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Dual-role factors in data envelopment analysis

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This paper presents a methodology for dealing with performance evaluation settings where factors can simultaneously play both input and output roles. Model structures are developed for classifying Decision-Making Units (DMUs) into three groups according to whether such a factor is behaving like an output, an input, or is in equilibrium, neither wanting to lose or gain any of the factors. We connect these ideas to those involving increasing, decreasing and constant returns to scale. Examples of factors that play this dual-role are: trainees in organizations, such as nurses, medical students, and doctoral students; awards to scholars or university departments; certain revenue—generating transactions in banks, and so on. We apply the model to the analysis of a set of university departments. In some settings, a dual-role factor may be one that can be reallocated, such as would be the case when DMUs are managed by a central authority. We develop the appropriate model structures to permit such a reallocation. We present two such structures, with the first involving reallocation from an existing allocation, and the second, a zero-base allocation.

1. Introduction

Data Envelopment Analysis (DEA) was developed by Charnes *et al.* (1978) to serve as a mechanism to evaluate the relative efficiencies of a set of similar decision-making units. A vast literature has grown out of this original work of both a methodological and applied nature. In the usual setting, Decision-Making Units (DMUs), for example, bank branches, hospitals, research projects, are evaluated relative to one another using a specified set of input and output factors. Outputs are meant to capture what the DMU generates; inputs represent the resources or circumstances that have led to the creation of those outputs.

In some situations there is a strong argument for permitting certain factors to simultaneously play the role of both inputs and outputs. Consider using the number of nurse trainees on staff in a study of hospital efficiency. Such a factor clearly constitutes an output measure for a hospital, but at the same time it is an important component of the hospital's total staff complement, hence it is an input.

In a completely different setting, consider the problem of evaluating researchers who receive grants from federal granting agencies. Such an evaluation might be undertaken as a means of identifying the highest-priority awardees, hence deriving an optimal allocation of research funds. In such a setting, while published research (articles in refereed journals, etc.) is likely the predominant output for evaluating the researcher, the extent to which the research contributes to the training of highly qualified personnel is also an important component in the evaluation. Thus, the total number of graduate students being trained is an output. On the input side one might argue that at least two inputs contribute to the generation of research publications: (i) research dollars available to support publication; and (ii) the number of graduate assistants participating in the awardee's research program. Hence, graduate students can be viewed as serving in a dual-role capacity, simultaneously as both an input and an output.

The idea of treating a factor as both an input and an output within the DEA framework is not entirely new. Beasley (1990, 1995), in a study of the efficiency of university departments, treated research funding on both the input and output sides. However, as will be shown later, this treatment was not entirely appropriate.

In Section 2, we develop the appropriate model structure for considering dual-role factors within a performance measurement setting. We show that the proper treatment of such factors calls for viewing them as nondiscretionary. Connections to returns to scale concepts are explored. Section 3 extends these ideas to developing an optimal distribution of such a factor across the set of DMUs for those

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situations where that factor is an allocatable resource. The model structure for accomplishing this can be transformed to a mixed integer programming problem. In Section 4 we illustrate these ideas using a subset of Beasley's data. Finally, Section 5 discusses further research directions.

2. Modeling dual-role factors

Consider a situation such as those discussed above where members k of a set of K DMUs are to be evaluated in terms of R outputs $Y_k = (y_{rk})_{r=1}^R$ and I inputs $X_k(x_{ik})_{i=1}^I$. In addition, assume that a particular factor (for example, research income) is held by each DMU in the amount w_k, and serves as both an input and output factor.

Adopting the *Constant Returns to Scale* (CRS) model of Charnes *et al.* (1978), the natural tendency is to view the measure of efficiency of DMU "o" as the solution to the problem:

problem (1):

$$\begin{aligned} \max\ & \left(\sum_{r=1}^{R} \mu_r y_{ro} + \gamma w_o\right) \Big/ \left(\sum_{i=1}^{I} \upsilon_i x_{io} + \beta w_o\right), \\ \text{subject to: } & \sum_{r=1}^{R} \mu_r y_{rk} + \gamma w_k - \sum_{i=1}^{I} \upsilon_i x_i - \beta w_k \le 0, \\ & \qquad k = 1, \ldots K, \\ & \mu_r, \upsilon_i, \gamma, \beta \ge 0. \end{aligned} \tag{1}$$

(We point out that herein we do not impose the non-Archimedian infinitesimal (ε) lower bound restrictions on the μ, υ multipliers. However, even if one did include such restrictions, the results discussed below are essentially the same.)

Problem (1) is essentially the model proposed by Beasley (1990, 1995), where in that setting, w_k would represent the level of research funding. This model would appear to be flawed from two perspectives:

The first flaw is that in the *absence* of constraints (e.g., AR or cone ratio) on the multipliers $\{\mu_r\}$ and $\{\upsilon_i\}$ (see Thompson *et al.* (1990), for example), each DMU will be 100% efficient. To show this we prove the following lemma and theorem.

Lemma 1. *If in the standard CCR model all DMUs* $k = 1,\ldots,K$ *possess the same amount of one of the given inputs* i_1 (*i.e.,* $x_{i_1 1} = x_{i_1 2} = \cdots\cdots\cdots = x_{i_1 K} = x$), *and the same amount of a given output* r_1 (*i.e.,* $y_{r_1 1} = y_{r_1 2} = \cdots\cdots\cdots\cdots = y_{r_1 K} = y$), *then all DMUs are efficient.*

Proof. According to Charnes *et al.* (1978), a DMU "o" is efficient if the following two conditions hold: (i) $\theta_o = 1$; and (ii) all slacks are zero. Consider a solution to the standard ratio CCR model in which all $\mu_r = 0, r \neq r_1$ and all $\upsilon_i = 0, i \neq i_1$. We now show that in the presence of such a solution, these two conditions are satisfied.

With this solution, the CCR problem reduces to the form:

problem (2):

$$\begin{aligned} \max\ & \mu_{r_1} y / \upsilon_{i_1} x, \\ \text{subject to: } & \mu_{r_1} y / \upsilon_{i_1} x \le 1, \quad k = 1 \ldots, K, \\ & \mu_{r_1}, \upsilon_{i_1} \ge 0. \end{aligned} \tag{2}$$

Since the K constraints are identical, then $K-1$ of these are redundant and may be dropped. Hence, this single-constraint fractional problem can be written as the two-constraint linear programming model:

problem (3):

$$\begin{aligned} \max\ & \mu_{r_1} y, \\ \text{subject to: } & \upsilon_{i_1} x = 1, \\ & \mu_{r_1} y - \upsilon_{i_1} x \le 0, \\ & \mu_{r_1}, \upsilon_{i_1} \ge 0, \end{aligned} \tag{3}$$

whose dual is given by:

problem (4):

$$\begin{aligned} \min\ & \theta, \\ \text{subject to: } & \theta x \ge \lambda x, \\ & y \le \lambda y, \\ & \lambda \ge 0, \quad \theta \text{ unrestricted in sign.} \end{aligned} \tag{4}$$

From problem (4) it is clear that $\theta \ge \lambda$ and $\lambda \ge 1$. Hence, $\theta^* = \lambda^* = 1$ is the optimal solution. Thus, both constraints in problem (4) are binding, meaning that all slacks are equal to zero. Therefore, both of the aforementioned conditions hold, proving the requisite efficiency of all DMUs. This completes the proof. ■

Theorem 1. *All DMUs under problem* (4) *are efficient.*

Proof. In problem (1) we may replace the ratio for each DMU by a scaled version in which for each k we divide both the numerator and denominator by w_k (assuming that $w_k > 0$ for all k). Thus, problem (1) is equivalent to the problem in which the dual-role factor for each DMU has $w_k = 1$. As a result, the model becomes a special case of that provided for by Lemma 1. Hence, each DMU is efficient in the sense of Charnes *et al.* (1978). This completes the proof. ■

It would appear that in the *presence* of multiplier constraints, the pathological efficiency results discussed above would not generally occur, hence more realistic outcomes might be expected. Problem (1) still, however, represents somewhat of a contradiction. The logic of this input-oriented structure is that in the case where a DMU has an efficiency score of θ, then all discretionary inputs, including w_o, are reduced by $1-\theta$. On the other hand, this factor is also included on the output side where we assume w_o will *not* be reduced. Thus, problem (1) treats w_o differently on the input than on the output side.

To correct for this apparent flaw in the logic of problem (1), let us view the treatment of the w_o variable in a somewhat different manner. Specifically, since w_o serves as an output, and is hence generally expected to remain at its current level in the input-oriented setting, we recommend treating it as being nondiscretionary on the input side. (Since on the output side, variables generally remain fixed in the optimization process of a model of the form problem (1), w_o can be viewed as nondiscretionary there as well). From this perspective, the proper form of the "dual-factor problem," and thus the requisite modification of problem (1) is that given by:

problem (5):

$$\max \left(\sum_{r=1}^{R} \mu_r y_{ro} + \gamma w_o - \beta w_o\right) \Big/ \left(\sum_{i=1}^{I} \upsilon_i x_{io}\right),$$
$$\text{subject to: } \sum_{r=1}^{R} \mu_r y_{r_k} + \gamma w_k - \beta w_k - \sum_{i=1}^{I} \upsilon_i \gamma_{i_k} \leq 0, \quad k = 1, \ldots, K,$$
$$\mu_r, \upsilon_i, \gamma, \beta \geq 0. \tag{5}$$

Banker and Morey (1986) were the first to introduce the concept of exogenously fixed or nondiscretionary variables. This idea often arises in situations where there are variables that have an impact on efficiency, but which cannot be controlled by the DMU manager (e.g., bank customer demographics). Thus, in the optimization, one does not want to proportionally reduce these factors as would be true of the discretionary inputs. The authors show that the proper way to model such inputs is to move them to the output side, but with the opposite sign.

In the usual manner, the linear programming form of problem (5) is:

problem (6):

$$e_o^* = \max \sum_{r=1}^{R} \mu_r y_{ro} + \gamma w_o - \beta w_o,$$
$$\text{subject to: } \sum_{i=1}^{I} \upsilon_i x_{io} = 1,$$
$$\sum_{r=1}^{R} \mu_r y_{rk} + \gamma w_k - \beta w_k - \sum_{i=1}^{I} \upsilon_i x_{ik} \leq 0, \quad k = 1, \ldots K, \tag{6}$$
$$\mu_r, \upsilon_i, \gamma, \beta \geq 0.$$

One of three possibilities exists in regard to the sign of $\hat{\gamma} - \hat{\beta}$, where $\hat{\gamma}, \hat{\beta}$ are the optimal values from problem (6); $\hat{\gamma} - \hat{\beta} > 0, = 0$, or < 0. Before we discuss the implications of these cases, we first prove the following property.

Property 1. *At the optimum of problem* (6), $\hat{\gamma}\hat{\beta} = 0$.

Proof. Since γ and β appear together in the same constraints, with multipliers that differ only in sign (w_k versus $-w_k$), indicating their linear dependence. Hence, they cannot be in the same basic feasible solution. Thus, at least one of the two variables is nonbasic at the optimum, meaning that, $\hat{\gamma}\hat{\beta} = 0$, completing the proof. ∎

2.1. *Input/output behavior and returns to scale*

The sign of $\hat{\gamma} - \hat{\beta}$ can have important implications when the dual-role factor is a resource that can be *allocated* across the DMUs. While we will examine this aspect in detail in the following section, it is useful to comment here on an interesting relationship that exists between the CRS model (problem (6)) and the standard (no dual-role factors present) Variable Returns to Scale (VRS) model of Banker *et al.* (1984). Specifically, the VRS model with outputs Y and inputs X is given by:

problem (7):

$$\max \left(\sum_{r=1}^{R} \mu_r y_{ro} - \mu_0\right) \Big/ \sum_{i=1}^{I} \upsilon_i x_{io},$$
$$\text{subject to: } \sum_{r=1}^{R} \mu_r y_{rk} - \mu_0 - \sum_{i=1}^{I} \upsilon_i x_{ik} \leq 0, \quad k = 1, \ldots K,$$
$$\mu_r, \upsilon_i \geq 0, \quad \mu_0 \text{ unrestricted in sign.} \tag{7}$$

In the constant returns to scale model, μ_0 is set to zero, and the supporting hyperplane to any facet of the frontier passes through the origin. Otherwise, μ_0 is a form of "Y-intercept," to use an analogy with regression techniques. It is well known that the sign of μ_0 identifies the "returns to scale" status of the DMU "o" under investigation. It is useful therefore to examine the three cases in regard to this sign, which will allow us to make important interpretations pertaining to the sign of the variable $\hat{\gamma} - \hat{\beta}$.

Case 1: If $\mu_0 > 0$ in problem (7), then the DMU "o" is said to be experiencing *decreasing returns to scale*. (Banker *et al.* 1984). Thus, the marginal return, in terms of output, is less than the amounts of input required to produce that output. In "returns to scale" terminology, this DMU is not operating at the *Most Productive Scale Size* (MPSS), and would benefit from a reduction in size (Banker and Morey, 1986). A somewhat similar interpretation can be made in problem (3) when $\hat{\gamma} - \hat{\beta} < 0$. Using research funding as the illustrative example, one might argue that DMU "o" would experience an improvement in efficiency with fewer research dollars. That is, in this particular university, this factor is at a level where diminishing returns have set in, hence less of this factor would improve its performance ratio. One can say that in this case, the dual-role factor is "behaving like an input."

Case 2: If $\mu_0 < 0$, then DMU "o" is experiencing *increasing returns to scale*, and again it is not at the MPSS. This case is analogous to $\hat{\gamma} - \hat{\beta} > 0$ in problem (6), meaning that this university's efficiency would benefit from increased research dollars. Specifically, this factor is at a level where it is "behaving like an output," hence more of the factor is better, and would lead to an increase in efficiency.

Case 3: If $\mu_0 = 0$, then the DMU is experiencing constant returns to scale, and the VRS model problem (7) reduces to the standard CRS model of Charnes *et al.* (1978). The DMU would then be operating at the MPSS. In a university setting, the analogous situation would be to have $\hat{\gamma} - \hat{\beta} = 0$, meaning that the funding is at an equilibrium or optimal level.

The above comparison of the efficiency implications arising from the sign of $\hat{\gamma} - \hat{\beta}$, to the returns to scale implications from the sign of μ_0 in problem (7), aids in understanding the impact of reallocation of a dual-role factor. Consider for a moment the situation involving two *inefficient* DMUs, where one DMU (k_1) has a positive factor $\hat{\gamma} - \hat{\beta}$, whereas the other ($k_2$) has a negative factor, and that we transfer a small amount of the resource from k_2 to k_1. Provided the amount transferred falls within the allowable sensitivity ranges (obtainable from the optimization), both DMUs will experience increases in efficiency. Also, if the DMUs are still inefficient after the transfer, then the efficiency scores of none of the other DMUs will have changed (since they are affected only by the frontier DMUs). In the case that the resource is truly allocatable, and assuming that this transfer does not have any negative impacts on other inputs and outputs, it would appear that the transfer should clearly be undertaken. Of course, if this transfer were to result in one or both of the DMUs k_1 or k_2 moving to the efficient frontier, or if either is a frontier unit already, then the scores of DMUs other than k_1 and k_2, may be affected negatively in the process. In this situation, one would need to answer the question as to whether the improvement in the efficiency of one DMU at the expense of another is worthwhile.

It would appear that the investigation of reallocation of a dual-role factor by looking at individual DMUs or pairs, thereof, is not very instructive. There is no obvious methodology to help in deciding how much of the resource to take from those with negative $\hat{\gamma} - \hat{\beta}$ values and give to those with positive values of this parameter. In the following section, we choose the route of working with the aggregate efficiency measure for the group of DMUs to facilitate this reallocation.

3. Optimal allocation of a dual-role factor

Let $\{e_k^*\}_{k=1}^K$ denote the set of optimal efficiency scores arising from problem (6) for a given current allocation $\{w_k\}_{k=1}^K$ of the dual-role factor. Furthermore, let K_1, K_2, K_3 denote those subsets of DMUs corresponding to cases 1, 2 and 3 above, respectively. Specifically, K_1 denotes those DMUs for which $\gamma - \beta < 0$, K_2, those for which $\gamma - \beta > 0$, and K_3 those where $\gamma - \beta = 0$. Note that $K = K_1 \cup K_2 \cup K_3$. Consider the situation in which the total amount W of the factor in question is such that it can be *allocated,* as would be the case involving the awarding of research funds to researchers. Also, to the extent that nursing programs are generally managed under provincial (or state) authority, the allocation of trainees to hospitals falls within the control of that authority. Viewed this way, it is natural to seek an optimal allocation of this factor.

We examine two forms of allocation of the dual-role factor. In the first model, we assume that given the distribution $\{w_k = \bar{w}_k\}$, the idea is to obtain a perturbation from the existing levels, possibly within fairly tight limits. In the second model, we assume that the distribution is such that there are no restrictions on the extent of reallocation allowed. Hence, in this case, we proceed as if there were no *a priori* distribution, taking a zero-base approach.

We recognize that in some jurisdictions resources such as nurse trainees may not presently be "allocated" by any central authority. In such settings, the current research may serve to motivate relevant authorities to consider viewing the situation more along the lines of the discussion herein. In the case of university research funding, the allocation is only partially controllable. Awards committees for research grants from a central government body such as NSERC or SSHRC in Canada, or NSF in the USA, could benefit from analysis of the type discussed herein, in making award decisions to faculty members or university departments.

To facilitate the above, we propose that rather than being concerned with the performance of each of the individual DMUs (e.g., universities), one should concentrate on the overall performance of the collection or aggregate of the DMUs. The question then becomes "what allocation of research funding to universities will result in the maximum efficiency score for the collection of those DMUs?"

3.1. *Model 1: Reallocation based on a perturbation from an existing distribution*

To address this question, we propose using an optimization model in which we maximize the ratio of aggregate output to aggregate (discretionary) input. In so doing, we propose to leave the $\bar{w}_k$, $k \in K_3$ at their current levels, since they are at an equilibrium position relative to problem (2). The idea is to reduce the levels of w_k in K_2 while increasing those in K_1. In addition, it is assumed that any multipliers $(\mu, \upsilon, \gamma, \beta)$ allowed in the evaluation must be such that when applied to any given DMU, the resulting efficiency score for that DMU will not exceed unity. We propose solving the following problem:

problem (8):

$$\max \sum_{k \in K_1} \left[\sum_r \mu_r y_{rk} + \alpha w_k \right] + \sum_{k \in K_2} \left[\sum_r \mu_r y_{rk} - \alpha w_k \right] + \sum_{k \in K_3} \left[\sum_r \mu_r y_{rk} \right], \tag{8a}$$

$$\text{subject to: } \sum_{k\in K}\left[\sum_i \upsilon_i x_{ik}\right] = 1, \quad (8b)$$

$$\sum_r \mu_r y_{rk} + \alpha w_k - \sum_i \upsilon_i x_{ik} \le 0, \quad k \in K_1, \quad (8c)$$

$$\sum_r \mu_r y_{rk} - \alpha w_k - \sum_i \upsilon_i x_{i_k} \le 0, \quad k \in K_2, \quad (8d)$$

$$\sum_r \mu_r y_{rk} - \sum_i \upsilon_i x_{ik} \le 0, \quad k \in K_3, \quad (8e)$$

$$\sum_{k\in K_1\cup K_2} w_k = \bar{W}, \quad (8f)$$

$$\bar{w}_k \le w_k \le w_k^{\cup}, \quad k \in K_1, \quad (8g)$$

$$w_k^{\mathrm{L}} \le w_k \le \bar{w}_k, \quad k \in K_2, \quad (8h)$$

$$\mu_r, \upsilon_i, \alpha, w_k \ge 0. \quad (8i)$$

In problem (8) we use the multiplier α in place of $\gamma - \beta$, and its value is the same in both groups K_1 and K_2. We argue that the marginal worth of one hospital trainee or one monetary unit of research funding is the same whether it is behaving more like an output than an input or *vice versa*. An alternative view of the dual-role factor, e.g., research income in Beasley's case, is to regard it as having two components: a positive component (to be considered as an output), and a negative component (to be treated as a nondiscretionary input). Hence, we may view this factor as a dual-component variable vector (a_k, b_k) for each DMU. For DMUs in K_1, $(a_k, b_k) = (w_k, 0)$, for those in K_2, (a_k, b_k)=(0, w_k), and in K_3, $(a_k, b_k) = (0, 0)$. Thus, we can argue that we are applying the same multiplier α to the vector component (a_k, b_k) in all DMUs.

The objective here is to optimize the ratio of aggregate outputs to aggregate inputs. In the usual manner, we replace the ratio by the aggregate output component of Equation (8) as the objective function, and restrict the aggregate inputs to equal unity, as per Equation (8). Constraints (8c), (8d), (8e) are the usual requirements that the efficiency scores for the individual DMUs not exceed unity, as discussed above. Constraint (8f) requires that the total amount of the factor in the two groups K_1 and K_2 is equal to $\bar{W} = W - \sum_{k\in K_3} \bar{w}_k$. Constraint (8g) restricts the amount of the factor allocated to members of group K_1 to be at least as much $\bar{w}_k$ as is currently the case, and no more than some upper limit $w_k^{\cup}$. We assume that an appropriate $w_k^{\cup}$ would be chosen by the organization. Constraint (8h) has a similar rationale for members of K_2. It might be argued that the upper bounds in the case of members of K_1, for example, should be a function of the ranges of optimality for the optimal values $\hat{\gamma} - \hat{\beta}$ computed. As discussed earlier, sensitivity analysis reports within standard optimization software would generally provide such ranges. Of course, even if an upper bound exceeds the limits of such ranges, this may not mean that efficiency scores will not still improve, since the new optimal $\hat{\gamma} - \hat{\beta}$ values may have the same sign. We have not bothered to invoke these ranges directly, but rather indirectly, in the analysis described in the following section.

It is noted that problem (8) is nonlinear through the products αw_k. To obtain a linear formulation, perform the change of variables:

$$\delta_k = \alpha w_k \quad (9)$$

At the same time, replace constraint (8f) by.

$$\alpha \sum_{k\in K_1\cup K_2} w_k = \alpha \bar{W} \Rightarrow \sum_{k\in K_1\cup K_2} \delta_k = \alpha \bar{W}.$$

Similarly, in Equations (8g) and (8h), multiply through by α. Thus, problem (8) becomes the linear programming model:

problem (9):

$$\max \sum_{k\in K_1}\left[\sum_r \mu_r y_{rk} + \delta_k\right] + \sum_{k\in K_2}\left[\sum_r \mu_r y_{rk} - \delta_k\right] + \sum_{k\in K_3}\left[\sum_r \mu_r y_{rk}\right],$$

$$\text{subject to: } \sum_k \sum_i \upsilon_i x_{ik} = 1,$$

$$\sum_r \mu_r y_{rk} + \delta_k - \sum_i \upsilon_i x_{ik} \le 0, \quad k \in K_1,$$

$$\sum_r \mu_r y_{rk} - \delta_k - \sum_i \upsilon_i x_{ik} \le 0, \quad k \in K_2,$$

$$\sum_r \mu_r y_{rk} - \sum_i \upsilon_i x_{ik} \le 0, \quad k \in K_3, \quad (10)$$

$$\sum_{k\in K_1\cup K_2} \delta_k = \alpha \bar{W},$$

$$\alpha \bar{w}_k \le \delta_k \le \alpha w_k^{\cup}, \quad k \in K_1,$$

$$\alpha w_k^{\mathrm{L}} \le \delta_k \le \alpha \bar{w}_k, \quad k \in K_2,$$

$$\mu_r, \upsilon_i, \alpha, \delta_k \ge 0.$$

We note that from the optimal solution $\hat{\mu}, \hat{\upsilon}, \hat{\delta}, \hat{\alpha}$ to problem (9), we obtain the optimal value of w_k by way of Equation (9), specifically:

$$\hat{w}_k = \hat{\delta}_k / \hat{\alpha}.$$

In Section 4 we apply problem (9) to the problem of (re)allocating research income across a set of university departments.

It must be pointed out that whereas members of K_3 were in an equilibrium position (neither wanting to gain or to lose resource w_k), such may not, in fact, be the case for these members *after* reallocation has occurred. The equilibrium status of these DMUs only held up under the condition that the members of K_1 and K_2 held their dual-role factor in amounts $\bar{w}_k$. Any changes to these amounts can, of course, impact on the optimal multipliers γ and β when evaluating members of K_3. It should also be pointed out that the efficiency rating for any given member $k \in K_1$, for example, may possibly worsen *after* reallocation, even though resources are added to units where the impact on the output side is more favorable than on the input side. Clearly, if a DMU $k \in K_1$ could gain an increase in w_k, *without* other DMU allocations changing, then the efficiency score for k could not get worse. However, those other allocations *can change.* This is generally due to the fact that the amount of resource reallocated into or out of a particular DMU may have exceeded the ranges of optimality as specified in the linear programming sensitivity reports, as discussed above. However, in adopting the objective of optimizing the aggregate score for the entire set of DMUs, we are guaranteed that this overall score will be at least as high as that generated by the existing allocation. At the same time, by moving resources from group K_2 to group K_1, we are moving in the obvious direction toward improvement in all DMU scores, even if some DMUs do suffer a setback in this process.

3.2. *Model 2: Zero-base allocation of a dual-role factor*

Suppose that despite the structure of an existing allocation of the factor in question, we wish to derive some form of best distribution. This is equivalent to starting from scratch, as if no distribution existed at all. If we adopt the same argument as above, and set out to obtain an allocation based on optimizing the aggregate score for the entire collection of DMUs, we face the problem of not knowing which DMUs will ultimately fall into groups K_1 and K_2. (Note that we will ignore K_3 here, but will discuss it later.) This is equivalent to saying that we wish not only to decide on the size w_k of the allocation, but as well, whether that value is the first or second component of the dual-component vector (a_k, b_k).

One might argue that rather than add the complication of having to decide a K_1 versus K_2 designation for each DMU, a version of model 1 (problem (8)) could be solved, taking the summation over the entire set of DMUs, and allowing α to take either a negative or positive sign that would be applied uniformly across the entire set. However, since ultimately the resulting allocations will be used in problem (10), where for each DMU the choice of output or input behavior is permitted, it is essential to imitate this behavior at the aggregate level as well.

To facilitate assignment of DMUs to the two groups, introduce the binary variable:

$$d_k = \begin{cases} 1, & \text{if DMU}k \text{ belongs to group } K_1, \\ 0, & \text{if DMU } k \text{ belongs to group } K_2. \end{cases}$$

Now, the term $\sum_{k\in K_1} \alpha w_k - \sum_{k\in K_2} \alpha w_k$ in the objective function, becomes $\sum_{k\in K} d_k \alpha w_k - \sum_{k\in K} (1-d_k)\alpha w_k$. This latter expression reduces to what we want if we can decide to which DMUs to assign $d_k = 1$, and to which $d_k = 0$. The following nonlinear mixed-integer programming model captures this idea:

problem (10):

$$\begin{aligned} \max\ & \sum_{k\in K}\sum_r \mu_r y_{rk} + \sum_{k\in K} d_k\alpha w_k - \sum_{k\in K}(1-d_k)\alpha w_k, \\ \text{subject to:}\ & \sum_{k\in K}\sum_i \upsilon_i x_{ik} = 1, \\ & \sum_r \mu_r r_{rk} + d_k\alpha w_k - (1-d_k)\alpha w_k \\ & \quad - \sum_i \upsilon_i x_{ik} \le 0, \quad k \in K, \qquad (11) \\ & \sum_{k\in K} w_k = W, \\ & w^{\mathrm{L}} \le w_k \le w^{\cup}, \quad k\in K, \\ & \mu_r, \upsilon_i, \alpha, w_k \ge 0, \quad d_k \in \{0,1\}, \end{aligned}$$

We have imposed lower and upper bounds w^{L}, $w^{\cup}$, respectively, on the variables w_k to ensure that each DMU has some reasonable level of the factor assigned to it. Neither here nor in the previous subsection, have we addressed the issue as to how the other outputs (the three student–related factors) might be influenced by changes in the level of the research income that a department receives. Clearly, if in the reallocation exercise it is the case that a department is left with fewex resources than is necessary to support those other outputs, the model results might be brought into question. The imposed bounds w^{L}, $w^{\cup}$ should then be set in a way that respects current levels of those other outputs.

Proceeding as before, and using the change of variables $\delta_k = \alpha w_k$, the expression $d_k\alpha w_k$ becomes $d_k\delta_k$. To remove the remaining nonlinearity, define a further transformation of variables:

$$\phi_k = d_k\delta_k, \qquad (12)$$

and impose the following restrictions:

$$\phi_k \le M d_k, \qquad (13a)$$
$$\phi_k \le \delta_k, \qquad (13b)$$
$$\delta_k \le \phi_k + M(1-d_k). \qquad (13c)$$

Here, M denotes a large positive number.

From Equations (13(a)–13(c)) we note that if $d_k = 0$, then $\phi_k = 0$, and if $d_k = 1$ then $\phi_k = \delta_k$. Hence, in problem (10) the term $d_k\alpha w_k - (1-d_k)\alpha w_k$ becomes $2d_k\delta_k - \delta_k$. If $d_k = 1$, this expression reduces to $2\phi_k - \delta_k = 2\delta_k - \delta_k = \delta_k$. If $d_k = 0$, then the expression reduces to $-\delta_k$. Thus, letting K_1 denote those DMUs for which $d_k = 1$, and K_2 the compliment of K_1, the expression:

$$\sum_{k\in K} d_k\alpha w_k - \sum_{k\in K}(1-d_k)\alpha w_k,$$

reduces to

$$\sum_{k\in K_1}\delta_k - \sum_{k\in K2}\delta_k.$$

The "zero-base" form of problem (9), namely the linearized version of problem (10), then becomes:

problem (11):

$$\max \sum_{k\in K}\sum_{r}\mu_r y_{rk} + 2\sum_{k\in K}\phi_k - \sum_{k\in K}\delta_k,$$

$$\text{subject to} : \sum_{k\in K}\sum_{i}\upsilon_i x_{ik} = 1,$$

$$\sum_{r}\mu_r y_{rk} + 2\phi_k - \delta_k - \sum_{i}\upsilon_i x_{ik} \le 0, \quad k \in K,$$

$$\sum_{k\in K}\delta_k = \alpha W, \tag{14}$$

$$\alpha w^{\mathrm{L}} \le \delta_k \le \alpha w^{\cup}, \quad k \in K,$$

$$\phi_k \le M d_k,$$

$$\phi_k \le \delta_k,$$

$$\delta_k \le \phi_k + M(1 - d_k),$$

$$\mu_r, \upsilon_i, \alpha, \delta_k, \phi_k \ge 0, \quad d_k \in \{0, 1\},$$

We point out that problem (11) is adaptable enough that other practical restrictions could also be imposed. One may, for example, wish to find a reallocation that would guarantee that each DMU has a resulting efficiency score not lower than some percentage (e.g., 95%) of its current standing. Such practical considerations are particular to the problem being studied, and will not be pursued herein.

In the following section, we use a portion of the data from Beasley (1990, 1995), in the context of the above development.

4. Dual-role factors: an illustration

4.1. *Background*

In this section we examine the modeling of dual-role factors in the context of comparing universities as discussed in Beasley (1990, 1995). Beasley studies both Chemistry and Physics departments at 50 UK universities, deriving efficiency scores that provide an overall ranking of those departments. A portion of the data for Physics departments, recreated from Beasley (1990, p. 174), is displayed here as Table 1. It is noted that for purposes of our analysis, we have excluded from the output set the four ratings labeled as "star," A+, A, A− in Beasley's original data. We use the three inputs, general expenditure, equipment expenditure, and research income. Outputs consist of research income, and the three student groups.

Our purpose is not to compare our efficiency results with those of Beasley, but rather to use his work as a backdrop for illustrating the required analysis when a dual-role factor is present. Beasley makes the compelling argument that research income constitutes both an output and an input. On the output side, it is a proxy for quality of the research program, and the quality of the faculty whose research resulted in them being able to acquire that income. On the input side, this factor supports the generation of other outputs (e.g., it provides support to graduate students).

Table 1. Data for Physics departments from Beasley (1990)

DMU	*Gen.exp (I1)*	*Equip.exp. (I2)*	*Res.inc. (I3,O1)*	*UG (O2)*	*PGT (O3)*	*PGR (O4)*
University 1	528	64	254	145	0	26
University 2	2605	301	1485	381	16	54
University 3	304	23	45	44	3	3
University 4	1620	485	940	287	0	48
University 5	490	90	106	91	8	22
University 6	2675	767	2967	352	4	166
University 7	422	0	298	70	12	19
University 8	986	126	776	203	0	32
University 9	523	32	39	60	0	17
University 10	585	87	353	80	17	27
University 11	931	161	293	191	0	20
University 12	1060	91	781	139	0	37
University 13	500	109	215	104	0	19
University 14	714	77	269	132	0	24
University 15	923	121	392	135	10	31
University 16	1267	128	546	169	0	31
University 17	891	116	925	125	0	24
University 18	1395	571	764	176	14	27
University 19	990	83	615	28	36	57
University 20	3512	267	3182	511	23	153
University 21	1451	226	791	198	0	53
University 22	1018	81	741	161	5	29
University 23	1115	450	347	148	4	32
University 24	2055	112	2945	207	1	47
University 25	440	74	453	115	0	9
University 26	3897	841	2331	353	28	65
University 27	836	81	695	129	0	37
University 28	1007	50	98	174	7	23
University 29	1188	170	879	253	0	38
University 30	4630	628	4838	544	0	217
University 31	977	77	490	94	26	26
University 32	829	61	291	128	17	25
University 33	898	39	327	190	1	18
University 34	901	131	956	168	9	50
University 35	924	119	512	119	37	48
University 36	1251	62	563	193	13	43
University 37	1011	235	714	217	0	36
University 38	732	94	297	151	3	23
University 39	444	46	277	49	2	19
University 40	308	28	154	57	0	7
University 41	483	40	531	117	0	23
University 42	515	68	305	79	7	23
University 43	593	82	85	101	1	9
University 44	570	26	130	71	20	11
University 45	1317	123	1043	293	1	39
University 46	2013	149	1523	403	2	51
University 47	992	89	743	161	1	30
University 48	1038	82	513	151	13	47
University 49	206	1	72	16	0	6
University 50	1193	95	485	240	0	32

4.2. *Analysis of efficiency*

4.2.1. *Current research income allocation*

Problem (6) was applied to the data of Table 1 to determine to which category, K_1, K_2, K_3, each university belongs.

Table 2. Efficiency scores and output/input behavior

DMU	*Efficiency*	γ	β
University 1	1.000	0.000 00	0.000 97
University 2	0.640	0.000 06	0.000 00
University 3	0.810	0.000 00	0.003 75
University 4	0.686	0.000 10	0.000 00
University 5	1.000	0.000 00	0.002 18
University 6	1.000	0.000 23	0.000 00
University 7	1.000	0.000 41	0.000 00
University 8	0.812	0.000 17	0.000 00
University 9	1.000	0.000 00	0.004 17
University 10	0.907	0.000 79	0.000 00
University 11	0.828	0.000 00	0.001 94
University 12	0.709	0.000 28	0.000 00
University 13	0.772	0.000 21	0.000 00
University 14	0.703	0.000 00	0.000 02
University 15	0.688	0.000 18	0.000 00
University 16	0.520	0.000 11	0.000 00
University 17	0.819	0.000 67	0.000 00
University 18	0.628	0.000 36	0.000 00
University 19	1.000	0.000 14	0.000 00
University 20	0.898	0.000 11	0.000 00
University 21	0.674	0.000 00	0.000 05
University 22	0.717	0.000 51	0.000 00
University 23	0.563	0.000 00	0.000 17
University 24	1.000	0.000 34	0.000 00
University 25	1.000	0.000 30	0.000 00
University 26	0.565	0.000 16	0.000 00
University 27	0.855	0.000 15	0.000 00
University 28	1.000	0.000 00	0.001 15
University 29	0.825	0.000 14	0.000 00
University 30	0.930	0.000 12	0.000 00
University 31	0.776	0.000 69	0.000 00
University 32	0.867	0.000 00	0.000 21
University 33	1.000	0.000 00	0.000 00
University 34	1.000	0.000 65	0.000 00
University 35	1.000	0.000 14	0.000 00
University 36	0.737	0.000 00	0.000 87
University 37	0.831	0.000 16	0.000 00
University 38	0.806	0.000 12	0.000 00
University 39	0.790	0.000 00	0.000 03
University 40	0.741	0.000 02	0.000 00
University 41	1.000	0.000 99	0.000 00
University 42	0.841	0.000 00	0.000 13
University 43	0.900	0.000 00	0.003 04
University 44	1.000	0.000 00	0.000 00
University 45	0.889	0.000 11	0.000 00
University 46	0.851	0.000 00	0.000 00
University 47	0.688	0.000 52	0.000 00
University 48	0.909	0.000 00	0.000 94
University 49	1.000	0.000 00	0.019 45
University 50	0.835	0.000 01	0.000 00

The results are displayed in Table 2. We note that $K_1 = 31$, $K_2 = 16$, and $K_3 = 3$. Recall that the DMUs in K_1 are those wherein the research income is behaving like an output, and where more of such income would improve the efficiencies of the members of that set. Those in K_2 could forfeit research income and in the process improve their efficiencies. The three universities in K_3 (namely universities 33, 44 and 46), are in equilibrium.

4.2.2. *Reallocation of research income*

Table 3 provides the results from applying problem (9). In the case of universities in group K_1, for example, we have imposed lower limits on research income equal to the current allocations (this prevents less income being assigned to those DMUs than they possess at present). In theory, upper limits should be imposed that reflect ranges of optimality as per the sensitivity analysis discussed earlier. However, as discussed, such ranges may be too restrictive, and may not properly reflect the full scope for changes in the reallocation process. As an alternative, we have imposed on the members of both K_1 and K_2 limits that permit up to a 10% change in the allocation of the research income.

The table displays the current income allocation and the corresponding efficiency score achieved for each DMU. As well, the proposed reallocation and the resulting efficiency are given. The ratio of the new and current efficiency scores has been computed, as a signal for the direction of any efficiency shift (increased, decreased, stayed the same) experienced by each university in the reallocation process. In most cases, the outcome has been either to leave the DMU at its existing level or result in an increase. In a few cases, however, e.g., university 12, the efficiency score actually decreased. Note that in this case, there was no change in its research income, yet due to the reallocation to other DMUs, its score has dropped slightly.

A further analysis was carried out, permitting a maximum of only a 5% change in the income allocation. Table 4 demonstrates that in this case, no DMU suffered in terms of its efficiency rating experiencing a drop. We have not provided the corresponding reallocation of the research income.

4.2.3. *Zero-base reallocation*

Table 5 displays the outcome from applying problem (11). The table shows the current and recommended research income levels needed to provide the optimal *aggregate* performance of the group. Under the column labeled "Binary *d*," the designation of each DMU as to its input versus output behavior appears. It is observed that 20% or 10 of the DMUs have $d = 1$ (output behavior), with the remainder exhibiting input behavior. Based on the recommended research incomes, individual DEA analyses were carried out using problem (6), and the resulting efficiency scores ("current efficiencies") are also displayed. It is noted that 80% of the efficiency scores either improved or remained the same, and 20% deteriorated under the reallocation. The average of the efficiency scores rose from 0.84 to 0.86. We have not bothered to impose lower and upper limits on the research income, as called for in the modeling, since these would

Table 3. Reallocation of research income (maximum 10%)

DMU	*Cur. inc.*	*Cur. effic.*	*Rec. inc.*	*New effic.*	*New effic./ cur. effic.*	*Income chg*
University 1	254	1.0000	254	1.0000	1.000	0
University 2	1485	0.6397	1485	0.6397	1.000	0
University 3	45	0.8098	40.5	0.8182	1.010	−4.5
University 4	940	0.6857	940	0.6857	1.000	0
University 5	106	1.0000	95.4	1.0000	1.000	−10.6
University 6	2967	1.0000	3181.2	1.0000	1.000	214.2
University 7	298	1.0000	298	1.0000	1.000	0
University 8	776	0.8119	776	0.8119	1.000	0
University 9	39	1.0000	35.1	1.0000	1.000	−3.9
University 10	353	0.9066	353	0.9066	1.000	0
University 11	293	0.8277	263.7	0.8783	1.061	−29.3
University 12	781	0.7093	781	0.7076	0.998	0
University 13	215	0.7721	215	0.7721	1.000	0
University 14	269	0.7029	269	0.7029	1.000	0
University 15	392	0.6883	392	0.6883	1.000	0
University 16	546	0.5197	546	0.5197	1.000	0
University 17	925	0.8195	925	0.8195	1.000	0
University 18	764	0.6278	764	0.6278	1.000	0
University 19	615	1.0000	676.5	1.0000	1.000	61.5
University 20	3182	0.8980	3182	0.8955	0.997	0
University 21	791	0.6736	791	0.6744	1.001	0
University 22	741	0.7167	741	0.7167	1.000	0
University 23	347	0.5627	312.3	0.5724	1.017	−34.7
University 24	2945	1.0000	2945	1.0000	1.000	0
University 25	453	1.0000	453	1.0000	1.000	0
University 26	2331	0.5654	2331	0.5654	1.000	0
University 27	695	0.8555	695	0.8546	0.999	0
University 28	98	1.0000	88.2	1.0000	1.000	−9.8
University 29	879	0.8250	879	0.8250	1.000	0
University 30	4838	0.9300	4838	0.9233	0.993	0
University 31	490	0.7759	490	0.7711	0.994	0
University 32	291	0.8675	261.9	0.8909	1.027	−29.1
University 33	327	1.0000	327	1.0000	1.000	0
University 34	956	1.0000	956	1.0000	1.000	0
University 35	512	1.0000	512	1.0000	1.000	0
University 36	563	0.7365	506.7	0.7961	1.081	−56.3
University 37	714	0.8308	714	0.8308	1.000	0
University 38	297	0.8064	297	0.8064	1.000	0
University 39	277	0.7896	277	0.7898	1.000	0
University 40	154	0.7414	154	0.7414	1.000	0
University 41	531	1.0000	531	1.0000	1.000	0
University 42	305	0.8410	274.5	0.8460	1.006	−30.5
University 43	85	0.9001	76.5	0.9121	1.013	−8.5
University 44	130	1.0000	130	1.0000	1.000	0
University 45	1043	0.8885	1043	0.8885	1.000	0
University 46	1523	0.8513	1523	0.8513	1.000	0
University 47	743	0.6884	743	0.6884	1.000	0
University 48	513	0.9094	461.7	0.9721	1.069	−51.3
University 49	72	1.0000	64.8	1.0000	1.000	−7.2
University 50	485	0.8355	485	0.8355	1.000	0

Table 4. Efficiency change (5% max reallocation)

DMU	*Cur. effic.*	*New effic.*	*New effic./cur. effic.*
University 1	1.000	1.000	1.000
University 2	0.640	0.640	1.000
University 3	0.810	0.814	1.005
University 4	0.686	0.686	1.000
University 5	1.000	1.000	1.000
University 6	1.000	1.000	1.000
University 7	1.000	1.000	1.000
University 8	0.812	0.812	1.000
University 9	1.000	1.000	1.000
University 10	0.907	0.907	1.000
University 11	0.828	0.854	1.031
University 12	0.709	0.709	1.000
University 13	0.772	0.772	1.000
University 14	0.703	0.703	1.000
University 15	0.688	0.688	1.000
University 16	0.520	0.520	1.000
University 17	0.819	0.819	1.000
University 18	0.628	0.628	1.000
University 19	1.000	1.000	1.000
University 20	0.898	0.898	1.000
University 21	0.674	0.674	1.000
University 22	0.717	0.717	1.000
University 23	0.563	0.566	1.005
University 24	1.000	1.000	1.000
University 25	1.000	1.000	1.000
University 26	0.565	0.565	1.000
University 27	0.855	0.855	1.000
University 28	1.000	1.000	1.000
University 29	0.825	0.825	1.000
University 30	0.930	0.946	1.017
University 31	0.776	0.776	1.000
University 32	0.867	0.874	1.008
University 33	1.000	1.000	1.000
University 34	1.000	1.000	1.000
University 35	1.000	1.000	1.000
University 36	0.737	0.767	1.041
University 37	0.831	0.831	1.000
University 38	0.806	0.806	1.000
University 39	0.790	0.790	1.000
University 40	0.741	0.741	1.000
University 41	1.000	1.000	1.000
University 42	0.841	0.843	1.002
University 43	0.900	0.906	1.007
University 44	1.000	1.000	1.000
University 45	0.889	0.889	1.000
University 46	0.851	0.851	1.000
University 47	0.688	0.688	1.000
University 48	0.909	0.933	1.026
University 49	1.000	1.000	1.000
University 50	0.835	0.835	1.000

need to be selected by those knowledgeable of the particular problem setting. As indicated in the earlier section, one could further restrict problem (11) to require that the score for each DMU not decrease. More generally, constraints can be imposed that would prevent scores from declining by more than say some desired percentage.

We emphasize that this analysis is not intended to represent an informed or in-depth study of the problem at hand,

Table 5. Zero-base allocation of research income

DMU	*Recom. inc.*	*Current inc.*	*Change*	*Binary d*	*Orig. effic.*	*Current effic.*
University 1	1375	254	1121	0	1.000	1.000
University 2	0	1485	−1485	1	0.640	1.000
University 3	105	45	60	0	0.810	0.706
University 4	602	940	−338	1	0.686	0.719
University 5	211	106	105	0	1.000	0.990
University 6	0	2967	−2967	0	1.000	1.000
University 7	252	298	−46	0	1.000	1.000
University 8	638	776	−138	0	0.812	0.820
University 9	1213	39	1174	0	1.000	0.726
University 10	795	353	442	0	0.907	0.910
University 11	1818	293	1525	0	0.828	0.748
University 12	195	781	−586	0	0.709	0.805
University 13	1316	215	1101	0	0.772	0.825
University 14	798	269	529	0	0.703	0.711
University 15	2128	392	1736	0	0.688	0.755
University 16	385	546	−161	1	0.520	0.543
University 17	5178	925	4253	0	0.819	0.970
University 18	294	764	−470	1	0.628	0.670
University 19	737	615	122	0	1.000	1.000
University 20	1706	3182	−1476	1	0.898	0.862
University 21	930	791	139	0	0.674	0.677
University 22	0	741	−741	0	0.717	1.000
University 23	183	347	−164	1	0.563	0.648
University 24	574	2945	−2371	1	1.000	0.485
University 25	469	453	16	0	1.000	1.000
University 26	40	2331	−2291	1	0.565	0.961
University 27	844	695	149	0	0.855	0.853
University 28	726	98	628	0	1.000	0.810
University 29	0	879	−879	0	0.825	1.000
University 30	407	4838	−4431	1	0.930	1.000
University 31	160	490	−330	0	0.776	1.000
University 32	386	291	95	0	0.867	0.871
University 33	887	327	560	0	1.000	1.000
University 34	1138	956	182	0	1.000	1.000
University 35	906	512	394	0	1.000	1.000
University 36	613	563	50	0	0.737	0.744
University 37	757	714	43	0	0.831	0.853
University 38	187	297	−110	0	0.806	0.938
University 39	698	277	421	0	0.790	0.814
University 40	340	154	186	0	0.741	0.741
University 41	179	531	−352	0	1.000	1.000
University 42	812	305	507	0	0.841	0.856
University 43	620	85	535	0	0.900	0.661
University 44	2733	130	2603	0	1.000	1.000
University 45	432	1043	−611	0	0.889	0.935
University 46	930	1523	−593	1	0.851	0.852
University 47	213	743	−530	0	0.688	0.701
University 48	279	513	−234	0	0.909	1.000
University 49	1595	72	1523	0	1.000	1.000
University 50	896	485	411	0	0.835	0.838
				Average	0.840	0.860

but rather to demonstrate the application of the model. The important feature of the models herein is that they provide the capability to aid managers in performing appropriate allocations of resources.

5. Conclusions

This paper has presented a methodology for dealing with those situations where a factor can simultaneously play both an input and output role. By treating such a factor on the input side as being nondiscretionary, the model developed here can be used to determine in which status that factor dominates within each DMU. Specifically, the model determines whether in a DMU the factor is behaving predominantly like an input, hence the DMU would benefit from having less of the factor, like an output where more of the factor is desirable, or where it is in equilibrium. We connect these ideas to those involving increasing, decreasing and constant returns to scale. Examples of factors that play this dual-role are: trainees in organizations, such as nurses, medical students, and doctoral students; awards to scholars or university departments; etc. We apply the model to the analysis of a set of university departments as per Beasley (1990, 1995).

We also develop the appropriate model structures for reallocation of such dual-role factors across DMUs in a manner that optimizes the aggregate efficiency of those DMUs. In some settings, reallocation of such a factor is at the discretion of a central body, and the models can aid in that reallocation exercise. In others, where there is no such central authority, the models can still serve to move towards a better allocation than presently exists. We present two such structures; the first involves reallocation from an existing allocation, and the second, a form of zero-base allocation.

We point out that the development herein pertains to a single dual-role factor. Extension to multiple factors is straightforward and hence is omitted.

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