the constant output case. Without the (implied) constraint of $\Sigma_{j=1}^n \lambda_j = 1$, their distance efficiency measure is of no value.

The statement ([6], p. 1032) "Unlike the Banker and Maindiratta [8], and Ali and Seiford [9] approaches, the distance efficiency measure does not need the addition of a constant term in the loglinear frontier, so it will rate fewer DMUs as being efficient" is erroneous for the following two reasons. First, this particular constant term is a dual variable for $\Sigma_{j=1}^n \lambda_j = 1$. Since they implied this convexity constraint, therefore, they also implied the constant term. Second, since (5) is equivalent to (6) for the constant output case, the frontier determined by (5) is the same as obtained from (1) or (3). Therefore, their distance efficiency measure does not rate fewer DMUs as being efficient. Table 1 provides the results of 12 DMUs in Chang and Guh ([6], p. 1035) when we employ (3). [The same results are obtained if we use (2).] It can be seen that no output slack is detected which in turn, implies that $\Sigma_{j=1}^n \lambda_j = 1$ at optimality.

The same inefficient DMUs are obtained, respectively, from their distance efficiency measure and the original variant loglinear frontier approach [4]. It can be seen that the variant multiplicative model (3) is, in fact, units invariant under the constant output case.

Furthermore, Chang and Guh [6] use $P_1$, $P_2$, $P_3$, $P_4$, and $P_5'$ in their distance efficiency measure, whereas they use $P_1$, $P_2$, $P_3$, $P_4$, $P_5$, and $P_6$ for the Banker and Maindiratta [8] and Ali and Seiford [9] approaches.† The different results obtained are caused by their use of two different set of DMUs. In particular, their footnote (p. 1036) is not valid. In fact, as seen from Table 1, Ali and Seiford's [10] approach only terms $P_1$ and $P_2$ efficient when we correctly consider the same data set of Chang and Guh [6], i.e., when we use $P_1$, $P_2$, $P_3$, $P_4$, and $P_5'$.

† The output in set of $P_6$, $P_2$, $P_3$, $P_4'$, and $P_5'$ is adjusted to a same amount of $10^1$ across all DMUs, whereas the amount of output in set $P_1$, $P_2$, $P_3$, $P_4$, and $P_5$ varies.
The above discussion demonstrates that Chang and Guh’s [6] distance efficiency measure has nothing to do with the invariance in ranking of DMUs, as their measure is limited to the case of a constant output, and the case of a linearly homogenous production technology. The core of units invariance in loglinear frontier DEA measures lies in the convexity constraint. Chang and Guh [6] fail to recognize that model (1) is units invariant, and (3) will be units invariant and their model (5) will be classification invariant in the case of constant output. There is nothing wrong with the existing loglinear DEA measures; Chang and Guh [6] just misunderstood and misinterpreted these previous works [3–5,8,9].

In addition, Chang and Guh [6] suggest using their remedy [11]† to overcome the non-zero slack problem. However, it is very difficult to determine all positive multipliers, and frequently a full dimensional efficient facet does not exist. In fact, models (2), (3) or the infinitesimal in DEA solves the non-zero slack problem very well (see Ali and Seiford [10]). More importantly, if a full dimensional efficient facet does not exist, the efficiency score obtained from the shortest vertical distance between

| Table 1. Efficiency results for the variant multiplicative model [model (3)] |
|------------------------|-----------------|-----------------|-----------------|-----------------|
|                      | \( u^* \) | \( v_1^* \) | \( v_2^* \) | \( s_1^* \) | \( s_2^* \) |
| \( P_1 \) = 10^4  | 5   | 1   | 1   | 0   | 0   |
| \( P_2 \) = 10^4  | 5   | 1   | 1   | 0   | 0   |
| \( P_3 \) = 10^3  | 5   | 1   | 1   | 0   | 0   |
| \( P_4 \) = 10^4  | 5   | 1   | 1   | 0   | 2   |
| \( P_5 \) = 10^4  | 5   | 1   | 1   | 0   | 2   |
| \( P_6 \) = 10^3  | 5   | 1   | 1   | 0   | 3   |
| \( P_7 \) = 10^1  | 5   | 1   | 1   | 0   | 1   |
| \( P_8 \) = 10^2  | 6   | 1   | 1   | 0   | 0   |
| \( P_9 \) = 10^3  | 6   | 1   | 1   | 0   | 0   |
| \( P_{10} \) = 10^4 | 6   | 1   | 1   | 0   | 0   |
| \( P_{11} \) = 10^3 | 6   | 1   | 1   | 0   | 0   |
| \( P_{12} \) = 10^4 | 6   | 1   | 1   | 0   | 2   |
| \( P_{13} \) = 10^3 | 6   | 1   | 1   | 0   | 4   |
| \( P_{14} \) = 10^4 | 6   | 1   | 1   | 0   | 4   |
| \( P_{15} \) = 10^3 | 6   | 1   | 1   | 0   | 3   |
| \( P_{16} \) = 10^4 | 6   | 1   | 1   | 0   | 1   |

Note:
(1) \( u^* \), \( v_1^* \) and \( v_2^* \) are optimal dual values in the dual to (3).
(2) The notation for each point (DMU) is the same as in Chang and Guh [6].

† Conceptual errors in Chang and Guh [11] were identified by Zhu and Shen [12], and Chang and Guh’s [11] remedy is equivalent to the constrained facet analysis [13].
each DMU and the referent facet, in contrast to the claim by Chang and Guh [6], is meaningless.

4. CONCLUSION

The current paper shows that with the assumption of a single constant output, Chang and Guh [6] developed their distance efficiency measure under an implicit convexity constraint of $\sum_{j=1}^{n} \lambda_j = 1$. Since their convexity constraint is the basis for maintaining units invariance in loglinear efficiency measures, the constant output assumption actually implies units invariance of the resulting loglinear efficient frontier. Loglinear efficient frontiers obtained from the distance efficiency measure [6] and the invariant multiplicative model [4] are identical under Chang and Guh’s [6] assumption of constant output. Consequently, their distance efficiency measure is a direct result and special case of the invariant multiplicative model.

Chang and Guh [6] fail to recognize that $\sum_{j=1}^{n} \lambda_j = 1$ serves as the basis for units invariance, and therefore make further erroneous claims and statements. It is not their distance efficiency measure, but their assumption of a single constant output and the associated implied constraint of $\sum_{j=1}^{n} \lambda_j = 1$ that preserves units invariance. As a consequence, their distance efficiency measure is of no value.

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