Decision Support

Incorporating performance measures with target levels in data envelopment analysis

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A B S T R A C T

Data envelopment analysis (DEA) is a technique for evaluating relative efficiencies of peer decision-making units (DMUs) which have multiple performance measures. These performance measures have to be classified as either inputs or outputs in DEA. DEA assumes that higher output levels and/or lower input levels indicate better performance. This study is motivated by the fact that there are performance measures (or factors) that cannot be classified as an input or output, because they have target levels with which all DMUs strive to achieve in order to attain the best practice, and any deviations from the target levels are not desirable and may indicate inefficiency. We show how such performance measures with target levels can be incorporated in DEA. We formulate a new production possibility set by extending the standard DEA production possibility set under variable returns-to-scale assumption based on a set of axiomatic properties postulated to suit the case of targeted factors. We develop three efficiency measures by extending the standard radial, slacks-based, and Nerlove–Luenberger measures. We illustrate the proposed model and efficiency measures by applying them to the efficiency evaluation of 36 US universities.

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1. Introduction

Data envelopment analysis (DEA) is an approach for evaluating the relative efficiency of a set of (homogeneous) peer entities, called Decision-Making Units (DMUs), whose performance is characterized by a set of multiple performance measures. In DEA, these multiple performance measures are classified into inputs and outputs. Based upon the observed values on the multiple inputs and multiple outputs, DEA determines or estimates a production frontier (or best-practice frontier) of the underlying technology. For a more detailed discussion on DEA models and applications, the reader is referred to Cooper et al. (2011).

In conventional DEA models, it is generally assumed that a higher output level and a lower input level are preferred. Therefore, one DMU is deemed more efficient than another if that DMU produces larger amounts of outputs using the same amounts of inputs, or produces the same amounts of outputs using smaller amounts of inputs. Generally, inputs are of smaller-the-better type and outputs are of larger-the-better type in conventional DEA models. We refer to these types of inputs and outputs as regular, as they do not need any transformation or special treatment prior to DEA application. DEA inputs and outputs that require special treatment include undesirable factors. For example, pollution is an undesirable output and needs to be reduced (Seiford and Zhu, 2002). However, despite of the special treatment, these performance measures can still be classified as outputs or inputs.

Our current study is motivated by situations where performance measures can be classified as neither inputs nor outputs. Such performance measures have target levels, and every DMU strives to achieve the target levels for that type of factors. If a performance measure is below its target level for some DMUs, that measure needs to be increased and should be treated as an input. However, if the measure is above the target level for other DMUs, that measure needs to be decreased and should be treated as an output. The standard assumption made in conventional DEA models for inputs and outputs does not hold for these types of performance measures. We will refer to this type of factors as targeted. For example, suppose we are comparing and evaluating different kinds of diets or animal feeds. In addition to the cost of each diet or animal feed (which can be treated as inputs), we have measures such as the amount of protein, the mount of vitamin C, and the ratio of fat to carbohydrate, among others, contained in each diet or animal feed. The amounts of protein and vitamin C usually have generally accepted target levels, e.g., 5 grams of protein and 100 milligrams of vitamin C, no more and no less. Furthermore, it is typically required that the ratio of fat to carbohydrate should
be close to certain desirable target level, e.g., the ratio of total fat to carbohydrate contained in a diet should be around 1:3. As a result, these measures cannot be classified as either inputs or outputs in a single DEA run. Another example can be found in product and process evaluation. Most parts in mechanical fittings have designed (or desirable) dimensions and characteristics such as thickness, length, and density. Furthermore, products and processes typically have certain performance standards or benchmarks, and any variations to the standards are undesirable. Therefore, when several product and process designs are competing with each other to be chosen and implemented, a DEA model which can handle targeted measures or factors is required. Other examples of targeted factors include journal acceptance rate which is a factor affecting the journal backlog and quality reputation. Performance evaluation with consistency-based performance measure (i.e., for measuring consistent quality) is another example. For example, there is a target value for the specification of “door seal resistance” in the House of Quality proposed by Hauser and Clausing (1988). In ice cream production and packaging, there are both cost and government regulations that require the ice cream weight meets the specifica-
tions, and any deviation (either below or above) can result in customer complaints/lost sale or increased cost.

Such factors with targeted levels are called “targeted response” in Liu et al. (2006). Although Liu et al. (2006) point to the fact that such type of factors should be treated properly, they do not provide a model for dealing with it. Cook et al. (2006) and Cook and Zhu (2007) discuss performance measures called “flexible measures” that can simultaneously play both input and output roles. Their models classify DMUs according to whether a flexible measure is behaving like an input or output. Namely, such dual-role flexible measures are still classified as inputs or outputs, and the status of input or output does not change for a particular DMU under evaluation. However, for targeted factors, it is desirable to change their values only up (or down) to target levels, and any deviations from the target levels are undesirable. Therefore, the status of the targeted factors changes from inputs to outputs, or vice versa. For example, a DMU only keeps the status of a performance measure as an output until that measure reaches a target level, and then the output status changes to input once that measure’s value is larger than the target level. In a similar manner, an input can turn into an output once the input is decreased to a target level.

Taking into account the fact that lots of potential DEA applications exist with targeted factors being present, it is worthwhile to develop a DEA model in which such factors can be accommodated. Note that targeted factors can be treated under the ordinary production possibility set of conventional DEA models by simply transforming them into ones of regular type which basically represent their absolute deviations from targets; i.e., we can treat these absolute deviations as regular inputs where “smaller-the-better” applies. While this deviation-based approach can be a technically equivalent alternative to the one developed in this paper, it is just an ad hoc approach and it is not based on an explicit production correspondence suited for targeted factors. Furthermore, the deviation-based approach can be used only under the assumption that deviations above and below targets are equally undesirable. For situations where this assumption cannot be made (the two types of deviations are not the same), an asymmetrical treatment of the production correspondence is required. To address these issues, we provide an axiomatic foundation to the deviation-based approach for ensuring its validity in this paper.

The remainder of the paper is organized as follows. In Section 2, the production possibility set is formulated by extending the standard DEA production possibility set under variable returns-to-scale (VRS) based on a set of properties postulated to suit the case of targeted factors being present. Section 3 is devoted to the development of three efficiency measures by extending the standard radial, slacks-based, and Nerlove–Luenberger measures. In Section 4, the proposed model and efficiency measures are illustrated by an application to the efficiency evaluation of 36 US universities. Concluding remarks are provided in Section 5.

2. Production possibility set with targeted factors

Assume that there are n DMUs and each DMU produces s regular outputs using m regular inputs. Formally, DMU \( J \ (j = 1, 2, \ldots, n) \) uses a vector of inputs \( x_j = (x_{j1}, \ldots, x_{jm}) \in \mathbb{R}^m \) to produce a vector of outputs \( y_j = (y_{j1}, \ldots, y_{js}) \in \mathbb{R}^s \). In addition, each DMU involves producing or using targeted factors and each of the targeted factors has its own specific target level, denoted by \( t_k \in \mathbb{R} \ (k = 1, \ldots, p) \), which is common across all DMUs (i.e., the target level of each targeted factor is agreed upon by all DMUs). A vector of \( z_j = (z_{j1}, \ldots, z_{jn}) \in \mathbb{R}^n \) denotes the levels of DMU \( J \)'s targeted factors, and a vector of \( d_j = (d_{j1} - t_1, \ldots, d_{jp} - t_p) \) denotes the deviations of the levels of DMU \( J \)'s targeted factors from the target levels. \( X = (x_1, y_1, z_1) \) and \( Z = (z_2, \ldots, z_n) \) denote the input, output, and targeted factor data matrices, respectively, where each column represents one of DMUs and each row represents the level of one of factors of the corresponding DMU. A function \( I: \mathbb{R}^m \rightarrow \mathbb{R}^n \) is defined to represent the sign pattern of a given vector; for any vector \( w \in \mathbb{R} \), each element of \( I(w) \in \mathbb{R} \) assumes the value of 1, 0, or −1 depending on the sign of the corresponding element of \( w \). To make our exposition simpler, we impose a regularity assumption on the data; \( x_j \geq 0, y_j \geq 0, z_j \geq 0, x_j \neq 0, y_j \neq 0, z_j \neq 0 \forall j, \) and \( z_{ij} - t_k \neq t_k \forall k, j \), but the latter assumption on \( z_{ij} \) can be easily relaxed.

Modifying the standard VRS technology set introduced by Banker et al. (1984), we postulate the properties of the production possibility set \( P \) as follows:

1. **A1** The observed activities \((x_j, y_j, z_j) \ (j = 1, \ldots, n) \) belong to \( P \).
2. **A2** Given two activities \((x_j, y_j, z_j) \) and \((x_k, y_k, z_k) \) in \( P \) with \( I(d_j) = I(d_k) \), any convex combination of the two activities belongs to \( P \).
3. **A3** If an activity \((x, y, z) \) belongs to \( P \), any semipositive activity \((\bar{x}, \bar{y}, \bar{z}) \) with \( \bar{x} \geq x, \bar{y} \leq y \) and \( |\bar{z}_k - t_k| \geq |z_k - t_k| \), \( \forall k \) is included in \( P \).

Property (A1) is one of the basic assumptions typically made in any DEA models in the literature. Property (A2) is a modification of the standard convexity axiom. It assumes that the entire production possibility set consists of mutually exclusive subsets, and convexity condition is separately imposed on each of the subsets, not on the entire set. If we allow convexity on the entire set, it becomes possible that a convex combination of two DMUs results in a DMU dominating the two, which cannot be accepted in the usual production theory. (A2) prevents such situation while ensuring convexity of each subset.

Property (A3) is an extension of the standard free disposability axiom to accommodate targeted factors. Note that (A3) assumes that if an activity is feasible then another activity with a larger absolute deviation from the target, ceteris paribus, is also considered feasible. This property may or may not be relevant depending on the given evaluation context. If the property is postulated, the production possibility set becomes symmetric; otherwise, asymmetric. In what follows, we discuss each of the two cases.

2.1. Symmetric production possibility set with property (A3)

We now proceed to establish the production possibility set based on all of the three properties. First, we expand the set of the observed activities to effect (A3). Centered on \((x, y, z) = (0, 0, t)\), the factor space \((R^{m+s+n})\) can be divided into 2^p orthants, where
\( t = (t_1, \ldots, t_p)^T \). Let us denote the entire set of the observed activities by \( G \), and denote the subset of the observed activities positioned in the \( h \)th orthant by \( G_h (h = 1, \ldots, 2^p) \), and define the associated index set \( J_h \) such that any activity \( j \) in \( G_h \) has \( z_{ij} < t_k \) if and only if \( k \in J_h \). Note that \( G_1 = \{(x, y, z) \in G: z_{ij} \geq t_k \quad \forall k \} \) with \( J_1 = \{1, \ldots, p\} \). Therefore, \( G_{2,2} = \{(x, y, z) \in G: z_{ij} < t_k \quad \forall k \} \) with \( J_2 = \{1, \ldots, p\} \). Furthermore, each observed activity \((x, y, z) \in G \) can be associated with \((x, y, d_j) \) where \( d_j = z_j - t \) measuring the deviation of \( z_j \) from the target \( t \).

While keeping \( G_1 \) positioned in the first orthant, the other observed activities belonging to the other orthants are reflected onto the first orthant. In other words, \( G_0 (h = 1, \ldots, 2^p) \) is transformed to \( G^0 \) using the following Synthesis Process, where the superscript of \( 1 \) indicates that the reflection is made onto the first orthant:

**Synthesis Process** (Construction of \( G^1 \))

1. **initialize:** \( G^0_l := \phi \).
2. **for each** \((x, y, z) \in G^0 \):
   - **for each** \( k \), \( z_{kj} := \begin{cases} 2t_k - z_{kj}, & k \in J_h \setminus \{k\} \\ z_{kj}, & k \not\in J_h \end{cases} \).
   - \( G^1_k := G^0 \cup \{(x, y, z') \} \).
3. **return** \( G^1 \).

Note that \( G^0_1 = G \). Combining \( G^0_1 (h = 1, \ldots, 2^p) \), an expansion of \( G \), denoted \( G^0 \), is obtained by \( G^0 = \bigcup_{h=1}^{2^p} G^0_h \). Further, by reflecting \( G^0 \) in the first orthant back onto the other orthants, respectively, using the following Diffusion Process, we obtain \( G^0_l (l = 2, \ldots, 2^p) \):

**Diffusion Process** (Construction of \( G^0_l \))

1. **initialize:** \( G^0_l := \phi \).
2. **for each** \((x, y, z) \in G^0 \):
   - **for each** \( k \), \( z_{kj} := \begin{cases} 2t_k - z_{kj}, & k \in J_l \setminus \{k\} \\ z_{kj}, & k \not\in J_l \end{cases} \).
   - \( G^0_k := G^0 \cup \{(x, y, z') \} \).
3. **return** \( G^0_l \).

Arranging the expanded data set \( G^0_l \) in matrices \( X^0_l = (x^0_l), Y^0_l = (y^0_l), \) and \( Z^0_l = (z^0_l) \), where \((x^0_l, y^0_l, z^0_l) \in G^0_l \), we define the production possibility subset \( P_l (l = 1, \ldots, 2^p) \) by

\[
P_l = \{(x, y, z) \in R^{m+p} : x \geq X^0_l, y \leq Y^0_l, z \geq Z^0_l, e^T \lambda = 1, \lambda \\
\geq 0 \}
\]

where \( e^T \) represents a component-wise vector inequality defined such that \( \alpha^T \) \((h = 1, \ldots, p)\) is either \( \leq \) for \( h \in J_l \) or \( \geq \) for \( h \not\in J_l \). Note that \( P_l \) can also be obtained by reflecting \( P_{l-1} \) back onto the \( l \)th orthant via Diffusion Process (and then taking only the nonnegative part).

Finally, the (entire) production possibility set satisfying (A1)–(A3) is obtained by

\[
P = \bigcup_{l=1}^{2^p} P_l
\]

2.1.1. Illustration

To illustrate how to construct a production possibility set given a set of observed activities, we use an example in which there are 12 DMUs producing two targeted factors (denoted \( z_{ij} \) and \( z_{ij} \)) using a single regular input (denoted \( x_{ij} \)). The data set is summarized in Table 1. We assume that the two targeted factors have the target levels of 7 and 7, respectively (i.e., \( t_1 = 7, t_2 = 7 \)). The 12 observed activities are plotted in Fig. 1, where \( z^1 \) and \( z^2 \) are variables (and used as names for the corresponding axes) measuring the amounts of the two targeted factors, the thin horizontal and vertical lines show graduations of size unity, and the two red dotted lines indicate the target levels of \( z_{1j} \) and \( z_{2j} \). Note that we ignore the single input because its level is the same for all DMUs. Centered on \((7,7)\), the \((z^1, z^2)\)-space is divided into four orthants (or quadrants) and the set of the observed activities is classified into four subsets: \( G_1 = \{A, B, C\}, \ G_2 = \{D, E, F\}, \ G_3 = \{G, H, I\}, \) and \( G_4 = \{J, K, L\} \). Different colors are used to indicate data points in different orthants. Reflecting \( G_h (h = 1, \ldots, 4) \) onto the first orthant by Synthesis Process, we get the following:

<table>
<thead>
<tr>
<th>DMU</th>
<th>( x_{1j} )</th>
<th>( z_{1j} )</th>
<th>( z_{2j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>J</td>
<td>1</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>K</td>
<td>1</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
<td>12</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table 1** Example data set 1.

Fig. 1. Plot of observed activities (data set 1).
indicates the efficient frontier in the first orthant, denoted by $E_1$. Next, we obtain $P_l (l = 2, \ldots, 4)$ by reflecting $P_1$ onto the corresponding orthant via Diffusion Process. Combining all four $P_l$s using (2), we get the entire production possibility set $P$ which is depicted in Fig. 3. Note that the thicker red lines indicate the efficient frontiers and DMUs A, F, G, and J contribute to their construction. Each efficient frontier in the $l$th orthant is denoted by $E_l (l = 1, \ldots, 4)$.

Next, we use another example to illustrate the above procedure for a case where a mix of regular and targeted factors is involved. Suppose that there are 10 DMUs producing a single regular output (denoted $y_{ij}$) using a single targeted factor (denoted $z_{ij}$). The data set is summarized in Table 2. We assume that the targeted factor has the target level of 7 (i.e., $t_1 = 7$). The 10 observed activities are plotted in Fig. 5, where $z^1$ and $y^1$ are variables (and used as names for the corresponding axes) measuring the amounts of the targeted and output factors, the thin horizontal and vertical lines show graduations of size unity, and the red dotted line indicates the target level of $z_{ij}$.

Centered on the value of 7 on the horizontal axis, the $(z^1,y^1)$-space is divided into two orthants and the set of the observed activities is classified into two subsets: $G_1 = \{F,G,H,I,J\}$ and $G_2 = \{A,B,C,D,E\}$. Different colors are used to indicate data points in different orthants. Reflecting $G_{h,5} (h = 1, 2)$ onto the first orthant by Synthesis Process, we get the following:

---

**Table 2**

<table>
<thead>
<tr>
<th>DMU</th>
<th>$z_{ij}$</th>
<th>$y_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>G</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>H</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>I</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>J</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

---

**Fig. 2.** $E_1 (h = 1, \ldots, 4)$ and $P_1$ (data set 1).

**Fig. 3.** Symmetric production possibility set and efficient frontiers (data set 1).

**Fig. 4.** Asymmetric production possibility set and efficient frontiers (data set 1).

**Fig. 5.** Plot of observed activities (data set 2).
where $A'$ through $J'$ are the data points plotted in Fig. 6. Note that DMUs F through J actually stay the same (i.e., $F' = F$, $G' = G$, ..., $J' = J$) since they are in the first orthant and $f_1 = \phi$. To illustrate this reflection transformation, let us see DMU B in $G_2$ for which $j_2 = \{1\}$. Applying Synthesis Process, DMU B (2, 5), is reflected to $B'$, $(2 \times 7 - 2, 5) = (12, 5)$.

Fig. 6 also shows the production possibility subset $P_1$ (shaded, enclosed by the solid and dotted red lines) in the first orthant, which is constructed using (1) for $l = 1$. The thicker red line indicates the efficient frontier in the first orthant, denoted by $E_1$. Next, $P_2$ is obtained by reflecting $P_1$ onto the corresponding orthant via Diffusion Process (and then taking only the nonnegative part). Combining all two $P$s using (2), we get the entire production possibility set $P$ which is depicted in Fig. 7. Note that the thicker red lines indicate the efficient frontiers, $E_1$ and $E_2$, and DMUs A, C, F, and J contribute to their construction.

2.1.2. Discussion

Notice that some observed DMUs positioned in one orthant may contribute to the formation of the production possibility subset in another orthant due to (A3) through Synthesis and Diffusion processes, and vice versa. This interrelation renders the production possibility set symmetric. The symmetry of the production possibility set implies that deviations above and below the targets are considered equally undesirable. In this case, as noted in Introduction, a technically equivalent alternative would be treating these absolute deviations as regular inputs and the corresponding production possibility set would look like the one given in Figs. 2 and 6. However, if deviations above and below the targets need to be treated differently (i.e., unequally), the symmetry cannot be maintained. Asymmetry of the production possibility set implies that DMUs in different orthants are governed by different production technologies. In this case, property (A3) cannot be postulated.

2.2. Asymmetrical production possibility set without property (A3)

If property (A3) is omitted, the production possibility set becomes asymmetric and production possibility subsets have different shapes. Synthesis and Diffusion processes are not needed in this case, and the process of building the production possibility set becomes much simpler as follows.

Arranging the data set $G_l$ in matrices $X_l = (x_{lj})$, $Y_l = (y_{lj})$ and $Z_l = (-z_{lj})$, where $(x_{lj}, y_{lj}, z_{lj}) \in G_0$, we define the production possibility subset $P_l$ ($l = 1, \ldots, 2^p$) by

$$P_l = \{(x, y, z) \in R_+^{m+p} : x \geqslant X_l \lambda, y \leqslant Y_l \lambda, z \geqslant Z_l \lambda, \quad e^T \lambda = 1, \quad \lambda \geqslant 0\},$$

where $e^T \lambda$ represents a component-wise vector inequality defined such that $\lambda_{hj}^\lambda (h = 1, \ldots, p)$ is either $\leqslant 0$ for $h \in J$ or $\geqslant 0$ for $h \notin J$. The (entire) production possibility set satisfying (A1) and (A2) is obtained by

$$P = \cup_{l=1}^{2^p} P_l.$$ 

Notice that the efficient frontier of a production possibility subset has a different shape (or different deviations from the targets) with that of another subset. This means that deviations above and below the targets are treated unequally. For an illustration, Figs. 4 and 8 depict the production possibility sets and the corresponding efficient frontiers for data sets 1 and 2, respectively.

3. Efficiency measures

In this section we develop several efficiency measures that suit the production possibility set developed in the previous section. It is accomplished by adapting existing well-known efficiency measures including radial measures (Banker et al., 1984), the slacks-based measure (Tone, 2001), and the Nerlove–Luenberger (NL) measure (Chambers et al., 1998). Note that the three different efficiency measures are differentiated by their different metrics for measuring distances of DMUs from the efficient frontier. While radial measures assume proportional changes of factors, the slacks-based measure handles input or output slacks directly and does not assume proportional changes of factors. The two types of measures induce radial and non-radial DEA models, respectively. On the other hand, the NL measure can be considered as a generalization of radial measures, which allows an arbitrary choice of directions of changes of factors. Selection among the alternative efficiency measures depends on the characteristics of the
production processes under evaluation. Regardless of which one of
efficiency measures is selected, they provide an indication of ef-
ciency or the degree of inefficiency.

Before we proceed, we should make a fundamental assumption
on efficiency measures, which is required because there are multi-
ple efficient frontiers in different orhtants:

**Assumption 1.** An efficiency of a DMU is measured with reference
to its closest efficient frontier.

This assumption can be alternatively restated as efficiencies of
DMUs in $G_0$ are measured with only reference to $E_0$. For example,
an efficiency of DMU $E \in G_2$ in Fig. 3 is measured by assessing
how distant it is from $E_2$, and it does not refer to the other efficient
frontiers $E_1, E_3$, and $E_4$, although some observed DMUs in the other
orhtants contribute to the construction of $E_2$.

### 3.1. Radial measures

Radial measures, based on Shephard’s distance function, repre-
sent how much proportional (or radial) contraction of inputs and/
or proportional expansion of outputs is allowed for a DMU under
evaluation consistent with the underlying production technology.

Regular factor-oriented (regular input-oriented or regular output-
oriented) measures under the current production technology can
be defined exactly in the same manner with their uses in the models
of Charnes et al. (1978) and Banker et al. (1984). However, a proper
modification is needed for a targeted factor-oriented measure. To
illustrate the basic idea, see how a radial z-oriented efficiency of
DMU $G$ in Fig. 7 can be measured based on its radial distance from
the efficient frontier. Observe that if the radial distance is measured
with respect to the vertical $y^1$-axis where $z^1 = 0$, its efficiency tends
to be overestimated. In fact, as the target value $t_1$ increases, the
overestimation becomes more intensified. To avoid this effect, it
is easy to conceive that the radial distance needs to be measured
with respect to the vertical axis where $z^1 = t_1$ (indicated by a
dotted vertical line). Notice that conventional input-oriented ra-
dial DEA models involving only regular inputs can be considered
to assume the target level of zero for inputs.

To effect the above-mentioned consideration, we develop a tar-
gated factor-oriented radial DEA model under VRS for measuring
an efficiency of DMU $o \in G_i$ as follows:

$$(VRS_{T}) \quad \min \{ \theta : (x_o, y_o, \theta z_o + (1 - \theta) t_1) \in P_i \}.$$  

An efficiency score of $\theta^*$ is given to DMU $o$, where $\theta^* \in (0, 1]$ is
the optimal value of model (VRS$_T$). If a DMU is given an efficiency score
of unity, it is deemed efficient.

For example, in an illustration of the above efficiency measurement, see
Figs. 9 and 10 which show how radial efficiency scores of DMU $E$ from
the data set 1 and DMU $G$ from the data set 2 are determined
based on model (VRS$_T$). DMU $E$ in Fig. 9 can reach the frontier
reference point $E^0$ on $E_2$ by expanding its level of the first targeted
factor $z_2$, and contracting its level of the second targeted factor
$z_{2E}$ toward $T(t_1, t_2)$ with the common ratio of $\theta^*$: $E^* = \theta^* E + (1 - \theta^*) T$. Specifically, DMU $E$ is given a radial efficiency score of $\theta^* = \frac{1}{2}$
and is deemed inefficient. In the same manner, DMU $G$ in Fig. 10 can
attain efficiency (reaching the frontier reference point $G^*$ on $E_1$) by
contracting its level of the single targeted factor toward $T(t_1, \theta^*)$
(keeping the level of $y^1$ the same) with the ratio of $\theta^*$: $G^* = \frac{1}{2} G + (1 - \frac{1}{2}) T$. Specifically, DMU $G$ is given a radial efficiency score of $\theta^* = \frac{1}{2}$
and is deemed inefficient.

### 3.2. Slacks-based measures

Slacks-based or additive measures can also be developed to suit
the case of targeted factors being present. While both oriented and
non-oriented slacks-based measures can be developed, we only
deal with non-oriented ones. A slacks-based DEA model under
VRS for measuring an efficiency of DMU $o \in G_i$ is formulated as follows:

$$(SBM_{T}) \quad \min \{ \rho(s^i, s^j, s^k) : (x_o - s^i, y_o + s^g, z_o - s^k) \in P_i, s^j \geq 0, s^g \geq 0, s^k \geq 0 \}.$$  

where $s^j \in R^n, s^g \in R^m$, and $s^k \in R^p$ are slack variables, and $s^i$ repre-
sents a component-wise vector inequality defined as $s^i_k$ for $k = 1, \ldots, p$
is either $\leq$ for $k \in J$ or $\geq$ for $k \notin J$. The objective function $\rho$ is a function of $s^i, s^g, s^k$ of which optimized value represents
an efficiency score of DMU $o$. Following Charnes et al. (1985),
$\rho$ can take the form of

$$\rho(s^i, s^g, s^k) = - \sum_{i=1}^m s^i_i + \sum_{j=1}^n s^g_j - \sum_{k \notin J} s^k_k + \sum_{k \in J} s^k_k,$$

where $A^c$ represents the complement of a set $A$. In this case, model
(SBM$_T$) is referred to as the additive DEA model with targeted
factors. The optimized $-\rho^*$ has no upper bound while it is nonnegative,
and a DMU is deemed efficient if it is given an efficiency score $-\rho^*$
of zero. Alternatively, following Tone (2001), $\rho$ can take the form of
where \(|A|\) represents the cardinality of a set \(A\). Note that targeted factor slacks corresponding to \(f_i\) are similar to output slacks and those corresponding to \(f_i'\) are similar to input slacks. This is the reason of the arrangement of targeted slacks in \(\rho\). With this objective function, model (SBM\(_T\)) is referred to as the slacks-based DEA model with targeted factors. The optimized \(\rho^*\) takes a value between zero and one, and a DMU is deemed efficient if it is given an efficiency score of unity.

For an illustration, Figs. 11 and 12 show how slacks-based efficiency scores of DMU E from the data set 1 and DMU G from the data set 2 are determined based on model (SBM\(_T\)) with (5). DMU E in Fig. 11 can reach the frontier reference point \(E'\) on \(E_1\) by subtracting \((s_1')\) and \((s_2')\) from its current levels of the two targeted factors. Note that \((s_1')\leq 0\) and \((s_2')\geq 0\) since DMU E is in \(G_2\) and \(J_2 = \{1\}\). Specifically, \((s_1') = 0\) and \((s_2') = \frac{1}{2}\) minimize the objective function (5); \(\rho((s')^*) = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3}\). Likewise, DMU G in Fig. 12 can attain efficiency (reaching the frontier reference point \(G'\) on \(E_1\)) by subtracting \((s_1')\) from its current level of the single targeted factor and adding \((s_1')\) to its current level of the single regular output factor. Note that \((s_1')\geq 0\) since DMU G is in \(G_T\) and \(J_1 = \phi\). Specifically, \((s_1') = 0\) and \((s_1') = \frac{1}{2}\) minimizes the objective function (5); \(\rho((s')^*) = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{2}\).

3.3. The Nerlove–Luenberger measure

The NL measure is based on the directional distance function introduced by Chambers et al. (1996) which is defined, with respect to a technology \(P\) and DMU \(o\) under evaluation, as

\[
D(x_o, y_o, g^*, g^o) = \max\{\beta : (x_o + \beta g^*, y_o + \beta g^o) \in P\}
\]

While the direction \((g^*, g^o)\) can be chosen in an arbitrary way, the NL measure uses \((-x_o, y_o)\). With targeted factors being present and assuming DMU \(o \in G_T\) under evaluation, the NL measure can be modified as follows:

\[
\text{(NL\(_T\))} \quad D(x_o, y_o, g^*, g^o, g^r, g^r') = \max\{\beta \cdot (x_o + \beta g^*, y_o + \beta g^o, z_o + \beta g^r) \in P\}
\]

with \((g^*, g^o, g^r)\) being chosen to be \((-x_o, y_o, t - z_o)\). An efficiency score of \(1 - \beta^*\) is given to DMU \(o\) where \(\beta^*\) is the optimal value, and a DMU is deemed efficient if it is given an efficiency score of unity.

For an illustration of the directional distance DEA model with the NL efficiency measure, see Figs. 13 and 14 showing how efficiency scores of DMU E from the data set 1 and DMU G from the data set 2 are determined based on model (NL\(_T\))\(^2\) DMU E in Fig. 13 can reach the frontier reference point \(E'\) on \(E_1\) by improving its levels of the two targeted factors along direction \(g^r = t - z_o\) with step length of \(\beta^*\). Specifically, \(g^r = (7, 7) - (3, 11) = (4, -4), \beta^* = \frac{1}{2}\), and DMU E attains an efficiency score of \(1 - \beta^* = \frac{1}{2}\). In the same way, DMU G in Fig. 14 can attain efficiency (reaching the frontier reference point \(G^*\) on \(E_1\)) by improving its levels of the single output factor and the single targeted factor along direction \((g^*, g^o) = (y_o, t - z_o)\)

\(^2\) Recall that the data set 1 involves one input factor (ignored in the preceding discussion) in addition to two targeted factors and all DMUs have the same level of the input factor. Therefore, it holds \(\beta^* = 0\) for all DMUs under VRS. To avoid this and illustrate the proposed NL measure more effectively, we take \(g^* = 0\) instead of \(g^* = -x_o\).
with step length of $\beta^*$. Specifically, $(\beta', \beta^*) = (4.7 - 10) = (4, -3)$, $\beta^* = \frac{2}{3}$, and DMU G attains an efficiency score of $\left(1 - \beta'\right) = \frac{2}{3}$.

It is interesting to notice that, with the proposed modification of the NL measure, regular inputs and regular outputs can be treated as targeted factors. Specifically, if regular inputs are treated as targeted factors with target levels of all zeros and regular outputs are treated as targeted factors with target levels of $2y_0$, then the same NL measure will result.

### 4. An illustrative application to university evaluation

In this section, we demonstrate how our proposed approach can be used to evaluate 36 top universities in the US in terms of efficiency and perceived research and teaching quality. Our data are based upon the 2011 rankings provided by the US News and World Report and 2010 ranking of top US research universities (http://mup.asu.edu/research_data.html). The data is provided in Table 3. Due to the data availability, we only obtain 36 universities data based upon the following selected performance measures.

**Inputs:**
1. Tuition. We use out-of-state tuition for state universities;
2. Student-to-Faculty ratio. This is the ratio of full-time-equivalent students to full-time-equivalent faculty during the fall of 2010. A lower student-to-faculty ratio scores higher in the US News and World Report ranking model than a higher ratio.

**Outputs:**
3. Total annual number of faculty awards.

**Targeted:**
4. Acceptance rate; and 5. Percentage of classes of sizes under 20.

These selected regular inputs and regular output have been used in other studies, see, e.g., Breu and Raab (1994) and Colbert et al. (2000). The targeted measures (iv–v) have never been used in other studies. The acceptance rate is equal to the total number of students admitted divided by the total number of applicants. A lower acceptance rate indicates a school is harder to get into and a higher acceptance rate indicates a school is easier to get into. While the US News and World Report ranks a university with lower acceptance rate higher, our survey of about 45 undergraduate students at a university indicates that not all students prefer a lower acceptance rate. If a university gets more applications, it can...

**Table 3** University data and efficiency scores.

<table>
<thead>
<tr>
<th>Univ.</th>
<th>Tuition</th>
<th>Student/faculty ratio</th>
<th>Faculty awards</th>
<th>Accept rate (%)</th>
<th>% of Classes under 20</th>
<th>Targeted factor-oriented radial SBM NL</th>
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Fig. 14. NL efficiency measure (for DMU G from data set 2).

These selected regular inputs and regular output have been used in other studies, see, e.g., Breu and Raab (1994) and Colbert et al. (2000). The targeted measures (iv–v) have never been used in other studies. The acceptance rate is equal to the total number of students admitted divided by the total number of applicants. A lower acceptance rate indicates a school is harder to get into and a higher acceptance rate indicates a school is easier to get into. While the US News and World Report ranks a university with lower acceptance rate higher, our survey of about 45 undergraduate students at a university indicates that not all students prefer a lower acceptance rate. If a university gets more applications, it can...
result in a low acceptance rate. Also, some universities encourage unqualified candidates to apply in order to lower acceptance rates. Therefore, we believe it is appropriate to treat acceptance rate as targeted. Like the journal acceptance rate, one would not expect the acceptance rate dropping very close to zero (as an input) or increasing to 100% (as an output) under the conventional DEA. The same can be said for the other measure. From an economic point of view, universities may wish to maintain a certain percentages of classes whose sizes are below 20. Since there is no generally accepted agreement on the desired levels of the two targeted factors, we just use 28% and 59.5% based upon arithmetic averages of the observed data as the target levels, which would suffice for our illustrative purpose.

We evaluate 36 universities using the three proposed efficiency measures with (A3) postulated, and the results are reported in Table 3. The names of the evaluated 36 universities are disguised with numbers in the first column, and the numbers indicate the published ranking (the lower, the better). It can be observed that there are certain differences between the calculated efficiency and published ranking. Note also that targeted factor-oriented radial and NL efficiency scores are high overall since the differences in the two targeted factors are not large among the universities.

5. Concluding remarks

In this paper, we have shown how performance measures with target levels can be incorporated in DEA. The production possibility set was formulated by extending the standard one under VRS based on a set of properties postulated to suit the case of targeted factors being present. We also developed three efficiency measures by extending the standard radial, slacks-based, and Nerlove–Luenberger measures. We illustrated the proposed model and efficiency measures by applying them to the efficiency evaluation of 36 US universities.

As hinted in Section 3, regular factors can be treated as targeted factors. Suppose that there are only regular inputs and regular outputs in a DEA model. If we regard regular inputs as targeted factors with target levels of all zeros, then (VRS) , (SBM) , and (NL) reduce to the standard input-oriented VRS, slacks-based, and directional distance DEA with the NL measure models, respectively. Likewise, models (VRS) , (SBM) , and (NL) reduce to the standard counterparts if we regard regular outputs as targeted factors with some specific target levels. Therefore, it can be said that regular inputs and outputs are special cases of targeted factors. Undesirable factors (Scheel, 2001; Seiford and Zhu, 2002) can also be treated as targeted ones since it can be considered that undesirable outputs and undesirable inputs are targeted factors whose target levels are zero and sufficiently larger numbers, respectively. Furthermore, according to our initial study, it is possible to explore a certain economic implication of general target levels of regular factors. For instance, general target levels (other than zero) of regular inputs could be interpreted as fixed investment. Although we get some initial results on this issue of generality of targeted factors, we do not pursue this further in the current paper and leave it as a possible future research topic.

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References