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A buyer–seller game model for selection and negotiation of purchasing bids: Extensions and new models

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Abstract

A number of efficiency-based vendor selection and negotiation models have been developed to deal with multiple attributes including price, quality and delivery performance. The efficiency is defined as the ratio of weighted outputs to weighted inputs. By minimizing the efficiency, Talluri [Eur. J. Operat. Res. 143(1) (2002) 171] proposes a buyer–seller game model that evaluates the efficiency of alternative bids with respect to the ideal target set by the buyer. The current paper shows that this buyer–seller game model is closely related to data envelopment analysis (DEA) and can be simplified. The current paper also shows that setting the (ideal) target actually incorporates implicit tradeoff information on the multiple attributes into efficiency evaluation. We develop a new buyer–seller game model where the efficiency is maximized with respect to multiple targets set by the buyer. The new model allows the buyer to evaluate and select the vendors in the context of best-practice. By both minimizing and maximizing efficiency, the buyer can obtain an efficiency range within which the true efficiency lies given the implicit tradeoff information characterized by the targets. The current study establishes the linkage between buyer–seller game models and DEA. Such a linkage can provide the buyer with correct evaluation methods based upon existing DEA models regarding the nature of bidding.

Keywords: Game models; Linear programming; Efficiency; Data envelopment analysis

1. Introduction

As pointed out by Wise and Morrison (2000), one of the major flaws in the current business-to-business (B2B) model is that it focuses on price-driven transactions between buyers and sellers, and fails to recognize other important vendor attributes such as response time, quality and customization. In fact, a number of efficiency-based negotiation models have been developed to deal with multiple attributes—inputs and outputs. For example, data envelopment analysis (DEA) is used by Weber and Desai (1996) and Weber et al. (1998) to develop models for vendor evaluation and negotiation. The efficiency is defined as the ratio of weighted outputs to weighted inputs. The weights reflect the tradeoffs among multiple outputs and multiple inputs. However, such tradeoffs are usually not completely known to the buyer. Using DEA, one can estimate a set of weights and obtains the relative efficiencies in the context of best-practice.

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Talluri (2002) develops an efficiency-based buyer–seller game model that evaluates alternative bids based upon the ideal target set by the buyer. In Talluri (2002), the efficiency of each vendor is minimized when the efficiency of the ideal target is set equal to one. That is,

$$\min \quad \frac{\sum_{r=1}^{v} a_r y_{rp}}{\sum_{s=1}^{u} b_s x_{sp}}$$

s.t.

$$\begin{aligned} & \frac{\sum_{r=1}^{v} a_r y_{r\,\mathrm{ideal}}}{\sum_{s=1}^{u} b_s x_{s\,\mathrm{ideal}}} = 1 \\ & \frac{\sum_{r=1}^{v} a_r y_{ri}}{\sum_{s=1}^{u} b_s x_{si}} \leqslant 1 \quad \forall i \\ & a_r, b_s \geqslant 0 \quad \forall r, s \end{aligned} \tag{1}$$

where a_r and b_s represent unknown output and input weights, respectively. p represents a specific vendor bid under evaluation among n bids ($i = 1, \ldots, n$). Each bid i has v bid outputs y_{ri} ($r = 1, \ldots, v$) and u bid inputs x_{si} ($s = 1, \ldots, u$). $y_{r\,\mathrm{ideal}}$ ($r = 1, \ldots, v$) and $x_{s\,\mathrm{ideal}}$ ($s = 1, \ldots, u$) represent the rth bid output and ith bid input for the ideal target set by the buyer. The ideal target represents the best values for each inputs and outputs (i.e., minimum input values and maximum output values) across all n vendor bids, and therefore dominates all the vendors.

Talluri (2002) states that "although our models are grounded in the general efficiency theory, they are different from a traditional DEA sense". In fact, model (1) is closely related to the CCR ratio model (Charnes et al., 1978) where the efficiency of each individual vendor is maximized. The next section shows that because the ideal target defined in Talluri (2002) dominates all vendors, the constraints on the vendors $((\sum_{r=1}^{v} a_r y_{ri} / \sum_{s=1}^{u} b_s x_{si}) \leqslant 1)$ are redundant and can be removed from both model (1) and the CCR ratio model. Note the fact that the efficiency of the ideal target must be equal to one in the CCR ratio model. Thus, the only difference between model (1) and the CCR ratio model is that the former minimizes the vendor efficiency and the latter maximizes the vendor efficiency.

Through efficiency minimization in model (1), the buyer identifies/selects the vendor who has the largest minimum efficiency with respect to the ideal target. Note that a vendor having the largest minimum efficiency does not necessary mean that the vendor is the best with respect to the ideal target. Because unlike the CCR ratio model, model (1) does not evaluate efficiency in the context best-practice.

Note that the constraint on the ideal target $((\sum_{r=1}^{v} a_r y_{r\,\mathrm{ideal}} / \sum_{s=1}^{u} b_s x_{s\,\mathrm{ideal}}) = 1)$ incorporates implicit tradeoff information into the efficiency evaluation. i.e., this constraint can be viewed as incorporation of tradeoffs or weight restrictions in DEA (see, e.g., cone-ratio DEA model by Charnes et al., 1990). Note also that the ideal target defined in Talluri (2002) may not be achievable by all the vendors, and therefore may be unrealistic. As a result, the implicit tradeoff information characterized by $(\sum_{r=1}^{v} a_r y_{r\,\mathrm{ideal}} / \sum_{s=1}^{u} b_s x_{s\,\mathrm{ideal}}) = 1$ may not be valid, and the vendor efficiency in model (1) may not be correctly characterized with respect to the unrealistic target/invalid tradeoffs. Therefore, alternative targets which are realistic may be used in evaluating and selecting the vendors. In fact, such targets represent the buyer's preferences over the weighting scheme. When multiple bid outputs and inputs are present, multiple targets may be needed in order to fully characterize the buyer's preferences over the bids.

The current paper extends the model (1) into situations where vendor efficiency is maximized so that the buyer evaluates and selects the vendors in the context of best-practice. The current paper also explicitly links the Talluri (2002) model to the DEA. Consequently, more buyer–seller game models can be obtained based upon DEA. For example, the Talluri (2002) model yields an invalid result in proposing negotiation strategies where the percentage of on-time delivery can exceed 100%. This invalid result can be circumvented by using a DEA model where the best-practice exhibits variable-returns-to scale.

Although the ideal target implicitly incorporates some sort of tradeoff information, the true efficiency of vendors is still not completely known to the buyer. With the current buyer–seller game model, the buyer can obtain an efficiency range where the true efficiency lies given the targets. This better helps the buyer in selecting the competitive bids.

The rest of the paper is organized as follows. The next section simplifies the models in Talluri (2002). We then develop new buyer–seller models. The new models are illustrated with numerical examples including a pharmaceutical company studied in previous studies. The final section concludes.

2. Simplified buyer–seller game model

Because the ideal target in Talluri (2002) is defined as dominating all vendors, constraints $(\sum_{r=1}^{v} a_r y_{ri} / \sum_{s=1}^{u} b_s x_{si}) \leqslant 1$ are redundant in model (1). Consequently, model (1) can be written as

$$\begin{aligned} \min \quad & \frac{\sum_{r=1}^{v} a_r y_{rp}}{\sum_{s=1}^{u} b_s x_{sp}} \\ \text{s.t.} \quad & \\ & \frac{\sum_{r=1}^{v} a_r y_{r\,\text{ideal}}}{\sum_{s=1}^{u} b_s x_{s\,\text{ideal}}} = 1 \\ & a_r, b_s \geqslant 0 \quad \forall r, s \end{aligned} \tag{2}$$

Because the ideal target dominates all the vendors, the DEA efficiency of the ideal target must be equal to one, and the DEA efficiency of vendors must be less than one. Consequently, if we change the objective from minimization to maximization in model (1), then we obtain the CCR ratio model. This finding links model (2) (or model (1)) to the DEA methodology. As a result, more buyer–seller game can be obtained by changing the objective in various DEA models.

In Talluri (2002), another buyer–seller game model is developed when all vendors' efficiencies are minimized simultaneously. That is

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \frac{\sum_{r=1}^{v} a_r y_{rp}}{\sum_{s=1}^{u} b_s x_{sp}} \\ \text{s.t.} \quad & \\ & \frac{\sum_{r=1}^{v} a_r y_{r\,\text{ideal}}}{\sum_{s=1}^{u} b_s x_{s\,\text{ideal}}} = 1 \\ & \frac{\sum_{r=1}^{v} a_r y_{ri}}{\sum_{s=1}^{u} b_s x_{si}} \leqslant 1 \quad \forall i \\ & a_r, b_s \geqslant 0 \quad \forall r, s \end{aligned} \tag{3}$$

Similarly, model (3) can also be simplified as

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \frac{\sum_{r=1}^{v} a_r y_{rp}}{\sum_{s=1}^{u} b_s x_{sp}} \\ \text{s.t.} \quad & \\ & \frac{\sum_{r=1}^{v} a_r y_{r\,\text{ideal}}}{\sum_{s=1}^{u} b_s x_{s\,\text{ideal}}} = 1 \end{aligned} \tag{4}$$

The contribution of the simplified models (2) and (4) lies in the fact that computational burden can be substantially reduced if the problem size is large. Also, the buyer–seller game model is linked to DEA.

3. New buyer–seller game models

Note that the ideal target in Talluri (2002) may not be achievable by all vendors. As a result, the target may not be realistic and the resulting vendor efficiency may not be correctly evaluated. In this section, we assume that the targets set by the buyer do not have to dominate all the vendors. Suppose the buyer selects G targets represented by y_{rk}^{target} and x_{sk}^{target} $(k = 1, \ldots, G)$. We modify model (1) into the following model where efficiency is maximized as in DEA [1]

$$\begin{aligned} \max \quad & \frac{\sum_{r=1}^{v} a_r y_{rp}}{\sum_{s=1}^{u} b_s x_{sp}} \\ \text{s.t.} \quad & \\ & \frac{\sum_{r=1}^{v} a_r y_k^{\text{target}}}{\sum_{s=1}^{u} b_s x_k^{\text{target}}} = 1 \quad k = 1, \ldots, G \\ & \frac{\sum_{r=1}^{v} a_r y_{rp}}{\sum_{s=1}^{u} b_s x_{sp}} \leqslant 1 \quad \forall i \neq p \\ & a_r, b_s \geqslant 0 \quad \forall r, s \end{aligned} \tag{5}$$

[1] Note that model (5) may be infeasible if the multiple targets set by the buyer cannot be fit into the same efficient facet. If that happens, the buyer has to adjust the targets. For example, we can require that $G \leqslant v + u - 1$. These G targets actually determines an efficient facet contains the origin. Therefore, we only need at most $v + u - 1$ points to determine such an efficient facet. Model (5) is also similar to the benchmarking model used in Zhu (2001) in measuring the quality-of-life of cities.

Model (5) maximizes each vendor's efficiency individually when the efficiencies of the targets are set equal to one. Model (5) differs from model (1) in three aspects. First, model (5) provides the best efficiency scenario for the vendors whereas model (1) calculates the worst scenario. Second, the efficiency of vendor can exceed one if the targets do not dominate all the n vendors. Third, if new vendors enter the negotiation process and some of the new vendors dominate the targets, new targets need not to be established.

If all the targets dominate the vendors, then $(\sum_{r=1}^{v} a_r y_{rp} / \sum_{s=1}^{u} b_s x_{sp}) \leqslant 1$ $(\forall i \neq p)$ are redundant and can be removed from model (5).

In fact, model (1) determines the minimum efficiency for vendor p, and model (5) determines the maximum efficiency when the efficiency of targets is assumed to be one. Let $\underline{h}_p^*$ and $\bar{h}_p^*$ be the optimal values to models (1) and (5), respectively. Let h_p be the true efficiency of vendor p with respect to the fact that the efficiencies of the targets are set equal to one. Then $\underline{h}_p^* \leqslant h_p \leqslant \bar{h}_p^*$. When both models (1) and (5) are calculated, the buyer can have an efficiency range within which the true vendor efficiency lies. This better helps the vendor in evaluating and selecting the vendors. [2]

We can develop the following buyer–seller game model when all vendors' efficiencies are maximized simultaneously.

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} w_i \frac{\sum_{r=1}^{v} a_r y_{rp}}{\sum_{s=1}^{u} b_s x_{sp}} \\ \text{s.t.} \quad & \\ & \frac{\sum_{r=1}^{v} a_r y_k^{\text{target}}}{\sum_{s=1}^{u} b_s x_k^{\text{target}}} = 1 \quad k = 1, \ldots, G \\ & a_r, b_s \geqslant 0 \quad \forall r, s \end{aligned} \tag{6}$$

where w_i $(i = 1, \ldots, n)$ are buyer-specified weights reflecting the preference over vendors' efficiencies.

By using the Charnes–Cooper transformation, all models (1)–(6) can be transformed into linear programs. For example, model (5) is equivalent to

$$\begin{aligned} \max \quad & \sum_{r=1}^{v} a_r y_{rp} \\ \text{s.t.} \quad & \\ & \sum_{r=1}^{v} a_r y_k^{\text{target}} = \sum_{s=1}^{u} b_s x_k^{\text{target}} \quad k = 1, \ldots, G \\ & \sum_{r=1}^{v} a_r y_{rp} \leqslant \sum_{s=1}^{u} b_s x_{sp} \quad \forall i \neq p \\ & \sum_{s=1}^{u} b_s x_{sp} = 1 \\ & a_r, b_s \geqslant 0 \quad \forall r, s \end{aligned} \tag{7}$$

4. Illustration

We first use the numerical example in Table 1 to illustrate the buyer–seller game models. We have 12 vendors (DMUs). Their performance is characterized by (1) distribution costs, including transportation and handling costs, (2) customer response time (days), the amount of time between an order and its corresponding delivery, (3) percentage of shipping errors (number of incorrect shipments made), (4) manufacturing cost, total cost of manufacturing, including labor, maintenance, and re-work costs, (5) number of items produced (in each month), (6) percentage of on-time delivery, (7) fill rate, proportion of order filled immediately, (8) weakly profit, and (9) number of distribution centers.

Usually smaller values on the first four measures and larger values on the remaining five measure indicate better performance of these vendors. We therefore select the first four measures as inputs and the remaining as outputs. The last row of Table 1 provides the target based upon the best values of the inputs and outputs across all 12 DMUs. The last column provides the efficiency scores obtained from the CCR ratio model.

It can be seen that model (1) indicates that DMU3 is the best DMU, because it has the largest minimum efficiency score. Model (5) indicates that DMU6 is the best DMU with respect to the target. This shows that the best DMU under model (1) does not necessarily mean that the DMU is the

[2] We may use the average of $\underline{h}_p^*$ and $\bar{h}_p^*$ as the efficiency of vendor bid p in the Talluri (2002) bid selection model.

Table 1
Numerical example

DMU	Distribution cost	Response time	Shipping errors (%)	Manufacturing cost	Items produced	On-time delivery (%)	Fill rate (%)	Profit	Distribution centers	Model (1)	Model (5)	CCR ratio
1	609	4	24	1340.55	45 673	7.29	7.21	131	125	0.187	0.878	1
2	613	4.50	18	634.70	40 990	3.19	4.94	33	47	0.092	0.755	1
3	558	3	18	657.50	39 079	5.15	8.50	104	62	0.283	0.720	1
4	580	3.75	20	882.40	38 455	3.184	4.48	71	94	0.133	0.694	0.977
5	625	3.75	33	3286.70	54 291	17.158	15.41	148	105	0.119	0.856	1
6	535	3.75	17	917.04	34 534	4.512	8.78	124	112	0.182	0.896	1
7	650	5	18	3714.30	41 984	12.195	8.82	98	77	0.070	0.730	1
8	740	6.75	20	2963.10	43 249	9.205	7.81	118	102	0.092	0.694	0.978
9	775	3.99	18	3240.75	43 291	5.825	10.05	102	45	0.066	0.753	1
10	888	4.25	34	2197.12	46 444	10.4	18.21	240	55	0.127	0.706	1
11	727	3.5	26	778.35	41 841	3.21	4.67	52	50	0.122	0.661	0.917
12	695	4	26	1245.75	40 221	2.365	3.58	37	37	0.070	0.570	0.795
Target	535	3	17	634.70	54 291	17.158	18.21	240	125			

Table 2
Data for the vendors

Variable	Vendor 1	Vendor 2	Vendor 3	Vendor 4	Vendor 5	Vendor 6	Ideal target
Price ($/units)	0.1958	0.1881	0.2204	0.2081	0.2118	0.2096	0.1881
% Acceptance	98.8	99.2	100	97.9	97.7	98.8	100
% OTD	95	93	100	100	97	96	100

best in the context of best-practice in model (5). Therefore, it is critical that we also use model (5) when selecting the best vendor. Finally, the efficiency ranges presented by the 11th and 12th columns yield more information regarding the performance of the DMUs.

We next use the data in Weber and Desai (1996), Weber et al. (1998) and Talluri (2002) to demonstrate the models. Table 2 presents the data where the price is used as the only input, and percentages of accepted items (measuring quality) and on-time deliveries (OTD) (measuring delivery performance) are used as the two outputs. The company is a division of a Fortune 500 pharmaceutical company. The last column reports the ideal target used in Talluri (2002) which by definition dominates all the vendors.

Table 3 presents the results obtained from model (2) with the last column reporting the efficiency scores ($\underline{h}_p^*$) obtained from model (1). Note that $\bar{h}_p^* = \underline{h}_p^*$ in vendors 3 and 4. Although Table 3 reports a set of optimal weights, we should point out that caution should be paid when using these optimal weights because of possible multiple optimal solutions. The same comment can be applied to Talluri (2002).

Now, we relax the assumption that the target must be composed by the best values of all attributes of the vendors. Suppose a new vendor enters the bidding. The last column of Table 4 reports the new vendor. The second column of Table 4 reports the efficiency scores obtained from model (5). If we use model (1), then a new ideal target will be determined via changing the input for the old ideal target to 0.1877. The third column of Table 4 reports the efficiency scores obtained from model (1) with the new ideal target. Both models (1) and (5) indicate that the new vendor is the best one.

We finally note that Talluri (2002) states "vendors 1, 3, 4, 5 and 6 must improve their percentage of OTD to 96.77%, 109.03%, 102.88%, 104.77%

Table 3
Vendor efficiency

	Efficiency ($\bar{h}_p^*$)	Weights			Efficiency ($\underline{h}_p^*$)
		Price	% Acceptance	% OTD	
Vendor 1	0.949	5.10725	0.00961	0	0.913
Vendor 2	0.992	5.31632	0.01000	0	0.930
Vendor 3	0.853	4.53721	0	0.00853	0.853
Vendor 4	0.904	4.80538	0	0.00904	0.904
Vendor 5	0.868	4.72144	0.00888	0	0.861
Vendor 6	0.887	4.77099	0.00897	0	0.862

Table 4
Vendor efficiency with a new vendor

	Efficiency ($\bar{h}_p^*$)	Efficiency ($\underline{h}_p^*$)	New vendor
Vendor 1	0.949	0.910	Price: 0.1877
Vendor 2	0.992	0.928	% Acceptance: 100
Vendor 3	0.853	0.852	% OTD: 98
Vendor 4	0.904	0.883	
Vendor 5	0.868	0.860	
Vendor 6	0.887	0.860	
New vendor	1.002	0.980	

and 103.57%, respectively." Obviously, one cannot have OTD exceeding 100%. As a result, the negotiation strategies proposed in Talluri (2002) are unpractical and invalid. Talluri (2002) does not demonstrate how these values are obtained. However, we should note the fact that we cannot use model (1) to generate these values, because model (1) does not evaluate efficiency in the context of best-practice.

Because of the linkage between DEA and the buyer–seller model established by the current study, one can use a variable-returns-to-scale DEA model to overcome this problem. i.e., the $\sum_{r=1}^{v} a_r y_{ri}$ should be replaced by $\sum_{r=1}^{v} a_r y_{ri} + u_o$, where u_o is unconstrained in sign. This u_o corresponds to the convexity constraint in the dual form to model (7), and this convexity constraint guarantees that the OTD will not exceed 100% (Banker et al., 1984). Note that Weber and Desai (1996) employ the correct DEA model with variable-returns-to-scale.

5. Conclusions

Talluri (2002) proposes minimizing efficiency to develop a buyer–seller game model for selection and negotiation of purchasing bids. The current paper proposes maximizing efficiency in developing the buyer–seller game model so that the performance is measured against best-practice. The current paper shows that the models developed in Talluri (2002) and in the current paper are closely related to DEA. As a result, new models based upon various DEA models can be established. For example, if we want to consider the returns to scale or treat uncontrollable bid output/input values, related DEA models can be used with respect to efficiency minimization and maximization. Finally, Zhu (2002) provides an Excel Add-In for calculating the models discussed in the current paper.

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