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O.R. Applications

Equivalence in two-stage DEA approaches

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Abstract

Data envelopment analysis (DEA) is a linear programming problem approach for evaluating the relative efficiency of peer decision making units (DMUs) that have multiple inputs and outputs. DMUs can have a two-stage structure where all the outputs from the first stage are the only inputs to the second stage, in addition to the inputs to the first stage and the outputs from the second stage. The outputs from the first stage to the second stage are called intermediate measures. This paper examines relations and equivalence between two existing DEA approaches that address measuring the performance of two-stage processes.

Keywords: Data envelopment analysis (DEA); Efficiency; Intermediate measure; Two-stage

1. Introduction

Data envelopment analysis (DEA) is an approach for measuring the relative efficiency of peer decision making units (DMUs) that have multiple inputs and outputs. While the definition of a DMU is generic and DMUs can be in various forms such as hospitals, products, universities, cities, courts, business firms, and others, DMUs can have a two-stage structure in many cases. For example, banks use labor and assets to generate deposits which are in turn used to generate load incomes. In such a setting, a DMU represents a two-stage process and intermediate measures exist in-between the two stages. The first stage uses inputs to generate outputs which become the inputs to the second stage. The first stage outputs are therefore called intermediate measures. The second stage then uses these intermediate measures to produce outputs. A key feature here is that the first stage's outputs are the only inputs to the second stage, i.e., in addition to the intermediate measures, the first stage does not have its own outputs and the second stage does not have its own inputs.

An usual attempt to deal with such two-stage processes is to apply the standard DEA model to each stage (see, e.g., Seiford and Zhu, 1999). However, as noted in Zhu (2003) and Chen and Zhu (2004), such an approach may conclude that two inefficient stages lead to an overall efficient DMU with the inputs of the first stage and outputs of the second stage. Consequently, improvement to the DEA frontier can be distorted, i.e., the performance improvement of one stage affects the efficiency status of the other, because of the presence of intermediate measures.

Based upon the variable returns to scale DEA model (Banker et al., 1984), Chen and Zhu (2004) develop a linear DEA type model where each stage's efficiency is defined on its own production possibility set. The two production possibility sets are linked with the intermediate measures which are set as decision variables for each DMU under

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evaluation. Chen and Zhu's (2004) model guarantees an overall efficient two-stage process when each stage is efficient. For inefficient DMUs, Chen and Zhu (2004) model provides a DEA projection with a set of optimal intermediate measures.

Kao and Hwang (2008), on the other hand, modify the standard DEA model by taking into account the series relationship of the two stages within the whole process. Under their framework, the efficiency of the whole process can be decomposed into the product of the efficiencies of the two sub-processes. Note that such an efficiency decomposition is not available in the standard DEA approach of Seiford and Zhu (1999) and the two-stage approach of Chen and Zhu (2004).

The current paper studies the relationship between the approaches of Chen and Zhu (2004) and Kao and Hwang (2008). Note that the approach of Kao and Hwang (2008) is developed under the assumption of constant returns to scale in the multiplier CCR DEA model of Charnes et al. (1978). We show that the CCR version of the Chen and Zhu (2004) model can be equivalent to the Kao and Hwang's (2008) model.

The rest of the paper is organized as follows. The next section presents the Kao and Hwang (2008) model and then the Chen and Zhu (2004) model. The relation and equivalence between the two approaches are then studied. Two data sets are then used to illustrate our discussion. Section 5 are given at last.

2. Two-stage DEA models

Consider a two-stage process shown in Fig. 1. Suppose, we have n DMUs, using the notations in Chen and Zhu (2004) and Kao and Hwang (2008), we assume that each DMU_j $(j = 1,2,\ldots,n)$ has m inputs to the first stage, x_{ij} $(i = 1,2,\ldots,m)$ and D outputs from the first stage, z_{dj} $(d = 1,2,\ldots,D)$. These D outputs then become the inputs to the second stage and are called intermediate measures. The outputs from the second stage are y_{rj} $(r = 1,2,\ldots,s)$. Based upon the CCR model, the efficiency scores of the two-stage process and the two individual stages can be expressed as

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \quad \theta_j^1 = \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \quad \text{and} \quad \theta_j^2 = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \tilde{w}_d z_{dj}}, \tag{1}$$

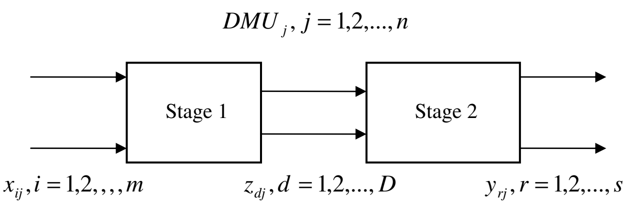

Fig. 1. Two-stage process.

where v_i, w_d, $\tilde{w}_d$, and u_r are unknown non-negative weights. Note that w_d can be equal to $\tilde{w}_d$.

Note that the intermediate measures of z_{dj} do not appear in θ_j. Kao and Hwang (2008) assume that $w_d = \tilde{w}_d$. As a result, for a specific DMU_{j_0}, $\theta_{j_0}^1 \cdot \theta_{j_0}^2$ becomes $\frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}$ which is the overall efficiency defined in the Kao and Hwang (2008) model:

$$\begin{aligned}
\text{Max } & \theta_0^1 \cdot \theta_0^2 = \frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}, \\
\text{s.t. } & a_j^1 \leqslant 1 \text{ and } \theta_j^2 \leqslant 1 \text{ for all } j, \\
& w_d = \tilde{w}_d \quad \text{for all } d.
\end{aligned} \tag{2}$$

Note that in the DEA terminology, model (2) is an input-oriented model. The equivalent output-oriented model can be expressed as

$$\begin{aligned}
\text{Min } & \frac{\sum_{i=1}^{m} v_i x_{ij_0}}{\sum_{r=1}^{s} u_r y_{rj_0}}, \\
\text{s.t. } & \theta_j^1 \leqslant 1 \text{ and } \theta_j^2 \leqslant 1 \text{ for all } j, \\
& w_d = \tilde{w}_d \quad \text{for all } d.
\end{aligned} \tag{3}$$

Model (3) is equivalent to the following linear program

$$\begin{aligned}
\text{Min } & \sum_{i=1}^{m} v_i x_{ij_0}, \\
\text{s.t. } & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leqslant 0, \quad j = 1,2,\ldots,n, \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j = 1,2,\ldots,n, \\
& \sum_{r=1}^{s} u_r y_{rj_0} = 1, \\
& w_d,\ d = 1,2,\ldots,D;\ v_i,\ i = 1,2,\ldots,m; \\
& u_r,\ r = 1,2,\ldots,s \geqslant 0.
\end{aligned} \tag{4}$$

The above model is an output-oriented version of Kao and Hwang's (2008) model. Note that constraints $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0$ are redundant in Kao and Hwang's (2008) model.[1]

Note that the Chen and Zhu (2004) model is developed under the condition of variable returns to scale. In order to make the comparison between the two approaches. We now write the Chen and Zhu (2004) model under the condition of constant returns to scale as follows

[1] Because $\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leqslant 0$ and $\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0$ imply $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0$.

$$\min_{\alpha,\beta,\lambda_j,\mu_j,\tilde{z}} \alpha-\beta,$$
s.t.
(stage 1)
$$\begin{aligned}
&\sum_{j=1}^{n}\lambda_j x_{ij}\leqslant \alpha x_{ij_o}, \quad i=1,\ldots,m,\\
&\sum_{j=1}^{n}\lambda_j z_{dj}\geqslant \tilde{z}_{dj_o}, \quad d=1,\ldots,D,\\
&\lambda_j\geqslant 0, \quad j=1,\ldots,n,\\
&\alpha\leqslant 1,\\
&\text{(stage 2)}\\
&\sum_{j=1}^{n}\mu_j z_{dj}\leqslant \tilde{z}_{dj_0}, \quad d=1,\ldots,D,\\
&\sum_{j=1}^{n}\mu_j y_{rj}\geqslant \beta y_{rj_0}, \quad r=1,\ldots,s,\\
&\mu_j\geqslant 0, \quad j=1,\ldots,n,\\
&\beta\geqslant 1.
\end{aligned} \tag{5}$$

If we add the convexity constraints of $\sum\lambda_j=\sum\mu_j=1$ into model (5), then model (5) becomes the original Chen and Zhu (2004) model under variable returns to scale assumption.

3. Relations

First, consider the following linear program

$$\begin{aligned}
\min_{\alpha,\beta,\lambda_j,\mu_j}\ & \alpha-\beta,\\
\text{s.t.}\ & \sum_{j=1}^{n}\lambda_j x_{ij}\leqslant \alpha x_{ij_0}, \quad i=1,\ldots,m,\\
& \sum_{j=1}^{n}\mu_j y_{rj}\geqslant \beta y_{rj_0}, \quad r=1,2,\ldots,s,\\
& \sum_{j=1}^{n}(\lambda_j-\mu_j)z_{dj}\geqslant 0, \quad d=1,2,\ldots,D,\\
& \lambda_j,\mu_j\geqslant 0, \quad j=1,2,\ldots,n,\\
& \alpha\leqslant 1, \quad \beta\geqslant 1.
\end{aligned} \tag{6}$$

Theorem 1. *Model (6)'s optimal solutions are optimal in model (5).*

Proof. Let model (5)'s optimal solutions be $\alpha^*,\beta^*,\lambda_j^*,\mu_j^*,\tilde{z}_{dj_0}^*$, and model (6)'s optimal solutions be $\alpha'^*,\beta'^*,\lambda_j'^*,\mu_j'^*$. Note that the feasible region of model (6) contains that of model (5). Thus, $\alpha'^*-\beta'^*\leqslant\alpha^*-\beta^*$. Note also that we always have

$$\sum_{j=1}^{n}\lambda_j'^* z_{dj}\geqslant \tilde{z}_{dj_0}'^*\geqslant \sum_{j=1}^{n}\mu_j'^* z_{dj}.$$

Therefore, $\alpha'^*,\beta'^*,\lambda_j'^*,\mu_j'^*,\tilde{z}_{dj_0}'^*$ are feasible in model (5). Thus, $\alpha'^*,\beta'^*,\lambda_j'^*,\mu_j'^*$ must be optimal in model (5) with $\alpha^*-\beta^*=\alpha'^*-\beta'^*$. □

Let $\lambda_j'=\frac{\lambda_j}{\alpha}$, $\mu_j'=\frac{\mu_j}{\alpha}$, and $\sigma=\beta/\alpha$, then model (6) becomes

$$\begin{aligned}
\text{Max}\ & \alpha(\sigma-1),\\
\text{s.t.}\ & \sum\lambda_j' x_{ij}\leqslant x_{ij_0},\\
& \sum\mu_j' y_{rj}\geqslant \sigma y_{rj_0},\\
& \sum(\lambda_j'-\mu_j')z_{dj}\geqslant 0,\\
& \lambda_j',\mu_j'\geqslant 0,\alpha\leqslant 1,\alpha\sigma\geqslant 1.
\end{aligned} \tag{7}$$

Next, consider the following model

$$\begin{aligned}
\text{Max}\ & \alpha(\sigma-1),\\
\text{s.t.}\ & \sum\lambda_j' x_{ij}\leqslant x_{ij_0},\\
& \sum\mu_j' y_{rj}\geqslant \sigma y_{rj_0},\\
& \sum(\lambda_j'-\mu_j')z_{dj}\geqslant 0,\\
& \lambda_j',\mu_j'\geqslant 0, \quad \alpha\leqslant 1, \quad \sigma\geqslant 1.
\end{aligned} \tag{8}$$

The only difference between models (7) and (8) is that model (8) sets $\alpha=1$ in the constraint of $\alpha\sigma\geqslant 1$ in model (7).

Theorem 2. *Model (8)'s optimal solutions are optimal in model (7).*

Proof. Let model (7)'s feasible solution be $\lambda_j',\mu_j',\alpha,\sigma\geqslant 0$ with $\alpha\leqslant 1$ and $\alpha\sigma\geqslant 1$. Then $\sigma\geqslant\frac{1}{\alpha}\geqslant 1$. Thus, $\lambda_j',\mu_j',\alpha,\sigma\geqslant 0$ are also feasible in model (8). This indicates that the feasible region of model (8) contains that of model (7).

Next let $\lambda_j'^C,\mu_j'^C,\alpha^C,\sigma^C\geqslant 0$ and $\lambda_j'^D,\mu_j'^D,\alpha^D,\sigma^D\geqslant 0$ be optimal solutions in models (7) and (8), respectively. We must have $\alpha^D(\sigma^D-1)\geqslant\alpha^C(\sigma^C-1)$.

Suppose $\alpha^D\sigma^D<1$. Note that $\sigma^D\geqslant 1$ in model (8). Therefore, we can find an α'^D such that $\frac{1}{\sigma^D}\leqslant\alpha'^D\leqslant 1$. We have $\alpha'^D\sigma^D\geqslant 1>\alpha^D\sigma^D$, namely, $1\geqslant\alpha'^D>\alpha^D$. This indicates that $\lambda_j'^D,\mu_j'^D,\alpha'^D,\sigma^D\geqslant 0$ are feasible in model (8) and $\alpha'^D(\sigma^D-1)>\alpha^D(\sigma^D-1)$. A contradiction to the fact that $\lambda_j'^D,\mu_j'^D,\alpha^D,\sigma^D\geqslant 0$ are optimal. Thus, $\alpha^D\sigma^D\geqslant 1$.

Therefore, $\lambda_j'^D,\mu_j'^D,\alpha^D,\sigma^D\geqslant 0$ are feasible in model (7). Recall that $\alpha^D(\sigma^D-1)\geqslant\alpha^C(\sigma^C-1)$. Thus, $\lambda_j'^D,\mu_j'^D,\alpha^D,\sigma^D\geqslant 0$ are optimal in model (7). □

Note that $\alpha\leqslant 1$ does not appear in other constraints of model (8). Therefore, at optimality, $\alpha^*=1$ and model (8) is equivalent to the following linear program

$$\begin{aligned}
\text{Max}\ & \sigma,\\
\text{s.t.}\ & \sum\lambda_j' x_{ij}\leqslant x_{ij_0}, \quad i=1,2,\ldots,m,\\
& \sum\mu_j' y_{rj}\geqslant \sigma y_{rj_0}, \quad r=1,2,\ldots,m,\\
& \sum(\lambda_j'-\mu_j')z_{dj}\geqslant 0, \quad d=1,2,\ldots,D,\\
& \lambda_j', \quad \mu_j'\geqslant 0, \quad \sigma\geqslant 1.
\end{aligned} \tag{9}$$

Model (9) is actually the dual to the Kao and Hwang's (2008) model (4). Therefore, we have

Theorem 3. *At optimality $\alpha^* = 1$ and $\beta^* = \sigma^*$, where α^* and β^* are optimal values of α and β in model (5) and σ^* is the optimal value of σ in model (9).*

Theorem 3 indicates that model (5) of Chen and Zhu (2004) is equivalent to the (output-oriented) model of Kao and Hwang (2008). The above discussion also indicates that the optimal α^* and β^* in model (5) do not represent the efficiency scores of individual stages under the condition of constant returns to scale. In fact, α^* is always equal to unity and β^* represents the overall efficiency of the two-stage process, i.e., model (4) can be used to measure the overall efficiency of the two-stage process under the condition of constant returns to scale.

4. Applications

We next apply the constant returns to scale version of Chen and Zhu (2004) model, i.e., model (5) to the data set used in Kao and Hwang (2008). Kao and Hwang's (2008) data set consists of 24 non-life insurance companies in Taiwan. The two inputs to the first stage (premium acquisition) are Operating expenses and Insurance expenses. The intermediate measures (or the outputs from the first stage) are Direct written premiums and Reinsurance premiums. The outputs of the second stage (profit generation) are Underwriting profit and Investment profit. The results of model (5) are reported in Table 1. As expected, $\alpha^* = 1$ for all the DMUs. The last column reports the inverse of β^* which is equal to the overall efficiency reported in Kao and Hwang (2008).

Table 1
Non-life insurance companies in Taiwan

	α	β	$1/\beta$
1	1	1.43014	0.69923
2	1	1.60059	0.62477
3	1	1.44923	0.69002
4	1	3.28709	0.30422
5	1	1.30381	0.76698
6	1	2.56621	0.38968
7	1	3.61559	0.27658
8	1	3.63412	0.27517
9	1	4.47868	0.22328
10	1	2.14611	0.46596
11	1	6.10054	0.16392
12	1	1.31652	0.75958
13	1	4.81209	0.20781
14	1	3.46452	0.28864
15	1	1.62912	0.61383
16	1	3.12354	0.32015
17	1	2.77770	0.36001
18	1	3.86339	0.25884
19	1	2.43191	0.4112
20	1	1.82966	0.54655
21	1	4.98058	0.20078
22	1	1.69630	0.58952
23	1	2.37903	0.42034
24	1	7.41785	0.13481

Table 2
IT data set

DMU	Fixed assets ($ billion)	IT budget ($ billion)	# of employees (thousand)	Deposits ($ billion)	Profit ($ billion)	Fraction of loans recovered
1	0.713	0.15	13.3	14.478	0.232	0.986
2	1.071	0.17	16.9	19.502	0.34	0.986
3	1.224	0.235	24	20.952	0.363	0.986
4	0.363	0.211	15.6	13.902	0.211	0.982
5	0.409	0.133	18.485	15.206	0.237	0.984
6	5.846	0.497	56.42	81.186	1.103	0.955
7	0.918	0.06	56.42	81.186	1.103	0.986
8	1.235	0.071	12	11.441	0.199	0.985
9	18.12	1.5	89.51	124.072	1.858	0.972
10	1.821	0.12	19.8	17.425	0.274	0.983
11	1.915	0.12	19.8	17.425	0.274	0.983
12	0.874	0.05	13.1	14.342	0.177	0.985
13	6.918	0.37	12.5	32.491	0.648	0.945
14	4.432	0.44	41.9	47.653	0.639	0.979
15	4.504	0.431	41.1	52.63	0.741	0.981
16	1.241	0.11	14.4	17.493	0.243	0.988
17	0.45	0.053	7.6	9.512	0.067	0.98
18	5.892	0.345	15.5	42.469	1.002	0.948
19	0.973	0.128	12.6	18.987	0.243	0.985
20	0.444	0.055	5.9	7.546	0.153	0.987
21	0.508	0.057	5.7	7.595	0.123	0.987
22	0.37	0.098	14.1	16.906	0.233	0.981
23	0.395	0.104	14.6	17.264	0.263	0.983
24	2.68	0.206	19.6	36.43	0.601	0.982
25	0.781	0.067	10.5	11.581	0.12	0.987
26	0.872	0.1	12.1	22.207	0.248	0.972
27	1.757	0.0106	12.7	20.67	0.253	0.988

We finally apply models (5) and (9) to the data set used in Wang et al. (1997) and then in Chen and Zhu (2004) in examining the information technology impact on productivity. Table 2 reports the data set which consists of 27 firms in the banking industry. The inputs for the first stage are fixed assets, number of employees, and IT investment. The intermediate measure is the deposits generated. The second stage's outputs are profit and fraction of loans recovered.[2]

Table 3 reports the results from model (5). It can be seen that $\alpha^* = 1$. The third column reports the β^* which is also the optimal value to model (9) and represents the overall efficiency score of Kao and Hwang (2008). Because model (9) is the output-oriented model, all β^* or σ^* are greater than one (except for DMU18), indicating that the two-stage process is inefficient. It can also be seen that both stages are efficient for DMU18 with $\beta^* = \sigma^* = 1$.

Columns 4 and 5 of Table 3 report the (output-oriented) CCR efficiency scores for the two stages, respectively. The last column reports the (output-oriented) CCR score for the overall performance with the first stage's inputs as the inputs and second stage's outputs as outputs.

[2] For detailed discussion on the data, the reader is referred to Wang et al. (1997).

Table 3
IT results

DMU	α^*	β^*	θ^1	θ^2	θ
1	1	2.099	1.565	1.341	1.357
2	1	1.966	1.537	1.279	1.246
3	1	2.498	1.931	1.294	1.559
4	1	2.339	1.671	1.400	1.000
5	1	2.487	1.800	1.382	1.096
6	1	2.285	1.316	1.737	1.554
7	1	1.737	1	1.737	1
8	1	2.265	1.868	1.212	1.465
9	1	2.521	1.600	1.576	2.053
10	1	2.804	2.015	1.391	1.866
11	1	2.813	2.022	1.391	1.876
12	1	2.515	1.496	1.681	1.139
13	1	1.228	1.054	1.165	1.095
14	1	2.941	1.701	1.729	2.001
15	1	2.518	1.519	1.657	1.721
16	1	2.339	1.505	1.554	1.606
17	1	1.769	1.393	1.270	1
18	1	1	1	1	1
19	1	2.072	1.228	1.687	1.377
20	1	1.442	1.442	1	1
21	1	1.424	1.415	1.006	1
22	1	1.965	1.259	1.561	1
23	1	1.833	1.282	1.430	1
24	1	1.507	1.075	1.402	1.119
25	1	2.448	1.595	1.535	1.347
26	1	1.941	1	1.941	1.267
27	1	1.772	1	1.772	1

It can be seen that DMU17 is inefficient in both stages while its overall CCR efficiency score equals to one. However, $\beta^* = 1.769$ indicates that DMU17 is inefficient with respect to the overall performance.

5. Conclusions

The current paper shows the equivalence between two DEA approaches for measuring the performance of two-stage processes. The two-stage process has a unique feature that the first stage's outputs are the only inputs to the second stage. It is shown that the constant returns to scale version of the Chen and Zhu (2004) model is equivalent to the output-oriented Kao and Hwang (2008) approach. Since the Kao and Hwang (2008) approach is based upon constant returns to scale, both the input- and output-oriented models yield equivalent results. As a result, the Chen and Zhu (2004) model is equivalent to the Kao and Hwang (2008) model in determining the overall efficiency score of the two-stage process. The Kao and Hwang (2008) approach further provides an efficiency decomposition for the two individual stages.

We note that under the condition of variable returns to scale, $\theta^1_{j_0} \cdot \theta^2_{j_0}$ no longer equals to $\frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}$, because of the free variable in the related DEA model. As a result, the variable returns to scale version of Kao and Hwang's (2008) model cannot be modeled as in model (2). The proven equivalence between the two approaches sheds lights on possible ways to developing the variable returns to scale version of Kao and Hwang's (2008) model.

We note also that both approaches assume that intermediate measures are the only inputs to the second stage. This imposes some limitations of these two approaches in use. It is likely that inputs (not as the outputs from the first stage) exist for the second stage, for example, in a supplier-manufacturer supply chain structure where the manufacturer (as the second stage) can have its own inputs. This can be a further research on how to extend the two approaches to deal with such situations.

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