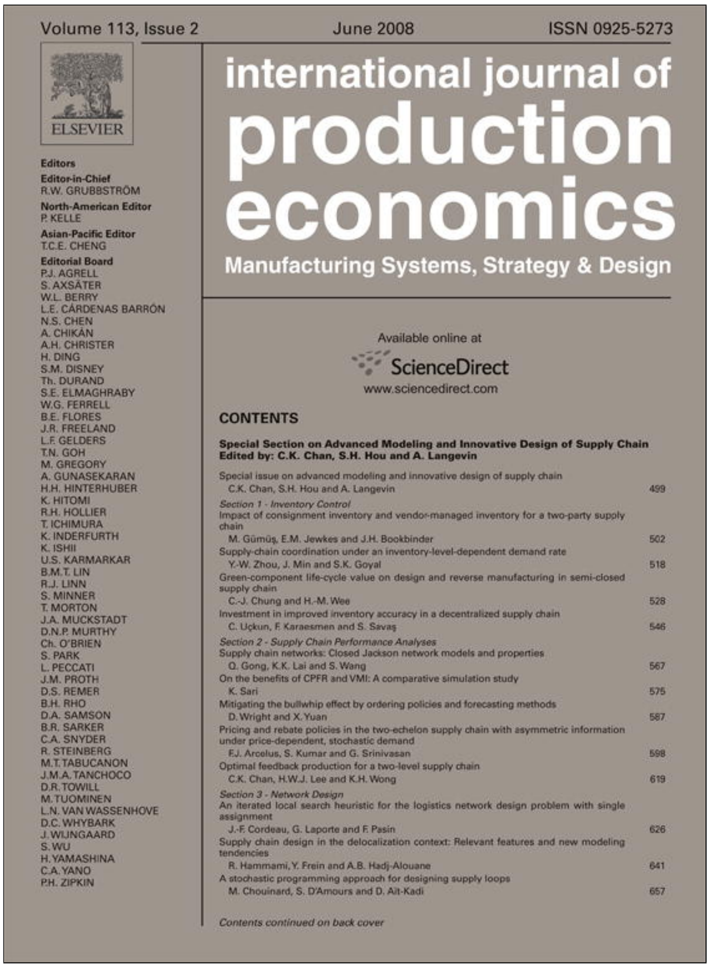

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Alternative secondary goals in DEA cross-efficiency evaluation

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Abstract

As an extension to data envelopment analysis (DEA), cross-efficiency evaluation not only provides a ranking among the decision-making units (DMUs) but also eliminates unrealistic DEA weighting schemes without requiring a priori information on weight restrictions. A factor that possibly reduces the usefulness of the cross-efficiency evaluation method is that the cross-efficiency scores may not be unique due to the presence of alternate optima. As a result, it is recommended that secondary goals be introduced in cross-efficiency evaluation. This paper seeks to extend the model of Doyle and Green [1994. Efficiency and cross efficiency in DEA: Derivations, meanings and the uses. Journal of the Operational Research Society 45 (5), 567–578], by introducing a number of different secondary objective functions. The models are illustrated with examples.

Keywords: Data envelopment analysis (DEA); Cross efficiency

1. Introduction

Data envelopment analysis (DEA) provides a relative efficiency measure for peer decision-making units (DMUs) with multiple inputs and outputs. While DEA has proved to be an effective approach in identifying the best practice frontiers, its flexibility in weighting multiple inputs and outputs and its nature of self-evaluation have been criticized. The cross-evaluation method was developed as a DEA extension tool that can be utilized to identify best-performing DMUs and to rank DMUs using cross-efficiency scores that are linked to all DMUs (Sexton et al., 1986). The main idea of cross evaluation is to use DEA in a peer evaluation instead of a self-evaluation mode. There are two principal advantages of cross evaluation: (1) it provides a unique ordering of the DMUs, and (2) it eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from application area experts (Anderson et al., 2002).

Cross-efficiency evaluation has been used in various applications, e.g., efficiency evaluations of nursing homes (Sexton et al., 1986), R&D project selection (Oral et al., 1991), preference voting (Green et al., 1996) and others. Some studies on other DEA issues are very relevant with the cross-efficiency concept (see, e.g., Nicole et al., 2002; Beasley, 2003; Mavrotas and Trifillis, 2006).

However, as noted in Doyle and Green (1994), the non-uniqueness of the DEA optimal weights possibly

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reduces the usefulness of cross efficiency. Specifically, cross-efficiency scores obtained from the original DEA are generally not unique, and depending on which of the alternate optimal solutions to the DEA linear programs is used, it may be possible to improve a DMU's (cross efficiency) performance rating, but generally only by worsening the ratings of others. Sexton et al. (1986) and Doyle and Green (1994) propose the use of secondary goals to deal with the non-uniqueness issue. They present *aggressive* and *benevolent* model formulations. In the case of the benevolent model, for example, the idea is to identify optimal weights that maximize not only the efficiency of a particular DMU under evaluation but also the average efficiency of other DMUs. In the case of the aggressive model, one seeks weights that minimize the average efficiency of those other units.

The purpose of the current paper is to extend the model of Doyle and Green (1994) by introducing various secondary objective functions. Each new secondary objective function represents an efficiency evaluation criterion. With these new models, one can compare the efficiency scores and obtain a better picture of cross-efficiency stability with respect to multiple DEA weights.

The rest of this paper is organized as follows. Section 2 presents the cross-efficiency evaluation approach. New models are introduced in Section 3. Section 4 demonstrates the models with two data sets. Conclusions are given in Section 5.

2. Cross-efficiency evaluation

Suppose we have a set of n DMUs, and each DMU$_j$ produces s different outputs from m different inputs. The ith input and rth output of DMU$_j$($j = 1,2,\ldots,n$) are denoted by $x_{ij}(i = 1,\ldots,m)$ and $y_{rj}(r = 1,\ldots,s)$, respectively. Cross efficiency is often calculated as a two-phase process. The first phase is calculated using the standard DEA model, e.g., the CCR model of Charnes et al. (1978).

Specifically, for any DMU$_d$ under evaluation, the efficiency score E^*_{dd} under the CCR model is given by the following optimization problem:

$$
\begin{aligned}
&E^*_{dd} = \text{Max } E_{dd} = \frac{\sum_{r=1}^{s} u_{rd} y_{rd}}{\sum_{i=1}^{m} v_{id} x_{id}} \\
&\text{s.t. } E_{dj} = \frac{\sum_{r=1}^{s} u_{rd} y_{rj}}{\sum_{i=1}^{m} v_{id} x_{ij}} \leqslant 1, \quad j = 1,2,\ldots,n, \\
&u_{rd} \geqslant 0, \quad r = 1,\ldots,s, \\
&v_{id} \geqslant 0, \quad i = 1,\ldots,m, \qquad (1)
\end{aligned}
$$

where v_{id} and u_{rd} represent the ith input and rth output weights for DMU$_d$.

The cross efficiency of DMU$_j$, using the weights that DMU$_d$ has chosen in model (1), is then

$$E_{dj} = \frac{\sum_{r=1}^{s} u^*_{rd} y_{rj}}{\sum_{i=1}^{m} v^*_{id} x_{ij}}, d, \quad j = 1,2,\ldots,n, \qquad (2)$$

where (*) denotes optimal values in model (1). For DMU_j ($j = 1,2,\ldots,n$), the average of all $E_{dj}(d = 1,2,\ldots,n)$, $\overline{E_j} = 1/n\sum_{d=1}^{n} E_{dj}$, referred to as the *cross-efficiency score* for DMU_j.

We point out that the DEA model (1) is equivalent to the following linear program:

$$
\begin{aligned}
&\text{Max } E_{dd} = \sum_{r=1}^{s} u_{rd} y_{rd} \\
&s.t. \quad \sum_{i=1}^{m} v_{id} x_{id} = 1, \\
&\sum_{r=1}^{s} u_{rd} y_{rd} - \sum_{i=1}^{m} v_{id} x_{ij} \leqslant 0, \quad j = 1,\ldots,n, \\
&u_{rd}, v_{id} \geqslant 0. \qquad (3)
\end{aligned}
$$

Model (3) can also be expressed equivalently in the following deviation variable form:

$$
\begin{aligned}
&\text{Min } \alpha_d \\
&\text{s.t.} \quad \sum_{i=1}^{m} v_{id} x_{id} = 1 \\
&\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} + \alpha_j = 0, \quad j = 1,\ldots,n, \\
&u_r, v_i, \alpha_j \geqslant 0 \quad \text{for all } r,i,j, \qquad (4)
\end{aligned}
$$

where α_d is the deviation variable for DMU$_d$ and α_j the deviation variable for the jth DMU. Under this model, DMU$_d$ is efficient if and only if $\alpha^*_d = 0$. If DMU$_d$ is not efficient, then its efficiency score is $1-\alpha^*_d$ (α_d can be regarded as a measure of "inefficiency"). We refer to the deviation variable α_j as the *d-inefficiency* of DMU j.

Note that optimal weights obtained from model (3) (or model (4)) are usually not unique. As a result, the cross efficiency defined in (2) is arbitrarily generated, depending on the optimal solution arising from the particular software in use (Despotis, 2002). To resolve this ambiguity, a secondary goal in cross-efficiency evaluation is introduced. As discussed above, Doyle and Green (1994) present benevolent and aggressive model formulations that seek to identify optimal weights that not only maximize the efficiency of a particular DMU under evaluation but also minimize (maximize) the average efficiency of other DMUs. One form of the benevolent model focuses on finding a multiplier bundle that maximizes the ratio of outputs to inputs for the "composite" DMU made up of $n-1$ peer units. (The composite DMU is created by aggregating the outputs and inputs for all $n-1$ peer units.) The aggressive form of this would involve minimizing the ratio for the composite unit.

Let us now examine various forms of secondary goals for aiding in cross evaluation. For purposes of presentation, it is convenient to use model (4) as a basis for this discussion.

3. Alternative secondary goals

Let the (CCR) inefficiency of DMU_d be α_d^*. We first consider the following model where the secondary goal is to minimize the sum of "inefficiencies".

3.1. Minimizing total deviation from the ideal point

The *ideal point* is defined as that multiplier bundle $(\hat{u}, \hat{v})$ for which every DMU is efficient, that is $\sum_{r=1}^{s} u_r^d y_{rj} / \sum_{i=1}^{m} v_i^d x_{ij} = 1$, or $\sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} = 0$. In the absence of such an ideal point, a reasonable objective is to treat α_j as goal-achievement variables, and for each DMU d, derive a multiplier set that is an alternative optimum for that DMU, and that at the same time minimizes $\sum_{j=1}^{n} \alpha_j$. Specifically, our ideal point model is the following goal programming problem:

$$
\begin{aligned}
&\text{Min} \sum\nolimits_{j=1}^{n} \alpha'_j \\
&\text{s.t.} \sum\nolimits_{r=1}^{s} u_r^d y_{rj} - \sum\nolimits_{i=1}^{m} v_i^d x_{ij} + \alpha'_j = 0, \quad j = 1, \ldots, n, \\
&\sum\nolimits_{i=1}^{m} v_i^d x_{id} = 1, \\
&\sum\nolimits_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*, \\
&u_r^d, v_i^d, \alpha'_j \geqslant 0, \text{ for all } i, r, j. \qquad (5)
\end{aligned}
$$

In model (5), minimizing the sum of the d-inefficiencies α'_j ($j = 1,\ldots,n$) is intuitively appealing, and in the spirit of all DMUs attempting to maximize their respective performances. This model may be especially applicable to a system consisting of a set of units that seek to maximize their efficiency, such as would be the case in a supply chain setting where a set of business entities are involved in the design, development, manufacturing and distribution of products. Here it is assumed that each member of the supply chain is acting in its own self-interest, without being concerned for the other members of the supply chain. When the DMUs are assumed to be in a non-cooperative and fully competitive mode, this approach to cross evaluation would be appropriate.

We state without proof the following theorem.

Theorem 3.1. *Model* (5) *is equivalent to a form of the Doyle and Green (1994) model.*

3.2. Minimizing the maximum d-efficiency score

Troutt (1997) developed a maximum efficiency ratio DEA model in an effort to further prioritize the efficient DMUs using a common set of weights. He further shows that such a model may be regarded as a maximum likelihood procedure for a family of expert performance densities. In this regard, one might consider as a secondary goal, solving the model:

$$
\begin{aligned}
&\text{Min Max } \alpha'_j \\
&\text{s.t.} \sum\nolimits_{r=1}^{s} u_r^d y_{rj} - \sum\nolimits_{i=1}^{m} v_i^d x_{ij} + \alpha'_j = 0, \quad j = 1, \ldots, n, \\
&\sum\nolimits_{i=1}^{m} v_i^d x_{id} = 1, \\
&\sum\nolimits_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*, \\
&u_r^d, v_i^d, \alpha'_j \geqslant 0, \text{ for all } i, r, j. \qquad (6)
\end{aligned}
$$

Model (6) can be expressed equivalently in the following form:

$$
\begin{aligned}
&\text{Min } \theta \\
&\text{s.t.} \sum\nolimits_{r=1}^{s} u_r^d y_{rj} - \sum\nolimits_{i=1}^{m} v_i^d x_{ij} + \alpha'_j = 0, \ j = 1, \ldots, n, \\
&\sum\nolimits_{i=1}^{m} v_i^d x_{id} = 1, \\
&\sum\nolimits_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*, \\
&\theta - \alpha'_j \geqslant 0, j = 1, \ldots, n, \\
&u_r^d, v_i^d, \alpha'_j \geqslant 0, \text{ for all } i, r, j. \qquad (6')
\end{aligned}
$$

In model (6), minimizing the maximal d-inefficiency α'_j is related to maximizing the minimal efficiency among n efficiencies as in Troutt (1997). The above model derives a multiplier bundle that affords the maximum possible score to the "worse"-performing DMU. In so doing, the resulting efficiencies of the other DMUs may be forced to be closer in value. Specifically, in attempting to show this worst-performing DMU in its best possible light, the scores of the other (better-performing) DMUs may decrease, hence leading to DMU performance levels that display less variation than was previously the case. This approach might be deemed appropriate in those settings where a more cooperative situation prevails; an example would be the evaluation of maintenance crews under a central authority, or bank branches under a single corporate head, where the worst-performing units would be given the least gap possible between where they are and where they need to be.

3.3. Minimizing the mean absolute deviation

In the spirit of seeking to minimize the variation among the efficiencies of the DMUs, we propose formalizing this concept through:

$$\begin{aligned}
&\text{Min } \frac{1}{n}\sum\nolimits_{j=1}^{n}|\alpha'_j-\overline{\alpha'}| \\
&\text{s.t. } \sum\nolimits_{r=1}^{s}u_r^d y_{rj}-\sum\nolimits_{i=1}^{m}v_i^d x_{ij}+\alpha'_j=0, \quad j=1,\ldots,n, \\
&\sum\nolimits_{i=1}^{m}v_i^d x_{id}=1, \\
&\sum\nolimits_{r=1}^{s}u_r^d y_{rd}=1-\alpha_d^*, \\
&u_r^d, v_i^d, \alpha'_j \geqslant 0, \text{ for all } i,r,j, \qquad (7)
\end{aligned}$$

where $\overline{\alpha'}=1/n\sum_{j=1}^{n}\alpha'_j$.

The objective function in Model (7) computes the mean absolute deviation of a set of data, namely, the average of the absolute deviations of data points from their mean. Therefore, minimizing the objective function tries to decrease the efficiency difference among DMUs, which to some extent demonstrates an equalitarian principle.

To show that this nonlinear model can be linearized, let $a'_j=\frac{1}{2}(|\alpha'_j-\overline{\alpha'}|+\alpha'_j-\overline{\alpha'})$ and $b'_j=\frac{1}{2}(|\alpha'_j-\overline{\alpha'}|-(\alpha'_j-\overline{\alpha'}))$. Then, model (7) becomes the following linear programming problem:

$$\begin{aligned}
&\text{Min} \frac{1}{n}\sum\nolimits_{j=1}^{n}(a'_j+b'_j) \\
&\text{s.t. } \sum\nolimits_{r=1}^{s}u_r^d y_{rj}-\sum\nolimits_{i=1}^{m}v_i^d x_{ij}+\alpha'_j=0, \quad j=1,\ldots,n, \\
&\sum\nolimits_{i=1}^{m}v_i^d x_{id}=1, \\
&\sum\nolimits_{r=1}^{s}u_r^d y_{rd}=1-\alpha_d^*, \\
&a'_j-b'_j=\alpha'_j-\frac{1}{n}\sum\nolimits_{j=1}^{n}\alpha'_j, \quad j=1,\ldots,n, \\
&u_r^d, v_i^d, a'_j, b'_j, \alpha'_j \geqslant 0, \text{ for all } i,r,j. \qquad (7')
\end{aligned}$$

Both models (6′) and (7′) are thus aimed at deriving a set of weights u^d, v^d for which the d-efficiency scores are as similar as possible. The latter model more directly aims at *equalizing* the various efficiency scores, as opposed to simply making the worst-performing unit as well off as possible. In some respects, this criterion would apply to the same settings as the model of Section 3.2, but more overtly tries to make all units as close as possible to being equally efficient. In a situation where there was an allocatable resource such as equipment for the maintenance crews, measuring efficiency via the model of this section might tend to result in the least amount of redistribution (to render the DMUs equally efficient) in regard to that resource.

In the above models, when DMU_d under evaluation is changed (i.e., $x_{id}, i=1,\ldots,m$; $y_{rd}, r=1,\ldots,s$ and α_d^* are changed in the constraints), different optimal solutions of $v_i{}^d$ and $u_r{}^d$ are obtained. We obtain n optimal weight vectors $W_d^*=(v_1^{d*},\ldots,v_m^{d*},u_1^{d*},\ldots,u_s^{d*})$, $d=1,\ldots,n$. Using this W_d^*, the cross efficiency for any $\text{DMU}_j (j=1,2,\ldots n)$ is then calculated as

$$E_j(W_d^*)=\frac{\sum_{r=1}^{s}u_r^{d*}y_{rj}}{\sum_{i=1}^{m}v_i^{d*}x_{ij}}, d, \quad j=1,2,\ldots,n. \qquad (8)$$

For $\text{DMU}_j (j=1,2,\ldots n)$, the average of all $E_j(W_d^*), d=1,\ldots,n$, namely

$$\overline{E_j}=\frac{1}{n}\sum\nolimits_{d=1}^{n}E_j(W_d^*), \quad j=1,2,\ldots,n \qquad (9)$$

is our new cross-efficiency score for DMU_j.

4. Illustration

4.1. Chinese cities

Table 1 provides 13 open coastal Chinese cities and 5 Chinese special economic zones in 1989. Two inputs and three outputs were chosen to characterize the technology of those cities/zones (see Zhu, 1998).

Input 1 (x_1): Investment in fixed assets by state-owned enterprises (10,000 RMB), where RMB is the Chinese monetary unit;

Input 2 (x_2): Foreign funds actually used (10,000 US$);

Output 1 (y_1): Total industrial output value (based on fixed prices of 1980) (10,000 RMB);

Output 2 (y_2): Total value of retail sales (10,000 RMB);

Output 3 (y_3): Handling capacity of coastal ports (10,000 tones).

The second and third columns of Table 2 report the CCR efficiency scores and rankings, respectively. For this data set, all our models yield identical cross-efficiency scores, reported in the fourth column of Table 2. This is a good indication that the cross-efficiency scores are unique or stable.

Table 1
Chinese city data

DMU no.	Cities/zones	Input 1	Input 2	Output 1	Output 2	Output 3
1	Dalian	2874.8	16,738	160.89	80,800	5092
2	Qinhuangdao	946.3	691	21.14	18,172	6563
3	Tianjin	6854.0	43,024	375.25	144,530	2437
4	Qingdao	2305.1	10,815	176.68	70,318	3145
5	Yantai	1010.3	2099	102.12	55,419	1225
6	Weihai	282.3	757	59.17	27,422	246
7	Shanghai	17,478.6	116,900	1029.09	351,390	14,604
8	Lianyungang	661.8	2024	30.07	23,550	1126
9	Ningbo	1544.2	3218	160.58	59,406	2230
10	Wenzhou	428.4	574	53.69	47,504	430
11	Guangzhou	6228.1	29,842	258.09	151,356	4649
12	Zhanjiang	697.7	3394	38.02	45,336	1555
13	Beihai	106.4	367	7.07	8236	121
14	Shenzhen	4539.3	45,809	116.46	56,135	956
15	Zhuhai	957.8	16,947	29.20	17,554	231
16	Shantou	1209.2	15,741	65.36	62,341	618
17	Xiamen	972.4	23,822	54.52	25,203	513
18	Hainan	2192.0	10,943	25.24	40,267	895

Table 2
Chinese city results

DMU	CCR score	Rank	Cross efficiency	Rank
1	0.46907	11	0.44608	10
2	1	1	1	1
3	0.27791	15	0.24359	15
4	0.50222	8	0.45216	9
5	0.63108	7	0.60498	6
6	1	1	0.97223	2
7	0.35804	12	0.30466	12
8	0.49594	9	0.45446	8
9	0.65766	6	0.56927	7
10	1	1	0.88699	3
11	0.30097	14	0.27884	14
12	0.78661	4	0.65762	4
13	0.75144	5	0.60856	5
14	0.1382	18	0.12883	18
15	0.18671	17	0.16646	16
16	0.47037	10	0.38768	11
17	0.30594	13	0.28193	13
18	0.19526	16	0.15127	17

4.2. Nursing homes

Sexton et al. (1986) considered a case of six nursing homes whose input and output data for a given year are reported in Table 3, where the input and output variables are defined as follows:

StHr($x1$): staff hours per day, including nurses, physicians, etc.

Supp($x2$): supplies per day, measured in thousands of dollars

MCPD($y1$): total medicare-plus medicaid-reimbursed patient days(0000)

PPPD($y2$): total privately paid patient days(0000).

Table 4 reports the results. The second column reports the CCR efficiency scores. Columns 3–5 report the cross-efficiency scores based upon models (5)–(7). It can be seen that models (5) and (7) yield identical results. Note that model (5) actually uses the benevolent criterion. This indicates that in this case, the benevolent criterion also tries to make all the cross-efficiency scores as closer as possible.

5. Conclusions

Because DEA weights are generally not unique, the related cross efficiency may not be unique either. It is this non-uniqueness phenomenon that can undermine the usefulness of the cross-evaluation method. This paper seeks to extend the model of Doyle and Green (1994), in which the ultimate cross efficiency of every DMU is achieved by introducing a secondary objective function. In this paper, different secondary objective functions are used to determine the ultimate cross efficiency, and the proposed models with their different

Table 3
Nursing home data

DMU	Inputs		Outputs	
	StHr (x_1)	Supp (x_2)	MCPD (y_1)	PPPD (y_2)
A	1.50	0.2	1.40	0.35
B	4.00	0.7	1.40	2.10
C	3.20	1.2	4.20	1.05
D	5.20	2.0	2.80	4.20
E	3.50	1.2	1.90	2.50
F	3.20	0.7	1.40	1.50

Table 4
Nursing home results

	CCR	Cross efficiency score based on		
		Model (5)	Model (6)	Model (7)
DMU_1	1	1	1	1
DMU_2	1	0.9547	0.9617	0.9547
DMU_3	1	0.8864	0.8759	0.8864
DMU_4	1	1	1	1
DMU_5	0.9775	0.9742	0.9748	0.9742
DMU_6	0.8675	0.8465	0.8499	0.8465

objective functions can be applied under different circumstances.

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