Using Operational and Stock Analytics to Measure Airline Performance: A Network DEA Approach

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ABSTRACT

The majority of extant studies that focus on performance and efficiency benchmarking of firms utilize only operational measures while neglecting to integrate stock market indicators in their methodological frameworks. Such an approach may lead to erroneous or biased conclusions given that operational and stock measures serve to capture different dimensions and attributes of an overall firm’s activities, health, and prospects. Thus, we build and implement a two-stage network data envelopment analysis process that utilizes both operational and stock market indicators in order to evaluate the performance of nine major international airline companies from 2006 until 2016. In our analysis, we show that there is heterogeneity in the performance of all airlines across time. Most notably, during the 2013–2014 European debt crisis and U.S. debt-ceiling crisis, we find that stock market-based performance scores declined significantly for all our sampled companies. We also show that while low cost carriers generally maintain higher operational-based performance scores than their full service counterparts, full service carriers earn higher performance scores based on stock market indicators. This finding lends support to our approach and our general premise that argues that performance evaluation methods can yield more comprehensive conclusions if both operational and stock market indicators are utilized. [Submitted: February 23, 2017. Revised: September 30, 2018. Accepted: December 3, 2018.]

Subject Areas: Airlines, Data Envelopment Analysis (DEA), Stock Market Indicators, and Two-stage Network.

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INTRODUCTION

The airline industry has experienced unparalleled changes in the last few decades. Liberalization and deregulatory initiatives have attracted many new firms into the industry and have facilitated the growing number of mergers and diverse collaborative schemes among firms (Brueckner, Lee, & Singer, 2013). As a result, the level of competition within the industry has grown immensely and prompted a growing area of research into how to measure efficiency and benchmark airline performance (Mallikarjun, 2015).

Measuring performance is a fundamentally important task from a regulatory standpoint, which is concerned with the social impact of airline operations on issues such as the environment, health, and safety (Lee, Yeo, & Thai, 2014). Performance benchmarking is also critical from a managerial and shareholder perspective, especially because upper management compensation schemes and CEO tenures are tied to operational and financial performance and efficiency (Davila & Venkatachalam, 2004; Mellat-Parast, Golmohammadi, McFadden, & Miller, 2015). Finally, in an efficient capital market, investors are constantly scanning the marketplace and vying to find the most efficient, sustainable, and healthy companies to invest in with the hopes that they will enjoy superior future capital gains (Eccles, Ioannou, & Serafeim, 2014).

There is already a voluminous body of empirical research that posits various methods and conceptual frameworks for measuring and capturing airline efficiency and performance. Research in the late 1970s and early 1980s established a conceptual framework that contains three elements pertaining to transit operations; specifically, resource inputs (e.g., number of employees, labor, fuel, etc.), service outputs (e.g., vehicle hour, vehicle mile, etc.), and service consumption (operating revenue, passenger-mile, etc.) (Fielding, Glauthier, & Lave, 1978; Fielding & Anderson, 1984).

These three elements served as a motivation for input–output methodologies designed to benchmark performance and efficiency. In the 1990s, several cost models emerged as a method for gauging performance (Windle, 1991; Oum & Yu, 1998; Liu & Lynk, 1999) as well as factor productivity methods (Bauer, 1990; Oum & Yu, 1995). Meanwhile, contemporary research has implemented various more advanced parametric as well as nonparametric models for performance benchmarking such as stochastic frontier analysis (Good, Nadiri, Roller, & Sickles, 1993; Baltagi, Griffin, & Rich, 1995) and data envelopment analysis (DEA; Barros & Peypoch, 2009; Ayanso & Mokaya, 2013; Barros & Couto, 2013; Talluri, DeCampos, & Hult, 2013).

Despite the growing sophistication in modeling over the years, currently, most studies seem to focus exclusively on operational indicators while neglecting to integrate measures pertaining to firms’ financial market performance, which can be extracted from stock market indicators. For example, a firm’s net income, capital gains, and market capitalization, to name only a few, give investors great insights as to the health and stability of the firm. Analysts and traders also utilize financial market data for individual companies in order to gain insights into the future prospects of the firm and to gauge the level of investor sentiment and attitude toward a firm.
Neglecting to include firm-level financial market measures sweeps important pieces of information under the rug and can ultimately lead to misleading and biased conclusions. From a managerial point of view, financial market measures can capture investor attitudes and sentiment toward their firm’s prospects and give upper management important feedback into the pulse of the market. For example, in the event shareholders and investors become pessimistic, as can be inferred from stock market indicators, this can seriously impede management’s ability to raise needed capital to fund their operations and projects. In a competitive industry such as the airlines industry, managers need to be acutely aware of not only their operational efficiency but also the sentiment, attitudes, and expectations of their shareholders and the stock market at large. As we discuss more rigorously later on in our article, these types of important financial market variables are included in our analysis.

By integrating stock market indicators into our analysis, we also align ourselves with literature in financial economics, which finds that investors trade based on market sentiment and fundamental factors (Koutmos, 2012; Chau, Deesomsak, & Koutmos, 2016).

In light of the aforementioned, this article thus makes a conceptual contribution to literature by integrating financial market indicators along with operational indicators into a two-stage network DEA model to study the performance of eight large and international airline companies.

Over the years, DEA has been proven an effective tool for performance evaluation and benchmarking. It allows us to integrate multiple performance measures into a single model and provide a performance index. The use of DEA is not restricted to estimating production frontiers. DEA is a technique for identifying best-practice frontiers (Cook, Tone, & Zhu, 2014). For example, Chiou and Chen (2006) evaluate airline performance based on air routes. They employ a DEA approach to evaluate the performance of domestic air routes from the perspectives of cost efficiency, cost effectiveness, and service effectiveness. Scheraga (2004) utilizes a sample of 38 airlines from North America, Europe, Asia, and the Middle East to investigate whether relative operational performance implied superior financial mobility (as defined by Donaldson, 1969). Other studies that successfully use DEA include, but are not limited to, Siregar and Norsworthy (2001), Barbot, Costa, and Sochirca (2008), Bhadra (2009), Wang, Lu, and Tsai (2011), and Tavassoli, Faramarzi, and Saen (2014), among others.

This article also demonstrates a second order cone programming (SOCP) technique that can be used to solve nonlinear network DEA models when the overall performance is defined as a weighted average of the two stages’ performance scores. Our approach improves upon the work of Chen and Zhu (2017) by addressing the symmetric issue in SOCP modeling. In addition, our two-stage network DEA framework models the internal structures of airlines (or decision-making units [DMUs]) and inner relations of performance metrics are considered. For example, Lu, Wang, Hung, and Lu (2012) explore the relationship between operating performance and corporate governance in 30 airline companies operating in the United States. In their study, the DMUs consist of two stages of production performance and marketing performance.
There are two types of approaches in modeling DMUs with two-stage network structures. One is called the additive approach, where the overall performance is defined as a weight average of the two-stage performance scores (Chen, Cook, Li, & Zhu, 2009). The other is called the multiplicative approach, where the overall performance is aggregated or decomposed as a product of the two-stage performance scores (Kao & Hwang, 2008; Liang, Cook, & Zhu, 2008). The current study is based upon the additive network DEA approach.

The remainder of this article is structured as follows: the second section provides a survey of the literature pertaining to the airline industry and DEA. The third section builds the two-stage network DEA methodology that is implemented. The fourth section describes the input–output variables and intermediate measures in our network DEA, which consist of operational and financial market variables. It then presents the major findings that we derive from our two-stage network DEA. Finally, the fifth section provides concluding remarks.

BACKGROUND LITERATURE ON THE AIRLINE INDUSTRY AND DEA

The importance of financial market indicators for decision-making is highlighted by many studies who take various approaches in examining the airline industry. Jenatabadi and Ismail (2014) use structural equation modeling with latent variables for estimating the financial and nonfinancial performance in airline companies. Their model includes independent, mediator, and dependent latent variables and comprises 214 airline companies. In a separate article, Ismail and Jenatabadi (2014) use firm age as a moderator and aim to investigate the moderating influence of firm age on the nature of the relationship between economic conditions and internal operations and ultimately its effect on the performance of the airline industry. They select 30 airline companies from the Asia-Pacific region and collect relevant data from 2006 to 2011. Their first step is to investigate the relationship among the economic situation, internal operation, and airline performance. They then establish the moderating effect of firm age on the relationship between the economic conditions and internal operations. Finally, they investigate the relationships between the three variables, which are investigated again by taking into account the moderating effect of the variable (firm age).

Riley, Pearson, and Trompeter (2003) examine the relative value relevance to investors of nonfinancial performance variables, traditional accounting variables (earnings and changes in abnormal earnings), and other financial statement information in the airline industry. Measures that do not strictly derive from financial variables, such as employee turnover and other variations in process services, are also examined by Tsikriktsis and Heineke (2004), who find that the relation between process variation and customer dissatisfaction is conditional upon firms’ average performance.

Feng and Wang (2000) construct a performance evaluation process for airlines with financial ratios taken into consideration. In their article, they use the grey relation analysis to select the representative indicators and the technique for order preference by similarity to ideal solution method for the ranking of airlines.
to overcome the problems of small sample size and the unknown distribution of the samples.

Rose (1990) analyzes the relationship between financial conditions and safety performance. This study uses data on 35 large scheduled passenger airlines from 1957 to 1986 in order to estimate the effect of profitability and other aspects of financial health on accident and incident rates.

Tsikriktsis (2007) studies the impact of operational performance on profitability in the context of the U.S. domestic airline industry. Their analysis demonstrates two main points. First, the relationship between operational performance and profitability is contingent on a company’s operating model. Second, focused airlines outperform the rest of the industry in terms of profitability.

In addition, Goll, Brown Johnson, and Rasheed (2008) focus on top management demographic characteristics, business strategy, and firm performance in the major U.S. airlines. They examine the relationships between management characteristics and business strategies. Finally, using a vector auto regression (VAR)—a model that has been widely implemented in financial economics literature—Guzhva and Pagiavlas (2004) explore how the September 11 terror attack affected the performance of the airline industry when controlling for aggregate economic conditions.

Some researchers use nonfinancial data to analyze airline performance because of different accounting or taxation rules. Liedtka (2002) extends the literature on nonfinancial performance measures (NFPMs) by assessing the information content of a broader set of NFPMs and whether NFPMs provide information not provided by financial performance measures (FPMs) from all previously identified FPM categories, rather than just earnings and book value.

Lapré and Scudder (2004) consider airline companies’ performance from an improvement aspect. Using a database on the 10 largest U.S. airlines for a period of 11 years, they test and validate some of the models presented in the operations literature. The 10 major airlines are separated into two groups for analysis: geographic specialists and geographic generalists. Other studies include revenue management decision making (Mukhopadhyay, Samaddar, & Colville, 2007) and option contracts with overbooking (Hellermann, Huchzermeier, & Spinler, 2013).

We now turn our attention to literature that implements DEA as a technique to measure performance of airlines. Bhadra (2009) uses DEA to examine intertemporal and peer group airline performance. This article indicates that airline performance is converging over time for the United States for the period 1985–2006. Wang et al. (2011) explore links between the operating performance of 30 airlines in the United States and corporate governance. Initially, DEA is used to assess the relative performance of airlines and to investigate the contribution of inputs and outputs that affect technical performance. Schefczyk (1993) utilizes DEA as a technique to analyze and compare operational performance characteristics of airlines, drawing on data from 15 airline companies. The study concludes with an analysis of strategic factors of high profitability and performance in the airline industry.

Prior research has attempted to utilize both DEA along with other measures (Total Factor Productivity [TFP], regression modeling, and financial mobility, to name only a few) in order to analyze the airline performance. Barros and Peypoch
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(2009) use both regression and DEA to evaluate the operational performance of a sample of Association of European Airlines from 2000 to 2005, combining operational along with financial variables. In the first stage, a DEA model ranks the airlines by their overall performance. In the second stage, a bootstrapped truncated regression is used to evaluate the drivers of performance. Barbot et al. (2008) analyze airline companies’ performance and productivity using two different methodologies: DEA and TFP, and they additionally investigate which factors account for differences in efficiency. Chiu and Chen (2006) evaluate airline performance based on air routes. They employ a DEA approach to evaluate the performance of domestic air routes from the perspectives of cost efficiency, cost effectiveness, and service effectiveness, and then examine a total of 15 routes operated by a Taiwanese domestic airline. Barbot et al. (2008) critique some of the shortcomings in Chiu and Chen (2006) and propose various remedies while Scheraga (2004) utilizes a sample of 38 airlines from North America, Europe, Asia, and the Middle East to investigate whether relative operational performance implies superior financial mobility (as is defined by Donaldson, 1969). DEA was utilized to derive performance scores for individual airlines. Their results indicate that the traditional framework developed in the literature still provides reasonable explanatory power for realized relative operational performance. However, relative operational performance did not inherently imply superior financial mobility. Siregar and Norsworthy (2001) investigate the effects of technology and their equity market impacts for major commercial airlines and the distributions of returns before, during, and after deregulation to see whether deregulation has increased or decreased risk. They also examine the relationships between stock returns and prices using DEA and TFP as measures of performance.

In recent years, two-stage network structures have been an important area of development in DEA. Under a network DEA framework, in addition to the inputs and outputs, a set of intermediate measures exist in-between the two stages. For example, Seiford and Zhu (1999) utilize a two-stage network structure to measure the profitability and marketability of U.S. commercial banks. Zhu (2000) applies the same two-stage network structure to the Fortune Global 500 companies. Kao and Hwang (2008) study a two-stage DEA to examine the operation of the nonlife insurance industry in Taiwan. Toloo, Emrouznejad, and Moreno (2017) extend a relational linear DEA model for dealing with measuring the performance score of two-stage processes with shared inputs in an additive manner. Limited studies, however, use a two-stage network DEA model to evaluate airline performance. Zhu (2011) uses a two-stage DEA process to measure airline operations performance. In this research, resources (fuel, salaries, and other factors) are used to maintain the fleet size and load factor in the first stage. In the second stage, the fleet size and load factors generate revenue. Lu et al. (2012) explore the relationship between operating performance and corporate governance in 30 airline companies operating in the United States. Their study applies a two-stage network DEA to evaluate the production performance and marketing performance of the airlines, and implements a truncated regression to explore whether the characteristics of corporate governance affect airline performance. Tavassoli et al. (2014) propose a slacks-based network DEA approach.
to measure both technical performance and service effectiveness of airlines. Their model represents both the nonstorable feature of transportation service and production technologies in a unified framework in the presence of shared input.

A TWO-STAGE NETWORK DEA APPROACH

We consider a general two-stage network structure shown in Figure 1. Each DMU \( j \) \((j = 1, 2, \ldots, n)\) has \( m \) inputs \( x_{ij} \) \((i = 1, 2, \ldots, m)\) to the first stage and \( P \) outputs \( y_{1p} \) \((p = 1, 2, \ldots, P)\) that leave the system. In addition to these \( P \) outputs, stage 1 has \( D \) outputs \( z_{dj} \) \((d = 1, 2, \ldots, D)\) called intermediate measures that become inputs to the second stage. The second stage has its own inputs \( x_{2h} \) \((h = 1, 2, \ldots, H)\). The outputs from the second stage are \( y_{r} \) \((r = 1, 2, \ldots, s)\).

We consider the variable returns to scale (VRS) case, because the constant returns to scale (CRS) can be regarded as a special case of VRS by removing the free variables in the VRS models. Please note that VRS and CRS here are used to distinguish the two different types of DEA best-practice frontiers (not production frontiers) and we do not study the issue of returns-to-scale. In particular, the DEA score is treated as a composite index, not an efficiency score from a production technology. This is due to the fact that DEA in the current study is used as a benchmarking (data analytics) tool.

Our performance measures include some ratio measures that are standard metrics used by the industry. The developed DEA models allow us to use the ratio measure in identifying the best practices, rather than a production function (see, e.g., Cook et al., 2014). As pointed out by Emrouznejad and Amin (2009) and Olesen, Petersen, and Podinovski (2015), ratio data do not satisfy the “convexity” condition typically required in estimating production technology. However, as pointed by Olesen et al. (2015), it is acceptable to include ratio data in the multiplier DEA models, whereas the use of ratio data can be only problematic if an application is concerned with production specifications.

Note that the standard DEA models can be presented in either envelopment or multiplier form. Obviously, we can use ratio data to define a new composite
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measure such as \( \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \). While in the DEA literature, the maximum value of \( \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \) is often called “efficiency.” In fact, the standard DEA model generates a composite index or measure and “efficiency” does not necessarily mean “production efficiency” in many DEA applications. “Efficiency” is a standard terminology in DEA to represent the optimal value to the DEA model. It can be seen that the above ratio can include only inputs or only outputs. Therefore, the standard DEA model is not necessarily a model of “production” or “technology.” In many applications of DEA, including the current application in this article, the latter is not considered essential or even required. Namely, herein in our analysis and like many other DEA applications (e.g., Paradi, Asmild, & Simak, 2004; Dula, 2009; and Shwartz, Burgess, & Zhu, 2016), we do not require the assumption that the observed DMUs are elements of some production technology.

We further note that it is in the dual to the multiplier DEA model (namely, the envelopment model), researchers discover the “convexity” and established a link between DEA and production function. Therefore, economic meaning is justified if DEA is used as a tool for estimating production functions. The DEA multiplier model the current article is based upon implies the “convexity” because of the duality to the envelopment model. However, this does not necessary mean that “convexity” is required in the multiplier model. The multiplier model optimizes ratios of weighted performance measures as defined below for a specific DMU0 under evaluation:

\[
e_0^1 = \frac{\sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0} + u^1}{\sum_{i=1}^{m} v_i x_{i0}} \quad \text{and} \quad e_0^2 = \frac{\sum_{r=1}^{s} u_r y_{r0} + u^2}{\sum_{d=1}^{D} \eta_d z_{d0} + \sum_{h=1}^{H} Q_h x_{h0}^2},
\]

where \( v_i, \eta_d, \lambda_p, u_r, \) and \( Q_h \) are weights that are assumed to be positive in the current study, by incorporating the small non-Archimedean \( \varepsilon \) in the DEA models. \( u^1 \) and \( u^2 \) are free variables associated with the VRS assumption. If we exclude \( u^1 \) and \( u^2 \), then we have a CRS model.

The following development is based upon the DEA multiplier model. Therefore, convexity is not assumed or required, and ratio data are allowed in our models. While under the standard DEA that is linear, the multiplier model is equivalent (or a dual) to the envelopment model, the multiplier and envelopment models are not equivalent and are not duals under network DEA. This is due to the fact that network DEA models are not linear and deal with intermediate measures (see, e.g., Chen, Cook, Kao, and Zhu (2013) where the authors show that the envelopment and multiplier network DEA models generate different results).
The VRS version of an additive performance with respect to Figure 1 can now be presented in the following way:

\[
\max \quad \alpha_1 \frac{\sum_{d=1}^D \eta_d z_{d0} + \sum_{p=1}^P \lambda_p y_{p0}^1 + u^1}{\sum_{i=1}^m v_i x_{i0}} \\
+ \alpha_2 \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{d=1}^D \eta_d z_{d0} + \sum_{h=1}^H Q_h x_{h0}^2 + u^2} \\
\text{s.t.} \quad \frac{\sum_{d=1}^D \eta_d z_{dj} + \sum_{p=1}^P \lambda_p y_{pj}^1 + u^1}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad \forall j \\
\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{d=1}^D \eta_d z_{dj} + \sum_{h=1}^H Q_h x_{hj}^2 + u^2} \leq 1, \quad \forall j \\
\alpha_1 + \alpha_2 = 1 \\
\eta_d, u_r, v_i, \lambda_p, Q_h \geq \varepsilon, \quad \forall d, r, i, p, h \\
u^1, u^2 \text{ free in sign,}
\]

where \( \alpha_1 \) and \( \alpha_2 \) are weights.

In the existing literature, there are two ways to solve model (1), which is highly nonlinear. One is to choose a special set of weights (\( \alpha_1 \) and \( \alpha_2 \)) to convert the objective function in problem (1) into a single linear fractional form (see, e.g., Cook, Zhu, Bi, & Yang, 2010). These special weights are actually variables related to the input sizes of the two stages. As shown in Despotis, Koronakos, and Sotiros (2016), such weights yield biased performance scores toward the second stage.

Guo and Zhu (2017) demonstrate a detailed approach to solving models similar to (1) by transforming problem (1) into a sequence of linear programming problems given different predetermined weights. Chen and Zhu (2017), on the other hand, show that for two-stage network DEA models, when the overall performance is expressed as a product of the two stages’ performance scores, the network DEA model can be solved using a SOCP technique. Note that SOCP can be solved by nonheuristic algorithms such as an interior point method. In the discipline of convex optimization, SOCP has already been a mature technology (Boyd & Vandenberghe, 2004).

In our study, using a SOCP technique, we propose an approach to solving model (1) where the overall performance is expressed as a weighted average of the two stages’ performance scores.
Model (1) is equivalent to the following:

\[
\begin{align*}
\min & \quad -\alpha_1 \frac{\sum_{d=1}^{D} \eta_d z_d 0 + \sum_{p=1}^{P} \lambda_p y^1_p 0 + u^1}{\sum_{i=1}^{m} v_i x_i 0} \\
& \quad - \alpha_2 \frac{\sum_{d=1}^{D} \eta_d z_d 0 + \sum_{h=1}^{H} Q_h x^2_h 0 + u^2}{\sum_{r=1}^{s} u_r y_r 0} \\
\text{s.t.} & \quad \frac{\sum_{d=1}^{D} \eta_d z_d j + \sum_{p=1}^{P} \lambda_p y^1_p j + u^1}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \forall j \\
& \quad \frac{\sum_{d=1}^{D} \eta_d z_d j + \sum_{h=1}^{H} Q_h x^2_h j + u^2}{\sum_{r=1}^{s} u_r y_{rj}} \leq 1, \quad \forall j \\
& \quad \alpha_1 + \alpha_2 = 1 \\
& \quad \eta_d, u_r, v_i, \lambda_p, Q_h \geq \varepsilon, \quad \forall d, r, i, p, h \\
& \quad u^1, u^2 \text{ free in sign}
\end{align*}
\]

Then, by an epigraph transformation, we obtain:

\[
\begin{align*}
\min & \quad -\theta_1 - \theta_2 \\
\text{s.t.} & \quad -\alpha_1 \frac{\sum_{d=1}^{D} \eta_d z_d 0 + \sum_{p=1}^{P} \lambda_p y^1_p 0 + u^1}{\sum_{i=1}^{m} v_i x_i 0} \leq -\theta_1 \\
& \quad -\alpha_2 \frac{\sum_{d=1}^{D} \eta_d z_d 0 + \sum_{h=1}^{H} Q_h x^2_h 0 + u^2}{\sum_{r=1}^{s} u_r y_r 0} \leq -\theta_2 \\
& \quad \frac{\sum_{d=1}^{D} \eta_d z_d j + \sum_{p=1}^{P} \lambda_p y^1_p j + u^1}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \forall j \\
& \quad \frac{\sum_{d=1}^{D} \eta_d z_d j + \sum_{h=1}^{H} Q_h x^2_h j + u^2}{\sum_{r=1}^{s} u_r y_{rj}} \leq 1, \quad \forall j \\
& \quad \alpha_1 + \alpha_2 = 1 \\
& \quad \eta_d, u_r, v_i, \lambda_p, Q_h, \theta_1, \theta_2 \geq \varepsilon, \quad \forall d, r, i, p, h \\
& \quad u^1, u^2 \text{ free in sign}
\end{align*}
\]
Finally, we can have the following SOCP problem (see Appendix for details on how this model is derived):

\[
\begin{align*}
\min \quad & -\theta_1 - \theta_2 \\
\text{s.t.} \quad & \frac{1}{2} \left[ \frac{1}{\theta_1^2} \left( \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0}^1 + u^1 \right) \\
& - \alpha_1^2 \left( \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0}^1 + u^1 \right) \right]_2 \\
& \leq \frac{1}{2} \left[ \frac{1}{\theta_2^2} \sum_{r=1}^{s} u_r y_{r0} - \alpha_2^2 \sum_{r=1}^{s} u_r y_{r0} \right]_2 \\
& \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{h=1}^{H} Q_h x_{h0}^2 + u^2 \\
& \leq \frac{1}{2} \left[ \frac{1}{\theta_2^2} \sum_{r=1}^{s} u_r y_{r0} + \alpha_2^2 \sum_{r=1}^{s} u_r y_{r0} \right]_2 \\
& \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0}^1 + u^1 \leq 1, \quad \forall j \\
& \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad \forall j \\
& \sum_{r=1}^{s} u_r y_{rj} \leq 1, \quad \forall j \\
& \alpha_1 + \alpha_2 = 1, \\
& \eta_d, u_r, v_i, \lambda_p, Q_h, \theta_1, \theta_2 \geq \varepsilon, \quad \forall d, r, i, p, h \\
& u^1, u^2 \text{ free in sign}
\end{align*}
\]

Thus, by solving model (4), we can determine stage efficiencies for a specific DMU under evaluation. We will demonstrate this in our empirical application in the following section.

**EMPIRICAL IMPLEMENTATION AND MAJOR FINDINGS**

In a contribution to existing literature, we show that integrating operational along with stock market indicators, which represent the financial prospects of a firm, into our two-stage network DEA framework can provide new insights into performance rankings and performance benchmarking for airline companies. As mentioned, neglecting to include stock market measures sweeps important firm-level information under the rug and can lead to erroneous or biased conclusions.

To empirically implement model (4), we utilize five key operational indicators and six financial market indicators. Our operational indicators are as follows: fuel cost per available seat miles (Fuel), number of employees (Employees),
operation costs per available seat miles (excluding fuel costs; ) (Operation Cost), sales revenue (Sales), and
revenue passenger kilometers (RPKs).i

**Fuel** is a unit of cost measurement that is derived by dividing fuel costs by available seat miles (ASM). ASM is a measure of airline companies’ passenger carrying capacity that is equivalent to the number of seats available to passengers multiplied by the number of miles (or kilometers) flown. Generally speaking, lower the Fuel is, lower the costs are for the airline company and, ceteris paribus, the higher the probability that the company will be profitable. Employees denotes the number of individuals that are employed by the airline. **Operation Cost** is another unit of cost measurement. It is computed by dividing operating costs by ASM. In general, lower the **Operation Cost** is, lower the operational costs are for the airline company and, ceteris paribus, the higher the probability that the company will be profitable. The reason why **Operation Costs** here is estimated excluding fuel costs, and the reason why this measure is important for airline companies, is because management is, among other methods, evaluated on company performance while isolating for macroeconomic factors that are beyond their direct control—such as oil price volatility, which is the result of a broad range of market forces. **Sales** captures income, which an airlines company generates for its services. Positive sales growth over time is a sign of rising market share and consumer demand. Finally, **RPKs** is a measure of traffic for an airline. The **RPKs** of an airline is the sum of the products obtained by multiplying the number of revenue passengers carried on each flight stage by the distance travelled. It can be regarded as airline “production.”

In terms of stock market and financial indicators, we use the following six indicators: market capitalization (**Market Cap**), the weighted average cost of capital (**WACC**), the short interest ratio (**Short Interest Ratio**), net income (**Net Income**), capital gains yield (**Capital Gains Yield**), and return on equity (**ROE**).

**Market Cap** is the total market value of a company’s outstanding shares and is computed by multiplying the number of outstanding shares by the market price for one of those shares. Market participants generally view this indicator as a measure for firm size whereby the larger **Market Cap** is, the larger the company is. Using **Market Cap** is a market indicator of size, which serves as an alternative to accounting measures of size such as total assets or sales figures. **WACC** reflects investors’ required rate of return on their investment.ii It is the rate of return that a company is expected to pay its security holders and is regarded as a useful proxy for the required rate of return in finance because it is determined by the market and not by a company’s management. **Short Interest Ratio** is a stock market indicator that reflects investor sentiment. It is computed by dividing short interest, or the quantity of shares sold short but not yet covered, by the average trading volume for a stock over a given period. When short interest rises, the **Short Interest Ratio**

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i The Bureau of Transportation Statistics (BTS) provides a comprehensive description some of the operational indicators we use in our article along with other commonly cited operations indicators. These can be accessed publicly on-line at this URL address: http://www.transtats.bts.gov/Glossary.asp?index=A.

ii Assuming that the company is financed with only debt and equity, the equation for **WACC** is as follows: 

$$WACC = \frac{D}{D+E} K_D + \frac{E}{D+E} K_E$$

whereby **D** and **E** represent total debt and total shareholder’s equity, respectively. **K_D** is the cost of debt while **K_E** is the cost of equity. When computing **WACC**, market values for debt and equity are conventionally used. We use market values in this article as well for each of the airline companies.
Figure 2: Two-stage network DEA structure of airline performance.

Zhang et al. (2023) discusses the operational and financial market indicators of airline performance. Net Income is the company’s total profit and is computed by subtracting costs of doing business (taxes, interest expenses, depreciation, employee salaries) from revenues. Capital Gains Yield is the percent change in a company’s stock price from one period to the next. This indicator excludes dividends paid on the stock by the company to its shareholders. Finally, the ROE is the amount of net income returned as a percentage of shareholder’s equity. This measure reflects profitability by showing profits that the company generated with money, which shareholders invested.

All operations and financial market indicators are quarterly—because they are extracted from quarterly disclosed statements and 10-Q filings with the Securities and Exchange Commission. Thus our sample frequency consists of quarterly observations for each of the aforementioned operational and financial market indicators for each of the eight respective airlines, which consists of Alaska Airlines, Air Canada, Delta, Hawaiian Airlines, Jet Blue, Southwest Airlines, Spirit Airlines, and United Continental Holdings.

With respect to the two-stage structure in Figure 1, our empirical study considers the operational and stock market performance in a unified way, as shown in Figure 2. $x_1$ represents Fuel, Employees, and Operation Cost. $y_1$ refers to Sales. $z_d$ refers to RPKs. $x_2^h$ corresponds to Short Interest Ratio, Market Cap, and WACC. $y_r$ refers to Net Income, Capital Gains Yield, and ROE.

Thus, the inputs in the first stage of the two-stage DEA network, as is illustrated in Figure 2, are Fuel, Employees, and Operation Cost. The output in the first stage is Sales. Our intermediate measure is Revenue Passenger Kilometers while inputs for the second stage are Short Interest Ratio, Market Cap, and WACC. The outputs in the second stage are Net Income, Capital Gains Yield, and ROE.

Our rationale for using the respective variables in the first and second stages, respectively, is as follows. Airline companies utilize Fuel, Employee, and
Operation Cost as inputs in order to generate Sales and Revenue Passenger Kilometers. Sales is also one of the final outputs from the first stage, whereas Revenue Passenger Kilometers is regarded as the intermediate measure, which is an output in the first stage and also used as an input in the second stage. Outside investors, traders, and other market participants then trade shares depending on how they feel about the future prospects of the firm. Thus, in the second stage, Short Interest Ratio, Market Cap, and WACC are used as inputs because they are what investors demand prior to a company undertaking a project. WACC is regarded as a proxy for the required rate of return in finance. Finally, Net Income, Capital Gains Yield, and ROE are regarded as outputs in the second stage because they can be viewed as the result of financial operations.

Using the two-stage DEA network and the aforementioned inputs, outputs, and intermediate measure (Figure 2), our study focuses on evaluating eight airline companies (Alaska Airlines, Air Canada, Delta Airlines, Hawaiian Airlines, JetBlue, Southwest Airlines, United Continental Holdings, and Spirit Airlines) in 10 rolling 1-year time windows for the total sample period of 1/1/2006–9/30/2016. Thus, the first time window is 1/1/2006–12/31/2007, whereas the next time window is 1/1/2007–12/31/2008, and so on. A dataset example for this rolling window approach is shown in Table 1.

We use our two-stage network DEA model to examine the data and to gauge airline performance. Table 2 reports results for the time window 1/1/2015–09/30/2016 of the overall performance and its decomposition.

Although the weights $\alpha_1$ and $\alpha_2$ can be viewed as the relative importance of the two stages, the current study is not able to use such information in this application for the following reasons. Because the overall performance is defined as the weighted average of the two stages’ performance scores, one would hope that different sets of stage weights lead to different individual stage performance scores. In the current study, we discover that multiple sets of stage weights lead to the identical individual stage performance scores; namely, each pair of stage performance scores can correspond to multiple stage weight combinations. Such a situation has already been observed in the literature (Guo et al., 2017). In fact, Guo et al. (2017) discover that sometimes the changes in the overall performance is due to the varying stage weights, while the individual stage performance scores remain unchanged. In such cases, a larger overall performance from one set of stage weights does not necessarily mean a better overall performance. This is because the larger overall score is caused by the stage weights, not the individual stage scores, which remain the same. We can perform sensitivity analysis to determine the exact range of stage weights that maintain the identical stage performance scores. Our analysis indicates that the current dataset yields a unique pair of individual stage performance scores for each DMU. Such scenarios have also been observed in the literature for other datasets and network DEA models (see, e.g., Chen et al., 2009, Chen et al., 2012, and Guo et al., 2017). In such a case, one can use any weight combinations to yield the overall performance scores. In the current study, we use average, indicating the two stages are viewed equally; namely, $\alpha_1$ and $\alpha_2$ are 0.5 in this current study.

For the overall performance, there are 71 airline units that are best practices (or efficient) out of total 518 airline units. There are 20 units from Hawaiian
Table 1: Data for one time window (1/1/2015–09/30/2016).

<table>
<thead>
<tr>
<th>Airline Companies</th>
<th>Date (Quarterly)</th>
<th>Cost per ASM (excluding fuel)</th>
<th>Employees (thousands)</th>
<th>Fuel Cost per ASM (cents)</th>
<th>Sales</th>
<th>Revenue Passenger Kilometers</th>
<th>WACC</th>
<th>Short Interest Ratio</th>
<th>Market Capitalization ($)</th>
<th>Net Income (millions)</th>
<th>Capital Gain Yield (%)</th>
<th>ROE</th>
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Continued
Table 1: Continued.

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<thead>
<tr>
<th>Airline Companies</th>
<th>Date (Quarterly)</th>
<th>Cost per ASM (excluding fuel)</th>
<th>Employees (thousands)</th>
<th>Fuel Cost per ASM (cents)</th>
<th>Sales</th>
<th>Revenue Passenger Kilometers</th>
<th>WACC</th>
<th>Short Interest Ratio</th>
<th>Market Capitalization ($)</th>
<th>Net Income (millions)</th>
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<td>8.91</td>
<td>49.58</td>
<td>2.26</td>
<td>4,977</td>
<td>29,727.97</td>
<td>9.93</td>
<td>2.98</td>
<td>28,004,290,560</td>
<td>1,996.00</td>
<td>142.40</td>
<td>350.86</td>
</tr>
<tr>
<td>Southwest</td>
<td>3/31/2016</td>
<td>8.59</td>
<td>50.91</td>
<td>2.42</td>
<td>4,826</td>
<td>28,408.16</td>
<td>10.32</td>
<td>2.49</td>
<td>28,585,535,488</td>
<td>1,971.00</td>
<td>133.96</td>
<td>351.10</td>
</tr>
<tr>
<td>Southwest</td>
<td>6/30/2016</td>
<td>8.38</td>
<td>52.30</td>
<td>2.36</td>
<td>5,384</td>
<td>32,707.69</td>
<td>10.02</td>
<td>1.15</td>
<td>25,042,911,232</td>
<td>2,280.00</td>
<td>116.67</td>
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</tr>
<tr>
<td>Southwest</td>
<td>9/30/2016</td>
<td>9.25</td>
<td>53.07</td>
<td>2.48</td>
<td>5,139</td>
<td>32,315.95</td>
<td>9.62</td>
<td>1.42</td>
<td>24,120,848,384</td>
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<td>129.18</td>
<td>350.04</td>
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<tr>
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<td>10.49</td>
<td>81.70</td>
<td>3.25</td>
<td>8,608</td>
<td>46,444.00</td>
<td>10.73</td>
<td>2.52</td>
<td>25,839,224,832</td>
<td>1,968.00</td>
<td>130.54</td>
<td>400.51</td>
</tr>
<tr>
<td>United Continental</td>
<td>6/30/2015</td>
<td>9.84</td>
<td>82.30</td>
<td>3.26</td>
<td>9,914</td>
<td>54,289.00</td>
<td>10.46</td>
<td>1.38</td>
<td>20,249,896,960</td>
<td>2,653.00</td>
<td>106.21</td>
<td>391.04</td>
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</table>

Continued
### Table 1: Continued.

<table>
<thead>
<tr>
<th>Airline Companies</th>
<th>Date (Quarterly)</th>
<th>Cost per ASM (excluding fuel)</th>
<th>Employees (thousands)</th>
<th>Fuel Cost per ASM (cents)</th>
<th>Sales</th>
<th>Revenue Passenger Kilometers</th>
<th>WACC</th>
<th>Short Interest Ratio</th>
<th>Market Capitalization ($)</th>
<th>Net Income (millions)</th>
<th>Capital Gain Yield (%)</th>
<th>ROE</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Continental</td>
<td>9/30/2015</td>
<td>9.70</td>
<td>82.40</td>
<td>2.90</td>
<td>10,306</td>
<td>57,160.00</td>
<td>9.37</td>
<td>1.72</td>
<td>20,265,177,088</td>
<td>6,276.00</td>
<td>130.08</td>
<td>425.40</td>
</tr>
<tr>
<td>United Continental</td>
<td>12/31/2015</td>
<td>10.34</td>
<td>82.10</td>
<td>2.64</td>
<td>9,036</td>
<td>50,718.00</td>
<td>9.84</td>
<td>1.75</td>
<td>21,888,684,032</td>
<td>2,283.00</td>
<td>137.71</td>
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</tr>
<tr>
<td>United Continental</td>
<td>3/31/2016</td>
<td>10.86</td>
<td>82.50</td>
<td>2.09</td>
<td>8,195</td>
<td>46,582.00</td>
<td>11.12</td>
<td>2.36</td>
<td>21,518,759,936</td>
<td>1,773.00</td>
<td>134.37</td>
<td>451.73</td>
</tr>
<tr>
<td>United Continental</td>
<td>6/30/2016</td>
<td>10.66</td>
<td>83.20</td>
<td>2.22</td>
<td>9,396</td>
<td>54,017.00</td>
<td>10.59</td>
<td>3.08</td>
<td>13,777,115,136</td>
<td>2,048.00</td>
<td>92.25</td>
<td>429.86</td>
</tr>
<tr>
<td>United Continental</td>
<td>9/30/2016</td>
<td>9.83</td>
<td>85.10</td>
<td>2.35</td>
<td>9,913</td>
<td>58,172.00</td>
<td>10.64</td>
<td>1.86</td>
<td>16,916,763,648</td>
<td>2,425.00</td>
<td>154.57</td>
<td>351.65</td>
</tr>
<tr>
<td>Spirit</td>
<td>3/31/2015</td>
<td>5.72</td>
<td>3.72</td>
<td>2.38</td>
<td>493</td>
<td>4,017.56</td>
<td>14.30</td>
<td>2.25</td>
<td>5,629,944,832</td>
<td>1,529.00</td>
<td>132.33</td>
<td>347.33</td>
</tr>
<tr>
<td>Spirit</td>
<td>6/30/2015</td>
<td>5.80</td>
<td>3.72</td>
<td>2.45</td>
<td>553</td>
<td>4,481.06</td>
<td>15.79</td>
<td>3.33</td>
<td>4,532,477,952</td>
<td>1,536.70</td>
<td>108.03</td>
<td>347.44</td>
</tr>
<tr>
<td>Spirit</td>
<td>9/30/2015</td>
<td>5.39</td>
<td>3.72</td>
<td>2.07</td>
<td>575</td>
<td>4,768.69</td>
<td>13.91</td>
<td>3.56</td>
<td>3,448,730,624</td>
<td>1,557.11</td>
<td>102.78</td>
<td>348.53</td>
</tr>
<tr>
<td>Spirit</td>
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<td>5.15</td>
<td>4.33</td>
<td>1.84</td>
<td>520</td>
<td>4,728.00</td>
<td>12.55</td>
<td>3.10</td>
<td>2,850,854,144</td>
<td>1,534.40</td>
<td>112.86</td>
<td>348.47</td>
</tr>
<tr>
<td>Spirit</td>
<td>3/31/2016</td>
<td>5.59</td>
<td>4.33</td>
<td>1.44</td>
<td>538</td>
<td>5,070.31</td>
<td>12.35</td>
<td>3.90</td>
<td>3,432,809,472</td>
<td>1,521.92</td>
<td>148.57</td>
<td>346.43</td>
</tr>
<tr>
<td>Spirit</td>
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<td>5.30</td>
<td>4.33</td>
<td>1.76</td>
<td>584</td>
<td>5,549.41</td>
<td>12.37</td>
<td>5.12</td>
<td>3,195,075,584</td>
<td>1,533.08</td>
<td>123.30</td>
<td>345.74</td>
</tr>
<tr>
<td>Spirit</td>
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<td>5.48</td>
<td>4.33</td>
<td>1.87</td>
<td>621</td>
<td>5,599.37</td>
<td>12.21</td>
<td>2.85</td>
<td>2,977,989,632</td>
<td>1,541.38</td>
<td>124.64</td>
<td>343.33</td>
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</table>
### Table 2: Example results for time window (1/1/2015–09/30/2016).

<table>
<thead>
<tr>
<th>Airlines Companies</th>
<th>Date</th>
<th>Operation Performance</th>
<th>Stock Market Performance</th>
<th>Overall Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alaska</td>
<td>3/31/2015</td>
<td>0.6642</td>
<td>0.9620</td>
<td>0.8131</td>
</tr>
<tr>
<td>Alaska</td>
<td>6/30/2015</td>
<td>0.6997</td>
<td>0.9246</td>
<td>0.8121</td>
</tr>
<tr>
<td>Alaska</td>
<td>9/30/2015</td>
<td>0.7225</td>
<td>1.0000</td>
<td>0.8612</td>
</tr>
<tr>
<td>Alaska</td>
<td>12/31/2015</td>
<td>0.7347</td>
<td>1.0000</td>
<td>0.8673</td>
</tr>
<tr>
<td>Alaska</td>
<td>3/31/2016</td>
<td>0.9368</td>
<td>0.9241</td>
<td>0.9304</td>
</tr>
<tr>
<td>Alaska</td>
<td>6/30/2016</td>
<td>0.8300</td>
<td>1.0000</td>
<td>0.9150</td>
</tr>
<tr>
<td>Alaska</td>
<td>9/30/2016</td>
<td>0.7644</td>
<td>0.9574</td>
<td>0.8609</td>
</tr>
<tr>
<td>Air Canada</td>
<td>3/31/2015</td>
<td>0.7933</td>
<td>1.0000</td>
<td>0.8967</td>
</tr>
<tr>
<td>Air Canada</td>
<td>6/30/2015</td>
<td>0.8228</td>
<td>1.0000</td>
<td>0.9114</td>
</tr>
<tr>
<td>Air Canada</td>
<td>9/30/2015</td>
<td>0.9594</td>
<td>1.0000</td>
<td>0.9797</td>
</tr>
<tr>
<td>Air Canada</td>
<td>12/31/2015</td>
<td>0.7646</td>
<td>0.9174</td>
<td>0.8410</td>
</tr>
<tr>
<td>Air Canada</td>
<td>3/31/2016</td>
<td>1.0000</td>
<td>0.8715</td>
<td>0.9358</td>
</tr>
<tr>
<td>Air Canada</td>
<td>6/30/2016</td>
<td>0.9333</td>
<td>0.8914</td>
<td>0.9124</td>
</tr>
<tr>
<td>Air Canada</td>
<td>9/30/2016</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Delta</td>
<td>3/31/2015</td>
<td>0.8866</td>
<td>0.7504</td>
<td>0.8185</td>
</tr>
<tr>
<td>Delta</td>
<td>6/30/2015</td>
<td>0.9890</td>
<td>0.7730</td>
<td>0.8810</td>
</tr>
<tr>
<td>Delta</td>
<td>9/30/2015</td>
<td>1.0000</td>
<td>0.8466</td>
<td>0.9233</td>
</tr>
<tr>
<td>Delta</td>
<td>12/31/2015</td>
<td>0.8911</td>
<td>0.9128</td>
<td>0.9020</td>
</tr>
<tr>
<td>Delta</td>
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<td>0.8832</td>
<td>0.7785</td>
<td>0.8309</td>
</tr>
<tr>
<td>Delta</td>
<td>6/30/2016</td>
<td>1.0000</td>
<td>0.8054</td>
<td>0.9027</td>
</tr>
<tr>
<td>Delta</td>
<td>9/30/2016</td>
<td>1.0000</td>
<td>0.8896</td>
<td>0.9448</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>3/31/2015</td>
<td>0.7227</td>
<td>1.0000</td>
<td>0.8613</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>6/30/2015</td>
<td>0.7317</td>
<td>1.0000</td>
<td>0.8659</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>9/30/2015</td>
<td>0.8010</td>
<td>1.0000</td>
<td>0.9005</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>12/31/2015</td>
<td>0.7840</td>
<td>1.0000</td>
<td>0.8920</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>3/31/2016</td>
<td>0.8984</td>
<td>1.0000</td>
<td>0.9492</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>6/30/2016</td>
<td>0.7840</td>
<td>1.0000</td>
<td>0.8920</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>9/30/2016</td>
<td>0.8113</td>
<td>1.0000</td>
<td>0.9056</td>
</tr>
<tr>
<td>Jet Blue</td>
<td>3/31/2015</td>
<td>0.6979</td>
<td>1.0000</td>
<td>0.8490</td>
</tr>
<tr>
<td>Jet Blue</td>
<td>6/30/2015</td>
<td>0.7376</td>
<td>0.9647</td>
<td>0.8512</td>
</tr>
<tr>
<td>Jet Blue</td>
<td>9/30/2015</td>
<td>0.7625</td>
<td>1.0000</td>
<td>0.8813</td>
</tr>
</tbody>
</table>

Airlines, 14 units from Spirit Airlines, 9 units from Southwest Airlines, 9 units from Air Canada, 7 units from Jet Blue, 5 units from Delta Airlines, 5 units from United Continental Holdings, and 2 units from Alaska Airlines.

In addition, airlines perform better in the second stage (stock market performance stage) than they do in the first stage (operation performance stage). In the second stage, we have 222 DMUs whose performance score is equal to one. However, there are only 126 DMUs in the first stage whose performance is aligned with best practices. Among them, Hawaiian Airlines has the vast majority of efficient units in the second stage, whereas Spirit Airlines has the vast majority of efficient units in the first stage.
Table 3: Performance score by carrier type.

<table>
<thead>
<tr>
<th>Carrier Type</th>
<th>Operation Performance</th>
<th>Stock Market Performance</th>
<th>Overall Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Cost Carrier</td>
<td>0.9258</td>
<td>0.8652</td>
<td>0.8955</td>
</tr>
<tr>
<td>Full Service Carrier</td>
<td>0.8954</td>
<td>0.9195</td>
<td>0.9075</td>
</tr>
</tbody>
</table>

We also conduct a comparison of the results between low-cost and the full-service carriers. A low-cost carrier is an airline that generally has lower fares and fewer comforts, whereas full-service airlines are usually regarded as having a higher level of customer service and more features. Table 3 summarizes the performance score for these two different airline carrier types.

According to Table 3, the full-service carriers perform better in the stock market stage than they perform in the operation stage, whereas the low-cost carriers perform better in the first stage than they do in the second stage. In addition, the low-cost carriers have a higher performance score in the operational performance stage than the full-service carriers have. On the contrary, full-service carriers perform better in the second stage relative to their low-cost counterparts. In terms of overall performance, full-service carriers appear to be more efficient with an average score of 0.9075.

Table 4 reports the results across all 10 time windows, whereas Table 5 reports final results and overall rankings for the airline companies for the entire sample period 1/1/2006–9/30/2016.

The 2008/2009 financial crisis and the European debt crisis and U.S. debt-ceiling crisis, both transpired during the 2013/2014 period, are associated with declines in overall airline performance. Note that the crisis during 2013/2014 has a greater impact on the airline industry where the average performance score for the stock market stage reduces to 0.7630. Following 2015/2016, however, the airlines appear to be recovering and performing better.

Table 4: Performance scores across all time windows.

<table>
<thead>
<tr>
<th>Time Window</th>
<th>Operation Performance</th>
<th>Stock Market Performance</th>
<th>Overall Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006–2007</td>
<td>0.9641</td>
<td>0.9880</td>
<td>0.9760</td>
</tr>
<tr>
<td>2007–2008</td>
<td>0.9544</td>
<td>0.9562</td>
<td>0.9553</td>
</tr>
<tr>
<td>2008–2009</td>
<td>0.9468</td>
<td>0.9125</td>
<td>0.9297</td>
</tr>
<tr>
<td>2009–2010</td>
<td>0.9340</td>
<td>0.9154</td>
<td>0.9247</td>
</tr>
<tr>
<td>2010–2011</td>
<td>0.9479</td>
<td>0.8822</td>
<td>0.9151</td>
</tr>
<tr>
<td>2011–2012</td>
<td>0.8957</td>
<td>0.9055</td>
<td>0.9006</td>
</tr>
<tr>
<td>2012–2013</td>
<td>0.8929</td>
<td>0.8913</td>
<td>0.8921</td>
</tr>
<tr>
<td>2013–2014</td>
<td>0.8811</td>
<td>0.7630</td>
<td>0.8220</td>
</tr>
<tr>
<td>2014–2015</td>
<td>0.8661</td>
<td>0.7906</td>
<td>0.8284</td>
</tr>
<tr>
<td>2015–2016</td>
<td>0.8736</td>
<td>0.9338</td>
<td>0.9037</td>
</tr>
</tbody>
</table>
Using Operational and Stock Analytics to Measure Airline Performance

Table 5: Performance scores for all airline companies.

<table>
<thead>
<tr>
<th>Airline Companies</th>
<th>Operation Performance</th>
<th>Stock Market Performance</th>
<th>Overall Performance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alaska</td>
<td>0.8394</td>
<td>0.9112</td>
<td>0.8753</td>
<td>6</td>
</tr>
<tr>
<td>Air Canada</td>
<td>0.9007</td>
<td>0.9335</td>
<td>0.9171</td>
<td>3</td>
</tr>
<tr>
<td>Delta</td>
<td>0.9404</td>
<td>0.8155</td>
<td>0.8780</td>
<td>5</td>
</tr>
<tr>
<td>Hawaiian</td>
<td>0.9098</td>
<td>0.9986</td>
<td>0.9542</td>
<td>2</td>
</tr>
<tr>
<td>Jet Blue</td>
<td>0.9188</td>
<td>0.8261</td>
<td>0.8725</td>
<td>7</td>
</tr>
<tr>
<td>Southwest</td>
<td>0.9046</td>
<td>0.8654</td>
<td>0.8850</td>
<td>4</td>
</tr>
<tr>
<td>United Continental Holdings</td>
<td>0.9181</td>
<td>0.7197</td>
<td>0.8189</td>
<td>8</td>
</tr>
<tr>
<td>Spirit</td>
<td>0.9952</td>
<td>0.9596</td>
<td>0.9774</td>
<td>1</td>
</tr>
</tbody>
</table>

The overall rank in Table 5 is constructed according to the value of overall performance of each airline. It appears that Spirit Airlines performs the best, whereas United Continental Holdings performs the worst. More than half of the airline companies perform better in terms of their operational performance relative to their stock market performance. Alaska Airlines and Air Canada perform worse in terms of their operational performance—this is an important finding considering that both of these companies are considered full-service carriers. In addition, Hawaiian Airlines performs best in the stock market performance stage and Spirit performs best in terms of operational performance.

Our results also indicate that there is no relationship between performance scores and capital gains. This finding lends support to a growing branch of behavioral finance literature, which shows that stock prices can behave in ways that do not reflect fundamental aspects of a firm, such as its operational performance, and can deviate from “fair” values indefinitely (Koutmos, 2015). They are prone to such deviations as a result of speculative buying and selling decisions that take place in the market and as a result of traders with heterogeneous trading styles.

CONCLUDING REMARKS

There have been significant methodological advances in efficiency and performance benchmarking. Despite this, the majority of extant studies, albeit with some exceptions, focus exclusively on operational indicators when drafting their conceptual and empirical frameworks. Stock market indicators are almost absent from empirical considerations. Such an approach may lead to biased or erroneous conclusions.

Our article makes a conceptual contribution to the literature by integrating operational with financial market data. Neglecting to include stock market and financial indicators into any empirical framework can lead to biased conclusions. From a managerial point of view, stock market measures can capture investor attitudes and give upper management important feedback into the pulse of the market. Given that the airlines industry is so competitive, managers need to be acutely
aware of not only their operational efficiency but also the sentiment, attitudes, and expectations of their shareholders and the stock market at large. By integrating financial market indicators into our two-stage DEA framework, we also align ourselves with financial economics literature, which finds that investors trade on sentiment and various market indicators.

Our article also makes a methodological contribution because it implements a two-stage network DEA with SOCP. Such a technique is novel in operations literature and is advantageous because it enables us to solve nonlinear DEA models without the need for calculating numerous parametric linear programs in an effort to estimate the global optimal solution.

Overall, we provide performance rankings of all the eight major airlines in our sample for the period 1/1/2006–9/30/2016. These performance rankings are unique in that they include both operational and financial stock market indicators. We also uncover some important findings regarding our full- and subsample analyses. Low-cost carriers generate higher performance scores in the operational performance stage than their full-service counterparts. On the contrary, full-service carriers perform better in the financial market stage relative to low-cost airlines.

The 2008/2009 financial crisis and the European debt crisis and United States debt-ceiling crisis, both transpired during the 2013/2014 period, are associated with declines in overall airline performance. Following 2013/2014, however, the airlines appear to be performing better.

The current study shows that there is no relationship between performance scores and capital gains. This finding lends support to an emerging body of behavioral finance literature, which shows that stock prices can behave in ways that do not reflect fundamental and objective aspects pertaining to a firm’s operations or growth prospects.

Note that the current study uses some industry standard ratio measures in our newly developed network DEA model. Although the use of ratio data can be problematic in the envelopment-type DEA model where convexity of performance measures needs to be satisfied, it is not a concern for the current study, which is based upon the multiplier-type model by generating a composite index of performance measures. Although under standard DEA models that are linear and where the multiplier model is equivalent (or a dual) to the envelopment model, the network multiplier and envelopment models are not equivalent and are not duals. This is due to the fact that network DEA models are not linear and Chen et al. (2013) show that the envelopment and multiplier network DEA models are developed independently and generate different results. If one wants to develop the network DEA models using the envelopment-type DEA models to estimate production functions, then one has to use the envelopment model for ratio data developed by Olesen et al. (2015). Due to the lack of duality in the network DEA, one cannot simply use the dual model of Olesen et al. (2015) or Emrouznejad and Amin (2009) to develop the multiplier network DEA models. This is clearly a future research topic that merits attention.
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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

APPENDIX

REFERENCES


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APPENDIX

In this Appendix, we provide the mathematics in deriving our SOCP model (4). Note that while our approach is based upon SOCP as in Chen and Zhu (2017), the underlying network DEA modeling (in particular, the overall performance score construct) is different. The Appendix provides more detailed discussion and shows an improvement to the Chen and Zhu (2017) approach by addressing the symmetric issue of the SOCP model developed in Chen and Zhu (2017).

Chen and Zhu (2017) develop a SOCP model for a two-stage network structure where the overall performance score is defined as the product of $e_1^0 = \frac{\sum_{d=1}^{D} \eta_d z_d + \sum_{m=1}^{M} \lambda_m y_{m0} + \alpha}{\sum_{s=1}^{S} v_{s0} \gamma_{s0}}$ and $e_2^0 = \frac{\sum_{r=1}^{R} \eta_r y_r + \alpha}{\sum_{s=1}^{S} v_{s0} \gamma_{s0}}$. Namely, their approach is a multiplicative two-stage DEA network model. Intuitively, one could use the same technique to develop a SOCP model when the overall score is defined as a weighted average of $e_1^0$ and $e_2^0$ as in model (1). However, after careful examination, we find that the positive semidefinite matrix to deal with multiplicative
two-stage DEA shown as follows may not be positive semidefinite because it is not symmetric.

\[
\begin{bmatrix}
\theta \left( \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0} + u^1 \right) & \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{h=1}^{H} Q_h x_{h0}^2 \\
\sum_{i=1}^{m} v_i x_{i0} & \sum_{r=1}^{s} u_r y_{r0} + u^2
\end{bmatrix} \succ= 0
\]

(A1)

Nevertheless, the symmetric problem of Chen and Zhu (2017) can be fixed by letting \( \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{h=1}^{H} Q_h x_{h0}^2 = k \sum_{i=1}^{m} v_i x_{i0} \), where \( k \) is a parameter that can be incorporated into other terms by algebraic manipulations and can be located by a bisection search method.

However, in the current article, which addresses additive two-stage network DEA, we have to develop a different technique to the symmetric problem because the numerator of stage score in an additive two-stage DEA includes only one linear combination. The new technique is summarized as follows.

Note that in model (3), because \( \alpha_1 \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0} + u^1 \geq \theta_1 \) is equivalent to the following:

\[
\alpha_1^2 \left( \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0} + u^1 \right)^2 \geq \theta_1^2
\]

(A2)

Then, we have

\[
\begin{bmatrix}
\frac{1}{\theta_1^2} \left( \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0} + u^1 \right) & \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{h=1}^{H} Q_h x_{h0}^2 \\
\sum_{i=1}^{m} v_i x_{i0} & \alpha_1^2 \left( \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{p=1}^{P} \lambda_p y_{p0} + u^1 \right)
\end{bmatrix} \succ= 0
\]

(A3)

Note that the symmetric matrix in (A3) is positive semidefinite because all of its principal minors are nonnegative.

Similarly, \( \alpha_2 \sum_{r=1}^{s} u_r y_{r0} \geq \theta_2 \) is equivalent to the following:

\[
\begin{bmatrix}
\frac{1}{\theta_2^2} \sum_{r=1}^{s} u_r y_{r0} & \sum_{d=1}^{D} \eta_d z_{d0} + \sum_{h=1}^{H} Q_h x_{h0}^2 \\
\sum_{d=1}^{D} \eta_d z_{d0} + \sum_{h=1}^{H} Q_h x_{h0}^2 + u^2 & \alpha_2^2 \sum_{r=1}^{s} u_r y_{r0}
\end{bmatrix} \succ= 0
\]

(A4)

Note that \( \alpha_1 \) in (A3) and \( \alpha_2 \) in (A4) can be provided by the decision maker, or searched for the optimal weight that yields the largest overall score as in Guo et al. (2017). Thus, if we can locate the true values of parameter \( \theta_1 \) in (A3)
and parameter $\theta_2$ in (A4), (A3) and (A4) correspond to the following two conic constraints, respectively.

\[
\left\|rac{1}{2} \left[ \frac{1}{\theta_1^2} \left( \sum_{d=1}^{D} \eta_d z_d^0 + \sum_{p=1}^{P} \lambda_p y_p^1 + u^1 \right) - \alpha_1^2 \left( \sum_{d=1}^{D} \eta_d z_d^0 + \sum_{p=1}^{P} \lambda_p y_p^1 + u^1 \right) \right] \right\|_2 \leq \frac{1}{2} \left[ \frac{1}{\theta_1^2} \left( \sum_{d=1}^{D} \eta_d z_d^0 + \sum_{p=1}^{P} \lambda_p y_p^1 + u^1 \right) + \alpha_1^2 \left( \sum_{d=1}^{D} \eta_d z_d^0 + \sum_{p=1}^{P} \lambda_p y_p^1 + u^1 \right) \right]
\]  
(A5)

\[
\left\| \sum_{d=1}^{D} \eta_d z_d^0 + \sum_{h=1}^{H} Q_h x_h^2 + u^2 \right\|_2 \leq \frac{1}{2} \left[ \frac{1}{\theta_2^2} \sum_{r=1}^{s} u_r y_r^0 - \alpha_2^2 \sum_{r=1}^{s} u_r y_r^0 \right] \right\|_2 \right\|_2 \leq \frac{1}{2} \left[ \frac{1}{\theta_2^2} \sum_{r=1}^{s} u_r y_r^0 + \alpha_2^2 \sum_{r=1}^{s} u_r y_r^0 \right]
\]  
(A6)

The above transformation is based on a simple fact that $AC - B^2 \geq 0$ is equivalent to $\sqrt{B^2 + \left( \frac{1}{2}(A - C) \right)^2} \leq \frac{1}{2}(A + C)$, where $A$, $B$, and $C \in \mathbb{R}$. Finally, we have our (parametric) SOCP problem (3) in the main text.

For model (3), we can search for the values of $\theta_1$ and $\theta_2$ in a convergent manner. Note that we can always find initial values of $\theta_1$ and $\theta_2$, denoted as $\theta_1^0$ and $\theta_2^0$, respectively, to satisfy the constraints of model (3). Otherwise, the original problem (1) is infeasible. Then, without loss of generality, we can fix $\theta_1^0$ and increase the value of $\theta_2$ from $\theta_2^0$ to $\theta_2^1$ by bisection method. Further, fixing $\theta_2^1$, we can adopt a bisection method again to increase the value of $\theta_1$ from $\theta_1^0$ to $\theta_1^1$. Repeatedly, the maximal values of $\theta_1$ and $\theta_2$ can be obtained because the searching sequence is monotonic in a compact set.

Thus, by solving model (3), we determine stage efficiencies for a specific DMU under evaluation.

The modeling language is CVX for Matlab. In CVX, we use the MOSEK package to implement an interior point algorithm for second order cone programming. The computational codes can be provided upon request.

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