Decision Support

Goal congruence analysis in multi-Division Organizations with shared resources based on data envelopment analysis

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\textbf{A B S T R A C T}

In multi-division organizations, goal congruence between different divisions and top management is critical to the success of management. In this paper, drawing upon a nonparametric framework to model production technology, we derive a necessary and sufficient condition for a firm with multiple divisions to be goal-congruent, and then extend it to a goal congruence testing measure, which coincides with a data envelopment analysis (DEA) model. The goal congruence measure not only shows empirically whether the firm is goal-congruent or not, but also provides a measurement for the degree of goal incongruence. To be goal-congruent, resources shared among divisions are suggested to be allocated so that the conditions for an optimizable operation are satisfied. In addition, goal-congruent firms are verified to be cost efficient. All findings in this research are examined and illustrated with a dataset of 20 bank branches with shared resources for service and sales divisions.

As a key issue in determining the fit between individuals and organizations, goal congruence has long been one of the central problems in the organization theory (Kristof, 1996; Vancouver, Mishap, & Peters, 1994). Under the setting of a large firm that consists of one central office and many functional divisions, management accounting literature has recognized the important role that the divergence of interests between top management and division managers and the private information of division managers concerning their work play in the budgeting process (Baiman & Rajan, 2002; Brown, Fisher, Sooy, & Sprinkle, 2014; Douthit and Stevens, 2015; Fisher, Maines, Peffer, & Sprinkle, 2002; Harris, Kriebel, & Raviv, 1982; Heine, Ross, & Saouma, 2014; Rajan & Reichelstein, 2004; Van der Stede, 2011). In particular, when resources are shared among functional divisions in large firms, budgeting becomes more complicated, because it is hard to evaluate a division's operation due to the fact that consumed shared resources cannot be observed. Studying the issues related to allocate the cost of shared resources will help budgeting systems function better in practice (Cook, Hababou, & Tuenter, 2000; Ding, Feng, Bi, Liang, & Khan, 2014).

Schaffer (2007) identifies four types of goal congruence in organizations, namely, constructive goal congruence, constructive goal incongruence, destructive goal congruence, and destructive goal incongruence. “Constructive” and “destructive” indicate whether goals are clearly communicated (i.e., constructive) or not

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(i.e., destructive). “Congruence” and “incongruence” mean whether front-line employees buy the top management’ goals (i.e., congruence) or not (i.e., incongruence). Two examples are shown here to offer a better understanding of “destructive” and “constructive” goal incongruence respectively. In one example, taken from Schaffer (2007), the top management of a company sets up a new goal for improving product quality, but the division management are still enjoying the initial success in reducing operating costs. Additionally, they may think achieving the new goal would incur additional costs. Without a clear emphasis on the goal changing when communicating to front-end employees, employees may still work for the initial goal. Messages can be misunderstood as a result. The other example is taken from Früitticher, Stroud, Laster, and Yakhou (2005) about American United Life company. The ultimate goal of the company is that the internal rate of return achieves 12%. The company, however, notices that division managers are not willing to spend $250,000 for mass advertising that would bring in $650,000 of annual premiums because it would lead to the overrun of the division’s operating budget. If division managers would have bought the goal and spent the money, the organization’s goal could be accomplished easily.

The role of goal congruence is nontrivial. For the top management, the designs or strategies developed, without the agreement and compliance of low-level members, are often deferred and rejected (McKelvey & Kilmann, 1975). Yamoah (2014) suggests that goal congruence is very important to attain organization’s strategic objectives and ensure the coordination and motivation of all employees concerned. If goal incongruence is not stopped in time, it will encourage organizational actors to pursue individual goals at the expense of the official organizational objectives. Johnson, Pfeffer, and Schneider (2013) mention that agency conflicts between divisions affect the design of capital charge rates and asset-sharing rules in the two-stage capital budgeting decisions for shared investments. Empirical studies show that (1) employees’ job satisfaction, commitment, and intentions to quit are related with goal congruence (Vancouver & Schmitt, 1991), (2) well-communication (Meyers, Ricucci, & Lurie, 2001), trust and shared values (Scott & Gable, 1997), among others, can help to improve the congruent level, (3) goal congruence is important to attain organization’s strategic objectives (Yamoah, 2014), and is able to augment employees’ productivity (Zhang, Wang, & Shi, 2012) and enhance organizational performances (Ayers, 2015), (4) goal congruence in top management team mediates the relationship between CEO transformational leadership and vice president’s attitude (Colbert, Kristof-Brown, Bradley, & Barrick, 2008), and (5) goal congruence moderates the relationship between organizational politics and job performances (Witt, 1998).

If the congruence issue is to be treated formally, a measure for the congruence between top management and divisional managers is warranted. According to Edwards (1994), most commonly used operational measures of goal congruence are bivariate congruence indices such as algebraic difference and absolute difference, and the index of profile similarity. Then, the directly measured goal congruence indices are treated as explanatory variables for other dependent variables relevant to the employee or organization. Edwards (1994) examines problems with these congruence indices and presents an alternative approach that overcomes these problems. The proposed approach (i.e., response surface analysis) is widely applied to organizational and psychological studies (e.g., Taafelín, Schwarz, & Hasson (2017) and Human, Dirks, Delongis, & Chen, 2016). In all the indices, goal congruence is always measured in a qualitative way. For example, supervisors and subordinates are invited to rate a range of goals using a 7-point Likert rating scale, and similarity is calculated to represent the degree of congruence (Ayers, 2015; Colbert et al., 2008). As subjective rating has limits, in this study we develop a new quantitative method to measure the level of goal congruence between top management and division managers based on observational objective input and output data and let data speak for themselves. Furthermore, the proposed measure is able to suggest the sources of the goal incongruence.

In this study, we shall start with the characterization of the optimal operation of the overall firm and the individual divisions in it that consume shared inputs. It should be noted that the difference between ‘optimizable operation’ and ‘optimum operation’ in this study is not trivial and worthy of discussion. To claim the operation is optimal, one should at least specify the goal or goals of the operation and the potential best level or levels of the operation. Then, the data of operation can be compared with the best potential to determine whether the operation is optimal. However, in some situations one might not have enough information to answer whether the operation is optimal. Instead, the data might be sufficient to determine a week version of optimality, i.e., the existing evidence is not against the claim that the operation is optimal. The current study dubs the weak version of optimality as ‘optimizable operation’. We propose that the operation of a firm or a division will be optimizable if it can achieve the maximum profit, which indicates that this study specifies profit maximization as an operational goal to introduce the idea of testing goal congruence. Notice that profit maximization is not the end but the means of the study. One may argue that an organization might not pursue the single goal of profit. This is true. In general cases, it is suitable to reformulate the idea of the profit function as a value function and the value of the function equals the level of utility (e.g., revenue) minimizes the level of disutility (e.g., cost). It shall be clear that the only requirement for the proposed method to be applicable in the general cases is that the utility and disutility functions characterizing the goals of divisional and organizational managers all have a linear and additive structure.

Then it follows with the formal definition of goal congruence that the goal congruence exists when a specific firm under evaluation and its divisions can meet some specific conditions to achieve the maximum profit at the same time. It will be proven in this study that an optimizable firm with goal-congruent multiple divisions is equivalent to being efficient by means of a DEA model. Hence, we leverage the DEA evaluation model as a testing technique for goal congruence. The efficiency will be viewed as a measure for assessing the degree of goal congruence. In addition, an optimal allocation scheme of shared inputs is obtained from this DEA-based evaluation model, together with a possible improvement plan for goal congruence. At last, the goal congruence testing measure is investigated to be a powerful one in terms of discriminating capability compared with the cost efficiency definition in the literature (Cherche, Rock, Dierynck, Roodhooft, & Sabbe, 2013), and based on more relaxed assumptions than the existing literature (Cherche, De Rock, & Walheer, 2016; Varian, 1984).

The current research contributes the literature by introducing a quantitative assessment method for the organizational goal congruence and extending the functionality of the DEA technique. Firstly, the insight that the suggestions for a goal congruence improvement in terms of resource allocation and cost sharing are endogenous to DEA models is missing in the conventional application fields of DEA models. In addition, the insight is gained through the methods that are based on more relaxed assumptions than similar researches in the DEA literature. Secondly, the proposed method is based on objective input and output data in evaluating goal congruence, rather than subjective ratings together with difference or similarity measures that are the prevalent practices in the organization behavior and management accounting literature. Last but not the least, though based on the profit maximization as an operational behavioral objective, the proposed method is applicable if the optimizing behaviors of decision makers are defined as other objectives such as utility maximization. The only requirement of
our analyses is that the measures that decision makers attempt to optimize can be formalized in a linear and additive structure.

The rest of the paper is organized as follows. Section 2 reviews issues related to shared inputs allocation for goal congruence faced by organizations. After analyzing the optimizing behavior and goal-congruent behavior of firms, a DEA-based model for measuring goal congruence is developed in Sections 3 and 4. The difference between the proposed goal congruence measure and the cost efficiency measure in literature is compared in Section 5. An empirical testing for a set of bank branches with shared inputs is provided in Section 6. Conclusions and discussions are presented in Section 7.

2. Literature review

In this paper, the goal congruence issue is investigated in an organization environment with shared resources. Since the literature about goal congruence has been introduced in the preceding section to motivate the current work, this section is dedicated to introduce the literature about shared resources allocation and the research technique in an effort to clarify our contribution to this line of research.

Shared resources are those that are simultaneously used for the multiple production units (divisions) with multiple outputs (Nehring & Puppe, 2004). Blanket advertisements issued by a corporation for all dealerships (Cook & Kress, 1999), and supporting staff for all bank branch divisions (Cook et al., 2000) are cases in point. Shared inputs allocation is a complex task, which can also be viewed as a goal congruence problem due to incompatible goals of the top managers and division managers. Top management optimizes the shared resource allocation among multiple divisions to attain the overall maximum profit. By contrast, the division managers are not willing to bear the cost of shared inputs for maximizing its own division’s profit. Let us consider a manufacturer who undertakes remanufacturing operation (Toktay & Wei, 2011), the division responsible for the new product manufacturing and selling is quite different from the one in charge of the subsequently product remanufacturing. Therefore, the question is how to allocate the cost of inputs, such as materials and parts, between two divisions when new products are produced. The goal of one division should be congruent with the other to achieve the firm-wide optimal solution.

In the literature, shared inputs are present in production systems with either multiple parallel components or multiple serial stages (e.g., Beasley, 1995; Chen, Du, David Sherman, & Zhu, 2010; Cook et al. 2000). Both in parallel and serial production divisions, shared inputs are split into multiple pieces and then used in all divisions respectively. Compared with parallel production systems, the only difference is that the output from one division becomes an input to a subsequent one in serial production systems. For instance, Chen et al. (2010) study banks with deposit and investment activities in serial. Deposit dollars generated from the deposit activity becomes the input of investment activity. At the same time, fixed assets, IT investment and employees are consumed by both stages. By contrast, Cook et al. (2000) investigate banks with two parallel production activities, sales and service, where the support and other staff are viewed as shared inputs. Another typical case is universities with two parallel components, i.e., research and teaching, modeled by Beasley (1995). In this scenario, the equipment expenditure is associated with both components.

DEA has been used extensively for resource allocation and cost sharing. DEA is a non-parametric frontier model which uses the mathematical programming approach to evaluate the relative efficiency of peer decision making units (DMU) with multiple inputs and outputs (Charnes, Cooper, & Rhodes, 1978). In terms of the cost allocation of shared inputs, Cook and Kress (1999) provide an equitable allocation of shared costs based on the theoretical Charnes et al. (1978) (also known as CCR) DEA framework. Cook and Zhu (2005) develop a fixed-cost allocation approach based on a DEA formulation, in which the fixed cost is modeled as an additional input variable. Both Cook and Kress (1999) and Cook and Zhu (2005) assume that the original efficiency of DMUs should remain unchanged after the allocation. In addition, Chen and Zhu (2011) propose a DEA approach for minimizing the risk during resource allocation process. However, most DEA models for resource allocation unvaryingly take a DMU as a whole, and aim primarily to maximizing overall efficiency without considering the behavioral issue within the DMU.

More recently, many studies focus on the behavioral issue in the resource allocation and cost sharing decisions. Du, Cook, Liang, and Zhu (2014) study allocating fixed cost in a competitive and cooperative situation based on cross-efficiency concept. In the study a cross-efficiency DEA-based iterative method is proposed and further extended into a resource allocation and generates an allocation scheme more acceptable to the players involved. Feng, Chu, Ding, Bi, and Liang (2015) point out that a centralized allocation plan suffers from an implementation difficulty in persuading DMUs into an agreement, and propose a new two-step method to mitigate this side effect. Bogetoff, Hougaard, and Smlilgren (2016) deal with the empirical computation of Aumann-Shapley cost shares, which is a cost allocation method designed for regulation of multi-production natural monopolies as well as for internal cost accounting and decentralized decision making in organizations. Afsharian, Ahn, and Thannassoulis (2017) propose a new DEA-based system for incentivizing operating units to operate efficiently for the benefit of the aggregate system of units to incentivize the units to operate efficiently.

Recognizing the limitations about only obtaining overall efficiency measures in DEA models, Cherchye et al. (2013) opened the “black box” of efficiency measurement with shared inputs. In their work, the overall efficiency value is decomposed into division-specific efficiency values, so that managers can clearly focus on remediating the observed inefficiency. Furthermore, Cherchye, Demuynck, De Rock, and De Witte (2014) adopted a DEA method to model the behavior of divisions about how to choose division-specific inputs and shared inputs to minimize the total costs of the firm. By leveraging the cost efficiency analysis framework in both Cherchye et al. (2013) and Cherchye et al. (2014), we derive a DEA model based goal congruence measure for firms with multiple divisions. We find that cost efficiency defined in the previous studies is a much weaker concept in terms of discriminating capability compared with the DEA model based goal congruence measure proposed in this study. Cherchye et al. (2016) extend their previous studies to the analyses of profit efficiency. First, the paper proposes a Banker et al., 1984 (also known as BCC) like model, which is not consistent with the basic assumptions of the current study. Importantly, the results of the current study are based on more relaxed conditions. Second, the paper focuses on the analyses of profit efficiency, while profit efficiency is the means but not the ends to analyze goal congruence of multidivisional organizations in the current paper. Finally, as Cherchye et al. (2016) has not been compared with Cherchye et al. (2013, 2016), the comparison in the current paper sheds some light on this.

Finally, the link of the current paper to Varian (1984) should be acknowledged here, which reviews the relevant results from both Afrafat (1972) and Hanoch and Rotschild (1972). The condition named WAPM in Varian (1984) specifies the condition of profit maximization that is similar to the result obtained in the current study. However, the result in the current paper relaxes the required premise of WAPM. Relying on the profit maximizing condition that operationalizes the behavioral goal of an organization, the current study gives conditions of goal congruency.
3. Methodology

This part attempts to develop a goal congruence measure for firms with multiple divisions. Before defining this measure, we investigate the conditions that optimizable operations of firms and their subordinate divisions should satisfy. Then, these necessary conditions will be utilized to devise techniques for achieving our goal of empirically investigating goal congruence within an organization.

3.1. Background and notations

We consider firms (denoted as DMUs) that consume both division-specific inputs and shared inputs to produce multiple outputs in D distinct divisions (sub decision making units, SDMUs). The production process of a DMU with D divisions is shown in Fig. 1. For n DMUs, each DMUj (j = 1, 2, … n) has D divisions, SDMUjd (d = 1, 2, … D). Let us take two divisions as an example. Cook et al. (2000) study the sales and service functions within the branches of a bank. The variable xjd = (xjd1, xjd2, …, xjdmjd) indicates mdj inputs dedicated to SDMUjd. yj = (yj1, yj2, …, yjdj) indicates l outputs produced by all SDMUjd (d = 1, 2, … D). ydj = (yjd1, yjd2, …, yjdmjd) indicates tdj outputs produced exclusively by SDMUjd (d = 1, 2, … D).

We also define the price vector for division-specific resources xjd as vjd = (vjdmjd) (d = 1, 2, … D). Vj = (Vj1, Vj2, …, Vjdj) is the price vector for shared inputs. The price vector for output products ydj is defined as udj = (ujd1, ujd2, …, ujdmjd). However, all prices for input and outputs are assumed to be unknown, and the shadow prices shall be determined in the sequel. Just as Chen and Zhu (2011) notice that price information can often be incomplete in practice. Thus, the models are particularly suitable for applications where price information is totally absent or partially available. In the latter case, the prices defined here can be viewed as the relative importance of the corresponding inputs (or outputs) with respective to the rest of the inputs (or outputs). For example, prices for inputs, e.g., transport, overheads costs of the studied company, are unknown in Cherchye et al. (2013), and prices for outputs of bank branches, e.g., retirement savings plan openings, mortgage accounts opened, are also not disclosed in Cook et al. (2000).

Production possibility set contains all input and output combinations such that the input (xjd, yj) that can produce ydj (d = 1, 2, … D) for DMUj. In this paper, we assume all observed data belong to the production possibility set. Furthermore, we assume that the production possibility set exhibits locally strong disposability, locally constant returns-to-scale (CRS), and convexity. Here, locally CRS implies (dx, dy) = δ(x, y) (δ > 0) is a possible direction, where e is a small positive quantity. Locally strong disposability means (dx, 0) and (0, dy) are feasible adjusting direction where dx ≥ 0 and dy ≤ 0. Below we refer to these assumptions as regular conditions.

The input requirement set of a DMUj (j = 1, 2, … n), given output vectors yj = (y1j, y2j, …, ynj), is defined as:

\[ I(y_j) = \{ x_{1j}, x_{2j}, \ldots, x_{nj}, X_j \} \]

In addition to the above regular conditions, for each output vector yj, the input requirement set is assumed to be closed. Before embarking on the modeling of optimizable operation for a multi-division firm, we firstly analyze a single division firm.

3.2. Single division firm

In firms with a single division, no shared inputs are involved, and top management is also the division manager. Thus, no incongruence issue within firm needs to be considered between the two parties.

The structure of a single-division production process is shown in Fig. 2. Here, xj = (xj1, xj2, …, xjmj) and yj = (yj1, yj2, …, yjtj) are inputs and outputs for DMUj. yj = (vj1, vj2, …, vjmtj) and uj = (uj1, uj2, …, ujmtj) are prices for inputs and outputs respectively.

In empirical applications, the production technology of DMUj is typically unobserved. In non-parametric production analysis, production technology is characterized through the observed set of input-output combinations. The input requirement set with respect to yj of DMUj in the one single division case is I(yj) = {xj ∈ Xj can produce yj}.

To simplify our analysis, we assume that profit maximization is the only goal of organizations. Even though an organization may emphasize multiple goals, such as profitability, market position, productivity, product leadership, personnel development, public responsibility (Feltham and Xie, 1994; Yamano, 2014), the main objective of a firm is profit maximization in the neoclassical theory of the firm (Beattie, Taylor, & Watts, 1985). We shall discuss the extension to other situations in the conclusion part of this paper. This assumption also holds in the analyses of the organizations with multiple divisions.

McFadden (1978) defined a revenue function as R(uj, xj) and a cost function as C(vj, yj), so the profit function is \( \pi = R(u_j, x_j) - C(v_j, y_j) \). Let us assume that revenue and cost functions are differentiable. Now let \( (x_0, y_0) \) be the input and output levels of DMU_0. If a DMU_0 gains the locally maximum profit, it follows that \( \partial \pi \mid_{x_0, y_0} = \partial R \mid_{x_0, y_0} - \partial C \mid_{x_0, y_0} \leq 0 \). This means that the profit can no longer be improved locally. We have the following theorem.

**Theorem 1.** With the regular conditions hold true, DMU_0 gains the maximum profit, if and only if there exist \( R_{x_0} = 0, C_{y_0} > 0 \), such that \( R_{x_0} x_0 - C_{y_0} y_0 = 0 \) and \( R_{x_0} x_j - C_{y_0} y_j \geq 0 \) hold for all \( (x_j, y_j) \) belonging to the production possibility set.
puts should

Proof. See the Appendix.

Note that the condition named WAPM from Varian (1984) says literally there exist a conical production set \( Y \) that \( p \)-rationalizes the data is equivalent to \( 0 = p_i y_i \geq p_i y_i \) (for all \( i, j = 1, \ldots, n \)). This is very similar to Theorem 1. However, the result in Varian (1984) is based on the definition of \( p_i y_i \geq p_i y_i \) (for all \( i, j = 1, \ldots, n \)) and the conical production set assumption, while Theorem 1 is based on the premise that \( d \pi x \leq 0 \) and DMU\(_0\) under examination possesses locally CRS and a convex production set. In other words, Theorem 1 requires the production bundle of DMU\(_0\) can be adjusted proportionately locally and the shape of the production possibility set including DMU\(_0\) and other DMUs is some convex set no matter it is a cone or not. Therefore, Theorem 1 relaxes the required premise in using WAPM. Now we proceed to define an optimizable operation of DMU\(_0\). Since attaining maximum profit is the primary organization-level goal, DMU\(_0\) is optimizable if its operation in terms of inputs and outputs is consistent with profit maximization, which is stated in detail in Definition 1.

Definition 1. For all the production units DMU\(_j\) (\( j = 1, 2, \ldots, n \)) with inputs-outputs combinations \( (x_j, y_j) \), we say that DMU\(_0\) is optimizable if it can achieve maximum profit, in other words there exists a price vector \( (R_{x_0}, C_{y_0}) \), that satisfies \( R_{x_0}x_0 - C_{y_0}y_0 = 0 \), \( R_{x_0}x_j - C_{y_0}y_j \geq 0 \) (\( j \neq 0 \)), \( R_{x_0} \geq 0 \), \( C_{y_0} \geq 0 \).

In the literature, theCCR DEA model for DMU\(_0\) is represented as 
\[
\max z = \frac{\sum_j \omega_j x_{0j}}{\lambda_0 \sum_j \frac{\mu_j y_{0j}}{\lambda_0}}, \text{ s.t. } \frac{\mu_j y_{0j}}{\lambda_0} \leq 1, j = 1, 2, \ldots, n, \text{ where } (\mu, \omega) \geq 0. \text{ Then, it is easy to see that the condition for the operation of a DMU to be optimizable is equal to the condition for the DMU to be } CCR \text{ efficient. It is worthy of noting that hereafter the "CCR efficient" qualification means the Farrell efficiency. It is also known as "weakly efficient".}

Proposition 1. An optimizable DMU\(_0\) is CCR efficient, vice versa.

Proof. See the Appendix.

In summary, we conclude that DMU\(_0\) is optimizable if it can achieve profit maximization, and if DMU\(_0\) achieves the maximal profit, DMU\(_0\) is CCR efficient or efficient in short. If a DMU cannot be optimized, it is inefficient in production. Proposition 1 provides a new perspective to the understanding of the CCR efficiency in the literature.

3.3. Multi-division firm

In multi-division firms, \( D \) distinct divisions consume both division-specific inputs and shared inputs to produce multiple outputs for the company. In turn, the cost of shared inputs, of course, should be shared among all divisions.

An example of multi-output production process is shown in Fig. 1. For the purpose of illustration, we use Fig. 3 where only the production process of one SDMU is shown. SDMUDMUs\(_d\) (\( d = 1, 2, \ldots, D \)) of DMU\(_j\) consumes both division-specific inputs \( x_{dj} \) and shared inputs \( X_j \) to produce outputs \( y_{dj} \). However, the cost of shared inputs \( \sum_{d=1}^{D} \frac{V_d^d X^s_j}{D} \) has to be allocated among all divisions, which is different from the single division case. Here \( V_d^d = (V_{d1}^d, V_{d2}^d, \ldots, V_{dj}^d) (d = 1, 2, \ldots, D) \) with \( \sum_{d=1}^{D} V^d_{dj} = V^d_j \) are the division-specific prices that SDMUDMUs\(_d\) of DMU\(_j\) would like to pay for the shared inputs. The total cost all SDMUs willing to pay equals to the market price of shared inputs from our modeling perspective. It should be noted that this implicit condition will be tested empirically in Section 4.

\[ V_d^d (d = 1, 2, \ldots, D) \] is also called implicit prices (Cherchye et al., 2013), which represents the fraction of the aggregated prices of the shared inputs that are borne by different production divisions. When consuming shared inputs, however, DMUs generally do not have the complete knowledge about the usage among divisions. Consequently, a mechanism is needed to split shared inputs across divisions. In this research, we use a ratio \( \beta_{dj} = (\beta_{d1}^j, \beta_{d2}^j, \ldots, \beta_{dj}^j) \) instead of \( V_d^d \), where \( V_d^d = \beta_{dj}^d V_d^d \) to represent the partial price that SDMUDMUs\(_d\) of DMU\(_j\) would like to pay for the shared input. The ratio is more flexible in a sense that when the quantity of a shared input, say \( X_j \), is of interest, we can apply allocation ratios to \( X_j \) to obtain the quantity that a SDMU consumes.

Assuming the profit function of a multi-division DMU takes the form \( \pi = \sum_{d=1}^{D} R^d_x X^d_j - \sum_{d=1}^{D} C^d_y Y^d_j, \) and is differentiable with respect to \( x_{dj}, y_{dj} \) given \( u_{dj}, v_{dj} \), and \( V^d_j \), we have Theorem 2 below by following the same logic of Theorem 1.

Theorem 2. With the regular conditions hold true, DMU\(_0\) gains the maximum profit, if and only if there exist \( R_{x_0}, C_{y_0}, \) \( R_{x_0} \geq 0 \), \( C_{y_0} \geq 0 \), \( C_{y_0} \geq 0 \) (\( d = 1, 2, \ldots, D \)) such that \( \sum_{d=1}^{D} R_{x_d} x_{0d} + R_{x_d} x_j - \sum_{d=1}^{D} C_{y_d} y_{0d} = 0 \), and \( \sum_{d=1}^{D} R_{x_d} x_{0d} + R_{x_d} x_j - \sum_{d=1}^{D} C_{y_d} y_{0d} \geq 0 \) hold for all \( (x_{dj}, y_{dj}) \) belonging to the production possibility set.

Here, \( R_{x_0}, R_{x_0}, C_{y_0} \) are first partial derivatives of the profit function with respect to \( X^d_j, x_{0j}, \) and \( y_{0j} \) respectively. On the basis of Theorem 2, we extend Definition 1 to a new definition of optimizable operation for a multi-division firm as follows.

Definition 2. For all the production units DMU\(_j\) (\( j = 1, 2, \ldots, n \)) with inputs-outputs combinations \( (x_{dj}, y_{dj}) (d = 1, 2, \ldots, D) \), DMU\(_0\) is optimizable if it can achieve maximum profit. In other words, there exists a price vector \( (R_{x_0}, R_{x_0}, C_{y_0}) (d = 1, 2, \ldots, D) \) that satisfies \( \sum_{d=1}^{D} R_{x_d} x_{0d} + R_{x_d} x_j - \sum_{d=1}^{D} C_{y_d} y_{0d} = 0 \), \( \sum_{d=1}^{D} R_{x_d} x_{0d} + R_{x_d} x_j - \sum_{d=1}^{D} C_{y_d} y_{0d} \geq 0 \) (\( j \neq 0 \)).

4. Measure of goal congruence

Suppose that the top management of a firm only cares about the maximum profit gained from the investment of shared resources, shared resources are allocated committing to this goal. However, due to divergence of interests between the top management and division managers and the fact that the benefits that a division derived from shared inputs is the private information of division managers, the allocation plan preferred by the top management might be in conflict with division managers' optimizable operation. In other words, goal incommensurate may be present between the top management and division managers.

To complete the task of testing quantitatively whether goal congruence between top management and division managers exists, our logic is to hypothesize that the division managers and top management are goal congruent in the first place. An implication of this hypothesis is that division managers are willing to accept any allocation plan in the best interest of the overall firm. Then, based on empirical evidence, we shall decide whether the congruence hypothesis is rejected or not. The definition of goal congruence between a DMU and its SDMUs is provided in Definition 3.
Definition 3. If a focal DMU and its SDMUs can achieve maximum profit at the same time, then they are goal congruent. SDMU\(_j\) with inputs-outputs combinations \((x_{d0}, x_{j0}^{X_1}, y_{j0}, x_{j0}^{X_2}, y_{j0}^{X_2})\) of DMU\(_j\) is optimizable if it can achieve maximum profit in the following sense: there exists a price vector \((R_{d0}, R_{j0}, C_{X_0})\) and \(\beta_{d0} = (\beta_{d0}^{X_1}, \beta_{d0}^{X_2}, \ldots, \beta_{d0}^{X_d})\) \((d = 1, 2, \ldots, D)\), such that \(R_{d0}x_{d0} + \beta_{d0}^{X_1}x_{d0}^{X_1} - C_{X_0}y_{d0} = 0, R_{j0}x_{j0} + \beta_{d0}^{X_1}x_{j0}^{X_1} - C_{X_0}y_{j0} \leq 0\) for \(j \neq 0\). \(\sum_{d=1}^{D} \beta_{d0}^{X_1} = 1, \beta_{d0}^{X_1} \geq 0\), for \(d = 1, 2, \ldots, D\). Here, \(\beta_{d0}^{X_1}x_{d0}^{X_1}\) indicates the partial cost that SDMU\(_d\) of DMU\(_j\) pay for the shared input.

Note that in the definitions of Definition 3, SDMU\(_d\) of DMU\(_j\) only compares with the corresponding SDMU\(_d\) \((j \neq 0)\) of other DMU\(_j\) \((j \neq 0)\). If the DMU and SDMUs are not goal congruent, there are two causes that prevent the DMU and SDMUs becoming optimizable simultaneously. The first is that the operations of SDMUs are not optimizeable themselves (denoted Cause 1 for latter reference). This is because they are inefficient in production. This explanation is the same as that in the single division setting. The other is that if the SDMUs can be optimizeable except for the overall DMU, and then there exist differences in input and output priorities and shared resources usage among the SDMUs and DMU (denoted Cause 2 for latter reference). We now proceed to models to capture quantitatively the magnitude of goal incongruence embodying those two causes.

Beasley (1995) proposes a DEA model assuming CRS for maximizing the overall efficiency, which can determine the joint efficiency of all production processes (see model (1)). Note that the term ‘overall efficiency’ here emphasizes it is the efficiency of the entire DMU consisting of many DMUs, while the term ‘joint efficiency’ highlights that the overall efficiency can be interpreted as the aggregation of SDMU efficiencies (i.e., the weighted average of SDMU efficiencies). Below, we explore the implication of this model for measuring the degree of goal congruence, which has not yet been discussed in the literature.

\[
\begin{align*}
\text{max } & \sum_{d=0}^{D} \omega_{d0}y_{d0} \\
\text{s.t. } & \frac{\mu_{d0}y_{d0}}{\beta_{d0}^{X_1}x_{d0}^{X_1} + \omega_{d0}x_{d0}^{X_1}} \leq 1, \quad j = 1, 2, \ldots, n \\
& \frac{\mu_{d0}y_{d0}}{\beta_{d0}^{X_1}x_{d0}^{X_1} + \omega_{d0}x_{d0}^{X_1}} \leq 1, \quad j = 1, 2, \ldots, n \\
& \vdots \\
& \frac{\mu_{d0}y_{d0}}{\beta_{d0}^{X_1}x_{d0}^{X_1} + \omega_{d0}x_{d0}^{X_1}} \leq 1, \quad j = 1, 2, \ldots, n \\
& \sum_{d=1}^{D} \beta_{d0}^{X_1} = 1 \\
& \omega_{0} \geq 0, \beta_{d0}^{X_1} \geq 0, \mu_{d0} \geq 0, \omega_{d0} \geq 0, \quad d = 1, 2, \ldots, D, \quad j = 1, 2, \ldots, n
\end{align*}
\]

(1)

In model (1), each DMU is evaluated in its most favorable light. Hence, each DMU is allowed to choose a set of \((\beta_{d0}, \omega_{d0}, \mu_{d0}, \omega_{d0})\) to maximize its efficiency score. The objective function of model (1) is an overall efficiency measure of DMU, the efficiency for each SDMU of DMU is measured by \(\mu_{d0}x_{d0}^{X_1} = \omega_{d0}x_{d0}^{X_1}\) \((d = 1, 2, \ldots, D)\). In other words, SDMU’s efficiency is defined based on a specific allocation of shared inputs among SDMUs. One should be cautious in understanding the phrase “SDMU’s efficiency”. As the realistic consumption data of shared inputs cannot be observed, it is difficult to evaluate the performance of a SDMU. Here, the efficiency of a SDMU is defined as a ratio of weighted outputs to weighted inputs. However, these weights are given aiming at putting the DMU as a whole, rather than the SDMU, in the best possible light. This is not quite consistent with the conventional definition of efficiency of a DMU in the DEA literature. Consequently, we refer to the objective value of model (1) as the overall efficiency of a DMU and the efficiency of a SDMU as the divisional efficiency of a SDMU. The use of overall efficiency instead of ‘CCR efficiency’ or efficiency in the multi-division case is to highlight the difference between model (1) and the classic CCR model. A DMU or SDMU will be overall or divisional efficient if the respective efficiency achieves 1.

By adopting “Charnes-Cooper” transformation (Charnes & Cooper, 1962), the objective function of model (1) can be expressed in a non-ratio form. In addition, we make the change of variables \(\tilde{\omega}_{0} = \beta_{d0}\omega_{d0}(d = 1, 2, \ldots, D)\), in which \(\tilde{\omega}_{0}\) has the same dimension with \(\omega_{0}\), and then model (1) is transformed into a linear model (2).

\[
\begin{align*}
\text{max } & \sum_{d=0}^{D} \bar{\omega}_{d0}y_{d0} \\
\text{s.t. } & \mu_{d0}y_{d0} \leq \tilde{\omega}_{d0}y_{d0}, \quad j = 1, 2, \ldots, n \\
& \mu_{d0}y_{d0} \leq \tilde{\omega}_{d0}y_{d0} \leq \omega_{d0}y_{d0}, \quad j = 1, 2, \ldots, n \\
& \vdots \\
& \mu_{d0}y_{d0} \leq \tilde{\omega}_{d0}y_{d0} \leq \omega_{d0}y_{d0}, \quad j = 1, 2, \ldots, n \\
& \omega_{0} \geq 0, \tilde{\omega}_{0} \geq 0, \mu_{d0} \geq 0, \omega_{d0} \geq 0, \quad d = 1, 2, \ldots, D, \quad j = 1, 2, \ldots, n
\end{align*}
\]

(2)

In model (2), it is clear that there are differences in input and output priorities and shared resources usage for the SDMUs (represented in inputs and outputs weights) if the DMU and SDMUs cannot be optimizeable at the same time. The relationship between a DMU’s overall efficiency and SDMUs’ divisional efficiencies is revealed in Lemma 1.

Lemma 1. A necessary condition for DMU\(_j\) to be overall efficient is that all SDMU\(_d\) of DMU\(_j\) are divisional efficient.

Proof. See Appendix.

Lemma 1 supports the Cause 1 why the DMU and SDMUs cannot be optimizeable at the same time. The necessity of Lemma 1 shows that SDMUs are not efficient in production when the overall DMU is not optimizeable. A single division DMU is optimizeable if it is CCR efficient according to Proposition 1. However, an additional condition is required for multi-division firms. We summarize the condition in Proposition 2.

Proposition 2. The sufficient and necessary condition for DMU\(_j\) to be overall efficient is that it is optimizeable and multiple divisions are goal-congruent with it.

The non-sufficiency of Lemma 1 and Proposition 2 verify that there are conflicts in determination of input and output priorities and the shared resources usage (represented by the inputs and outputs weights in model (2)) even though the two SDMUs are efficient, which supports the Cause 2 why the DMU and SDMUs cannot be optimizeable at the same time. Therefore, if a DMU is optimizeable and its SDMUs are goal-congruent with it, both the DMU and all its SDMUs are able to attain their maximum profit. Then both the DMU and all its SDMUs are overall efficient. In turn, if a DMU is overall efficient, and then all its SDMUs are divisional efficient. Furthermore, the DMU and its SDMUs are congruent with each other. Hence, DMU’s overall efficiency is a good indicator to show whether its SDMUs are goal-congruent with it.
In addition to serving as an indicator of whether goal congruence is achieved, a DMU’s overall efficiency measure by model (1) is a measure of the degree of congruence. As a DMU’s overall efficiency is a weighted average of each DMU’s efficiency (under a specific shared input allocation scheme) (Beasley, 1995), SDMUs’ low efficiency level will lead to a smaller overall efficiency score. In this case, it is obviously that SDMUs and the DMU are goal incongruent. Furthermore, the lower the efficiencies of SDMUs, the lower the degree of congruence is.

The overall efficiency proposed here sheds light on the source of goal incongruence. For example, in model (1), the variables \( \beta_{0j} (d = 1, 2, \ldots, D) \) are treated as DMU-specific variables. It will be at the discretion of the DMU under evaluation to allocate shared inputs among its divisions. However, as long as one SDMU is not efficient under DMU’s allocation scheme for achieving the profit maximization goal, the overall DMU cannot be efficient. If there is no allocation scheme available to reconcile the profit optimizing behaviors of both SDMUs and DMU as a whole, goal in-congruence follows.

If SDMUs are to adjust their inputs in goal incongruent situation to achieve a goal congruent state, the extent to which a SDMU needs to make adjustment in order to be overall efficient is reflected in model (1). Treating the overall DMU efficiency as a quantitative measure for goal congruence constitutes a contribution of this research. The goal congruence measure, other than efficiency measure, offers a new insight to the optimal value of the weighted ratio efficiency in model (1). In the course of maximizing overall efficiency of a DMU, a set of optimal \( \beta_{ij} (d = 1, 2, \ldots, D; j = 1, 2, \ldots, n) \) is obtained. As a result, a measure of goal congruence between the DMUs and their SDMUs is provided, together with an optimal allocation scheme of the shared resources.

5. Relationship between goal congruence measure and cost efficiency

The allocation of shared inputs among divisions of a DMU has already been studied by Cherchye et al. (2013) and Cherchye et al. (2014). They propose a structural DEA approach to model cost minimizing behavior at the overall firm level when allocating shared inputs among divisions. A notion of cost efficiency is developed in these two studies. Adopting the notations in the current paper, we re-write the cost efficiency definition as Definitions 4 and 5 for both the single division and multi-division firms. Cost minimization and cost efficiency is an important topic of budgeting and management control as well. Hence, the difference between the overall efficiency indicated in model (1) and cost efficiency is further discussed in this section.

5.1. Single-division firm

Based on the cost efficiency defined by Cherchye et al. (2013), we have the following cost efficiency definition.

**Definition 4.** Let \( v_0 \) be the prices for \( y_0 \) and \( x_0 \) respectively, DMU\(o\) with a single division is cost efficient among all DMU\(j\) (\( j = 1, 2, \ldots, n \)) if and only if there exists prices \( v_0 \) that satisfy:

1. \( x_j \in l(y_j), \ j = 1, 2, \ldots, n; \)
2. \( v_0x_0 = \min_{j \neq o} v_0x_j, \) where \( j \notin \Phi_0 = \{ j | y_j \geq y_0 \} \).

In this definition, the first constraint guarantees that the inputs \( x_j \) can effectively produce the outputs \( y_j \) under price \( v_0 \), which indicates technology feasibility. \( v_0x_0 \) is the minimum cost of producing \( y_0 \) at a minimal cost under price \( v_0 \). For these DMU\(j\) with \( y_j < y_0 \), by definition, the cost \( v_0x_j \) incurred to produce \( y_j \) is less than or equal to \( v_0x_0 \) for producing \( y_0 \), otherwise more inputs for DMU\(j\) indicates inefficiency. Both of these two cases infer that DMUs with \( y_j < y_0 \) are meaningless for comparison purpose. Such is also in line with the free output disposability of production functions that less output never requires more input. Therefore, the set \( \Phi_0 = \{ j | y_j \geq y_0 \} \) is built to capture those DMU\(j\) that produce at least the output \( y_0 \). We know that an optimal DMU is CCR efficient according to Proposition 1. Furthermore, if a DMU is CCR efficient, it follows the Proposition 3 below.

**Proposition 3.** If DMU\(0\) is CCR efficient, and then DMU\(0\) is cost efficient.

**Proof.** See the Appendix.

However, if DMU\(0\) is cost efficient, CCR efficiency cannot always be inferred. For those DMU\(j\) having \( y_j \geq y_0 \), by setting a small input \( u_0 (u_0 \geq 0) \), which satisfies \( u_0y_j - u_0y_0 \leq (\min_{j \neq o} v_0x_j) - v_0x_0 \) ( \( j \in \Phi_0 \)), we have \( v_0x_0 - u_0y_0 \geq v_0x_j - u_0y_j \). There is no evidence that \( v_0x_0 - u_0y_0 = 0 \). Therefore, DMU\(0\) is CCR efficient cannot hold.

5.2. Multiple-division firm

**Definition 5.** For a multiple-division DMU\(o\) with inputs-outputs combinations \( (x_0, X^o_0, y_0) \) \( (d = 1, 2, \ldots, D) \), it is multi-output cost efficient among all DMU\(j\) \( (j = 1, 2, \ldots, n) \) if and only if there exist a set of prices \( \{ v_0, v^o_0 \} \) and \( \beta_{0} = (\beta_{10}, \beta_{20}, \ldots, \beta_{D0}) \) \((d = 1, 2, \ldots, D)\) that satisfy:

1. \( (x_0, X^o_0) \in l(y_0) \)
2. \( v_0x_0 + \beta_{0} V^o_0 \leq \min_{j \neq o} (v_0x_j + \beta_{0} V^o_j), \sum_{d=1}^{D} \beta_{d0} = 1 \)

where \( j \notin \Phi_0 = \{ j | y_j \geq y_0 \}, \ d = 1, 2, \ldots, D \).

In this definition, the first constraint guarantees that the inputs \( (x_0, X^o_0) \) can effectively produce the outputs \( y_0 \) under the price \( (v_0, \beta_{0} V^o_0) \), which indicates technology feasibility. The second constraint ensures that every SDMU\(j\) of DMU\(0\) produce \( y_0 \) at a minimal cost under the price \( (v_0, \beta_{0} V^o_0) \). The constraint \( \sum_{d=1}^{D} \beta_{d0} = 1 \) confines the money paid for shared inputs to meet the total price requirement. Same as that in Definition 4, the set \( \Phi_0 = \{ j | y_j \geq y_0 \} \) indicates that only these SDMUs that produce at least \( y_0 \) are compared with SDMU\(0\). Note that each division SDMU\(j\) of DMU\(0\) should satisfies the second constraint of Definition 5. Thus there are actually \( D \) sub-constraints within the second constraint. If all SDMUs of DMU\(0\) are measured as cost efficient, it is obviously that DMU\(0\) is cost efficient. The cost efficiency measure proposed in Cherchye et al. (2013) is shown as follows.

\[
CE_0 = \max \sum_{d=1}^{D} \beta_{d0} c_{d0}^2 \\
\text{subject to} \quad v_0x_0 + \sum_{d=1}^{D} \beta_{d0} V^o_0 \leq \sum_{j \neq o} (v_0x_j + \beta_{0} V^o_j), \sum_{d=1}^{D} \beta_{d0} = 1
\]  

(3)

Note that parameters \( c_{d0}^2 \ (d = 1, 2, \ldots, D) \) in model (3) represent \( \min_{\beta_{d0}} (v_0x_0 + \beta_{0} V^o_0) \). It is easy to verify that cost efficiency measurement for a DMU in model (3) is equivalent to the cost efficient definition in Definition 5 when the DMU is cost efficient. First, if a DMU\(0\) is cost efficient as defined in Definition 5, then all its SDMUs are cost efficient, and we get \( c_{d0}^2 = v_0x_0 + \beta_{0} V^o_0 \). Thus the objective function of model (3) equals to one, and all constraints of model (3) are satisfied. Second, if \( CE_0 \) of model (3) equals to one, and then \( \sum_{d=1}^{D} c_{d0}^2 = \sum_{d=1}^{D} v_0x_0 + \beta_{0} V^o_0 \). Suppose there exists one \( c_{d0}^2 \) for SDMU\(j\) equal to \( v_0x_0 + \beta_{0} V^o_0 \) other than \( v_0x_0 + \beta_{0} V^o_0 \). It follows that \( c_{d0}^2 < v_0x_0 + \beta_{0} V^o_0 \), and \( \sum_{d=1}^{D} c_{d0}^2 = \sum_{d=1}^{D} v_0x_0 + \beta_{0} V^o_0 \), which contradicts that \( CE_0 \) of model (3) equals to one. Through the contradiction, we know that
if the objective function value of model (3) equals to one, for every
\( c_i \), we get \( c_i^d = \omega_0 X_{d0} + \beta_d X_{d1} \), which infers that DMU_0 is cost
efficient according to Definition 5.

Since an optimizable DMU with goal-congruent multiple
divisions has already proven to be overall efficient in Proposition 2,
the relationship between the overall efficiency and cost efficiency
for multi-division DMU is further revealed in Proposition 4.

**Proposition 4.** If DMU_0 with multiple divisions is overall efficient,
and then DMU_0 is cost efficient.

**Proof.** See the Appendix.

For the same reason with Proposition 3, cost efficiency cannot
derive overall efficiency. In summary, overall efficiency is equiva-
lent to the optimization both for DMUs with a single division and
multiple divisions. And the cost efficiency defined in Definitions 4
and 5 can be inferred from the overall efficiency, but the con-
verse is not true. Therefore, the cost efficiency defined in Cherchye
et al. (2013) is a much weaker concept compared with the overall
efficiency.

6. Empirical application

In this section, we take a dataset of 20 branches from a ma-
jor Canadian bank as in Cook et al. (2000). All branches are cho-
sen from one district. The transactions of banks are assumed to
be separated into two distinct classes: service and sales functions
(or called divisions in this study). Among them, examples for the
service function are counter transactions, and the sales function
involves the opening of the mutual funds for instance. What should
be emphasized is that the exact split of input resources between
these transactions, however, is not always possible. For example,
the technology investment of a bank branch serves both service
and sales functions. Each branch produces multiple outputs and
consumes multiple inputs including the shared resources. In Cook
et al. (2000), a best resource split is found by optimizing each
branch’s overall efficiency, while measuring the efficiencies of both
service and sales functions. In the current research, we not only
calculate the efficiencies of branches and their sub-functions, but
also arrive at a goal congruence measure for each branch. Whether
a branch shows optimized operation and its cost efficiency are all
discussed in this section.

The structure of the banking process is the same as that in Fig.
1. Each DMU_1 represents a bank branch with two SDMUs. SDMU_1
represents the service function, and SDMU_2 represents the sales
function. The inputs for DMU_1 are the number of service staff
\((x_{1j} = (x_{1j}^s))\), the number of sales staff \((x_{2j} = (x_{2j}^s))\), the number
of support staff \((X_{1j})\), and the number of other staff \((X_{2j})\). The
number of service and sales staff are dedicated inputs for service
and sales function, respectively, and the number of support staff and
other staff are shared inputs for both functions \((X_j = (X_{1j}, X_{2j}))\).

The outputs include the number of counter level deposits \((y_{1j}^c))
the number of transfers between accounts \((y_{1j}^t))\), the number of
retirement savings plan openings \((y_{1j}^r))\), and the number of mort-
gage accounts opened \((y_{1j}^m))\). The first two outputs are the prod-
ucts of the service function \((y_{1j} = (y_{1j}^c, y_{1j}^t))\), and the other two are
the outcomes of the sales function \((y_{2j} = (y_{2j}^c, y_{2j}^t))\). All inputs
and outputs of 20 branches are collected in one year period and
are displayed in Table 1.

6.1. Measure the degree of goal congruence

Based on Proposition 2, a DMU’s overall efficiency by model
(1) is not only an indicator of goal congruence, but also is a

<table>
<thead>
<tr>
<th>Branch</th>
<th>Dedicated inputs</th>
<th>Shared inputs</th>
<th>Service outputs</th>
<th>Sales outputs</th>
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<tr>
<td>x_{1j}</td>
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<td>X_{1j}</td>
<td>y_{1j}</td>
<td>y_{2j}</td>
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</table>
and sales functions are quite low, so they are far away from top management’s expectation. As a consequence, the goal-congruence value is 0.31, which is quite low. Moreover, an allocation plan of two shared inputs among two functions is shown in the last four columns of Table 2. Take branch 1 as an example, 9% of the support staff cost and 33% of the other staff cost are allocated to service function. Correspondingly, 91% of the support staff cost and 67% of the other staff cost are allocated to the sales function. Obviously, $\beta_{10}^1 + \beta_{10}^2 = 1$ and $\beta_{20}^1 + \beta_{20}^2 = 1$. This indicates that the total money the two functions would like to pay equals the market prices of shared inputs. The allocation scheme of branch 1 leads to the efficiency values 0.5 and 1.0 for the service and sales function respectively, followed by a value of goal congruence measure at 0.96.

### 6.2. Improvement of the degree of goal congruence

In Table 2, the last four columns are the ratios of shared inputs that could be allocated to service function and sales function respectively. These numbers are generated by Model (2). Under the allocation plan in Table 2, some branches are evaluated as efficient, but most of them are not. In other words, there is a goal incongruence issue between the division management and the top management in most branches. Goal-incongruent branches can get improved in the degree of goal congruence under a preferred allocation of shared inputs using the benchmarking point approach. It is an ideal allocation of the shared inputs so that the congruent state is able to achieve for each branch.

In this study, benchmarking points of each branch’s divisions are selected based on the dual of Model (2). Especially for the consumption of shared inputs in the benchmarking points, it is computed by multiplying the ideal consumption of shared inputs by the allocation ratios in Table 2. The specific amount of shared inputs allocated for each function division based on the ratios in the last four columns of Table 2 are shown in the second to fifth columns of Table 3. Correspondingly, the ideal benchmarking points for the functions of each branch are identified in the last six columns of Table 3.

Since each service (sales) function is only compared with other service (sales) functions, the benchmarking points are identified among homogeneous functions. For those branches that are not overall efficient or not goal congruent, they are suggested to make the adjustment of consumption of the dedicated inputs and/or shared inputs. Let us take the branch 1 as an example. The benchmarking points of service and sales function consume an equal number of dedicated inputs $x_{11}$ and $x_{21}$ as with their original amounts. Each function of the branch 1 does not need to adjust the dedicated inputs, but the shared inputs. For the branch 6, the service function is suggested to reduce the consumption of dedicated input from 0.883 to 0.315, and a reduction from 1.474 to 0.526 for the sales function. The allocation of shared inputs also needs to be adjusted according to the benchmark points. By contrast, for the original CCR efficient branches, i.e., 4, 10, 11, 15, they are not required to modify the consumption of dedicated inputs as shown in Table 3. In addition, they do not have to adjust the consumption of allocated shared inputs by the top management as well. In summary, if each branch function adjusts its consumption of dedicated inputs and/or shared inputs as that of the benchmarking point, both the service and sales functions, together with the overall branch will be efficient. Consequently, two functions are goal-congruent with the branch.

### 6.3. Measure of cost efficiency

We further analyze the relationship between the goal congruence measure and the cost efficiency. The cost efficiencies computed by Model (3) for these branches and their sub functions are shown in Table 4.

In Table 4, if both service and sales functions are cost efficient, then the branch is cost efficient. Comparing the second column of Tables 2 and 4, we find that if a branch is overall efficient, it will be cost efficient, but not vice versa. This finding is consistent with Proposition 4. Comparing the last four columns of Tables 2 and 4, we find that the allocation plans derived from models (1) and (3) are quite different. Since the goal congruence measure is a stronger concept compared with the cost efficiency measure, the shared inputs allocation scheme provided by model (1) is more applicable especially for a profit maximization goal rather than a cost minimization one.

### 7. Conclusions

In this research, we have attempted to shed light on the issues of goal congruence measure within multi-division organizations. The development of goal congruence measure becomes more complicated when production resources are shared among divisions. This research simplifies the organization goal as only pursuing profit maximization. Therefore, the optimizable behavior of
a firm is defined as being able to attain the maximum profit. Furthermore, goal congruence depicts a state when the overall firm and all divisions achieve the maximum profit at the same time. Finally, this goal-congruent state, under a nonparametric analysis framework, is tested to be equivalent to a DEA model with CRS assumption. Our goal congruence measure is represented by the overall DEA efficiency in this study. It is a quantitative measure that shows not only whether goals between top management and division managers are congruent, but also to what extent they are divergent.

Except for the quantitative goal congruence measure developed in this study, this research extends the functionality of DEA models. Goal congruence measure, in addition to efficiency measure, becomes a new explanation for the optimal value of weighted ratio efficiency in the original DEA model. The shared resource allocation issue is also successfully handled in the process of pursuing goal congruence. Goal congruence within organizations makes budgeting easier, and then top-down management control becomes possible. Employees, top management, and the firm itself will benefit a lot by measuring the degree of goal congruence in organization. Goal-congruent employees are motivated to work towards the top management’s strategic objectives, which make the success of the firm at last.

The goal congruence measure developed in this paper can be adapted to measure value congruence (Hoffman, Bynum, Piccolo, & Sutton, 2011), and person-organization fit (Kristof, 1996), which shall be pursued in the future. The current paper uses a profit function to characterize DMUs’ behavioral goal. In many non-profit situations, however, this assumption seems too strong. This shortcoming can be addressed by using a utility function instead of a profit function. The only requirement for this extension is that the utility function can be described in a linear and additive structure. The incorporation of utility function will make the goal congruence measure method proposed in this study more widely applicable.

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Appendix

A.1. Proof of Theorem 1

Proof. [ Sufficiency]

(1) If DMU0 shows locally DRS, i.e., (dx, dy) = δ(x, y)(δ ∈ (−ε, 0), ε is a small positive quantity), is a possible direction, then Rnx − Cny ≥ 0. (In this case, if Rnx − Cny ≠ 0, then Rnx + Cny(ε) > 0); If DMU shows locally IRS, i.e., (dx, dy) = δ(x, y)(δ ∈ (0, ε)) is a possible direction, then Rnx − Cny ≤ 0. (In this case, if Rnx − Cny ≠ 0, then Rnx + Cny(ε) < 0). To sum up, if DMU0 gains locally maximum profit, and the production technology of DMU0 exhibit CRS locally according to the regular condition, then there exist a feasible adjusting direction (dx, dy) which is proportional to (x, y). Therefore, it follows Rnx0 − Cny0 = 0 for DMU0.

(2) By the convexity of the production possibility set, any (xj, yj) of DMU0(j ≠ 0) and (x0, y0) of DMU0 can be connected by a line consisting of feasible points. This property implies that any observational datum (xj, yj) is equal to k(dx, dy) + (x0, y0), where k > 0 and (dx, dy) is a feasible changing direction at point (x0, y0). Hence, Rnxj − Cnyj = Rnx0 + k(dx, dy) − Cny0 = k(Rnxj − Cnyj) + (Rnx0 − Cny0) = Rnx0 − Cny0 ≤ 0. Note that for any feasible input and output bundle (x, y), (dx, dy) are feasible adjusting direction due to locally strong disposability where dx ≥ 0 and dy ≤ 0 according to the regular condition. Hence, if DMU0 gains the locally maximum profit, i.e., Rnx − Cny ≤ 0, then for all (dx, dy) we have Rnx = Rnx0 = 0, Cny = Cny0 = 0 satisfying Rnx − Cny ≤ 0. Therefore, it follows that there exist Rnx0, Cny0 ≥ 0 such that Rnx0 − Cny0 ≥ 0. This is equivalent to claim that there exist Rnx0, Cny0 ≥ 0 such that Rnxj − Cnyj ≥ 0.

[Necessity]

(1) If the production technology of DMU0 exhibit locally CRS according to the regular condition, then (dx, dy) is proportional to (x0, y0), which means (dx, dy) = δ(x0, y0)(δ ∈ (−ε, ε), ε is a small positive quantity), is a possible direction. Since Rnx0 − Cny0 = 0 for DMU0, it follows that Rnx + Cny = 0.

(2) Suppose (dx, dy) is any feasible changing direction at point (x0, y0) of DMU0, we obtain that any (xj, yj) of DMU0(j ≠ 0) equal to k(dx, dy) + (x0, y0), where k > 0, on the basis of convexity assumption of the production possibility set. Hence Rnxj − Cnyj = k(Rnx0 − Cny0). Since Rnx0 − Cny0 ≥ 0 when Rnx0, Cny0 ≥ 0, it is equivalent that there exist Rnx0, Cny0 ≥ 0, Rnx0 − Cny0 ≥ 0 such that Rnxj − Cnyj = Rnx0 − Cny0 ≤ 0. Thus there exist Rnx0, Cny0 ≥ 0, such that Rnxj − Cnyj ≥ 0. On summary, we derive that DMU0 gains the locally maximum profit. □

A.2. Proof of Proposition 1

Proof. [Optimizable ⇒ CCR efficient] If DMU0 is optimizable, according to Definition 1, we have Rnx0 − Cny0 = 0, Rnxj − Cnyj ≥ 0(j ≠ 0), Rnx0 ≥ 0, Cny0 ≥ 0. Then it is obviously that (Cny0, Rnx0) is the optimal solution for (μ∗, ω∗) in CCR DEA model, and z∗ = 1. Hence, DMU0 is CCR efficient.

[CCR efficient ⇒ optimizable] If DMU0 is CCR efficient among all DMUs, then there exists (μ∗, ω∗) ≥ 0 such that we have maxz=μ∗yω∗x=1, s.t. μ∗yω∗x≤1, j = 1, 2, ..., n. Hence, μ∗yω∗x = 0, and μ∗yω∗x ≥ 0. Therefore, there exists Rnx = α and Cny = μ∗, so that Rnx = Cny ≥ 0, Rnx = Cny ≥ 0 if (x0, y0). Rnx ≥ 0, Cny ≥ 0. According to Theorem 1, we know that DMU0 gets the maximum profit. In other words, DMU0 is optimizable. □

A.3. Proof of Lemma 1

Proof. The objective function of model (1) is maxz=0D=1μ∗αxω∗x=1, s.t. μ∗αxω∗x≤1, j = 1, 2, ..., D. When maximizing the objective function of DMUs, efficiencies of individual SDMU0 are measured by $z_0^D = \frac{\text{SDMU}_0}{\text{PR} \text{DMU}_0}$, where $z_0^D \alpha \omega_0 x^* + \omega_0 x^* > 0$, $d = 1, 2, ..., D$. According to findings of Beasley (1995), we have z = $\sum_{d=1}^{D} y_j^d z^d$, where $y_j^d = \frac{\text{PR} \text{DMU}_0}{\text{PR} \text{DMU}_0}$ and $\sum_{d=1}^{D} y_j^d = 1$. Since $z_0^D + \omega_0 x^* > 0$, all $y_j^d > 0$ for $d = 1, 2, ..., D$. Obviously, $\min_{j} z^d \geq z^d \geq \max_{j} z^d$.

We have a simple conclusion that the overall efficiency can always realize maximization, being equal to the higher one of the SDMUs' efficiencies, through controlling the values of $y_j^d$. Suppose $z^j_k = \max_{j} z^d$, if we want $z^d_k = z^j_k$, then there are two sets of solutions of $y_j^d$. One is that $y_j^d = 1$, and $y_j^d = 0(d \neq j)$.

A.4. Proof of Proposition 2

Proof. (1) [Optimizable DMU + goal congruent ⇒ Overall efficient] If DMU0 is optimizable, according to Definition 2, we have $D=1 ^{\sum D=1} R_{nx} x^D + R_{ny} x^D = 0$, $D=1 ^{\sum D=1} C_{nx} x^D \geq 0(d \neq 0), R_{nx} x^D + R_{ny} x^D = 0$. Since DMU0 and its SDMUs are goal congruent, i.e., SDMUs are also optimizable, we also get $D=1 ^{\sum D=1} R_{nx} x^D = 0$, $D=1 ^{\sum D=1} C_{nx} x^D = 0$, $D=1 ^{\sum D=1} R_{nx} x^D + R_{ny} x^D = 0$, $D=1 ^{\sum D=1} C_{nx} x^D = 0$.

(2) [Overall efficient ⇒ optimizable DMU + goal congruent] If DMU0 is overall efficient among all DMUs, then there exists $\beta_{d 0}, \mu_{d 0}, \omega_{d 0}$ $(d = 1, 2, ..., D)$ and $\sum_{d=1}^{D} \beta_{d 0} = 1$, such that we have maxz=0D=1μ∗αxω∗x=1, s.t. μ∗αxω∗x≤1, d = 1, 2, ..., D, j = 1, 2, ..., n. Hence, $D=1 ^{\sum D=1} R_{nx} x^D + R_{ny} x^D = 0$, $D=1 ^{\sum D=1} C_{nx} x^D = 0$, and $\beta_{d 0} = \beta_{d 0} x^D + \omega_{d 0} x^D = 0$. If the second constraints are summarized over all SDMUs, we get $D=1 ^{\sum D=1} R_{nx} x^D + R_{ny} x^D = 0$, $D=1 ^{\sum D=1} C_{nx} x^D = 0$, and $\beta_{d 0} = \beta_{d 0} x^D + \omega_{d 0} x^D = 0$. Therefore, there exist $C_{nx} x^D = 0$, $R_{nx} x^D = 0$, and $\beta_{d 0} = \beta_{d 0} x^D$. Combined
with \( R_{y}X_{y} + \beta_{0}y_{0}X_{0} - C_{g}y_{0} \geq 0 (d = 1, 2, \ldots, D) \), all DMUs of \( D \) are concordant. It is obviously that \( D \) and its SDMU are goal congruent. □

A.5. Proof of Proposition 3

Proof. If DMU0 is CCR efficient, then there exists \((\mu^{*}, \omega^{*}) \geq 0\) such that we have \( \max \{ \mu^{*}, \omega^{*} \} = 1, \forall j \mid \sum_{i} \lambda_{ij} = 1, j = 1, 2, \ldots, n \). Put it differently, \( \mu^{*} y_{j} - \omega^{*} x_{j} \leq \mu^{*} y_{0} - \omega^{*} x_{0} (j \neq 0) \) or \( \omega^{*} x_{j} - \mu^{*} y_{j} \leq \mu^{*} y_{0} - \omega^{*} x_{0} \). Besides, for \( j \in \Phi = \{ j \mid y_{0} \geq y_{j} \} \) we have \( \mu^{*} y_{j} - \omega^{*} x_{j} \leq \mu^{*} y_{0} - \omega^{*} x_{0} \) or \( \mu^{*} y_{0} - \omega^{*} x_{0} \leq \mu^{*} y_{j} - \omega^{*} x_{j} \). Hence \( \omega^{*} x_{j} - \mu^{*} y_{j} \leq 0 \) for these \( j \in \Phi \). Thus these exits \( y^{0} = \omega^{*} \) and \( u = \mu^{*} \) such that \( (x_{0}, y_{0}) = \arg \min_{j \in \Phi} y_{j} \), where \( j \in \Phi \) which means \( D \) is cost efficient. □

A.6. Proof of Proposition 4

Proof. Based on Lemma 1, we know that if DMU0 is CCR efficient, and then all SDMUj of DMU0 are CCR efficient. According to model (1), we have \( \omega y_{j} + \sum_{d} \sigma_{d} u_{d} \) and \( \omega_{0}y_{0} + \sum_{d} \sigma_{d} u_{d} = 0 \) and \( \beta_{0}y_{j} + \sum_{d} \sigma_{d} u_{d} = 0 \) for all SDMUj of DMU0. The constraints of model (1) are \( \beta_{0}y_{j} + \sum_{d} \sigma_{d} u_{d} = 0 \) for all \( d = 1, 2, \ldots, D \), and \( j = 1, 2, \ldots, n \), where \( \sum_{j} \sigma_{d} y_{j} = 0 \). For any \( d \in \{ 1, 2, \ldots, D \} \), we have \( \beta_{0}y_{j} + \sum_{d} \sigma_{d} u_{d} = 0 \) for any \( \mu_{0} \). There exist a set of prices \( (v_{0}, t_{0}, u_{j}) \) replacing \( (\omega_{0}, \mu_{0}, \lambda_{0}) \) such that \( \beta_{0}y_{j} + \sum_{d} \sigma_{d} u_{d} = 0 \), for all \( j \in \Phi = \{ j \mid y_{0} \geq y_{j} \} \) under a specification of \( (v_{0}, t_{0}, u_{j}) \). □

References


