Best-performing US mutual fund families from 1993 to 2008: Evidence from a novel two-stage DEA model for efficiency decomposition

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\textbf{A B S T R A C T}

When analyzing relative performance, especially at the institutional level, the traditional data envelopment analysis (DEA) models do not recognize vastly different and important activities as separate functions and therefore cannot identify which function may be the main source of inefficiency. We propose a novel two-stage DEA model that decomposes the overall efficiency of a decision-making unit into two components and demonstrate its applicability by assessing the relative performance of 66 large mutual fund families in the US over the period 1993–2008. By decomposing the overall efficiency into operational management efficiency and portfolio management efficiency components, we reveal the best performers, the families that deteriorated in performance, and those that improved in their performance over the sample period. We also make frontier projections for poorly performing mutual fund families and highlight how the portfolio managers have managed their funds relative to the others during financial crisis periods.

\section{1. Introduction}

The mutual fund industry in the US is by far the largest such industry in the world, managing US$11.2 trillion in assets by the end of 2009. In this paper, we propose a novel data envelopment analysis (DEA) model to investigate the relative performance of 66 large (in terms of total funds) mutual fund families in the US over the 16-year period 1993–2008. Research on individual mutual fund performance is vast and is spread widely across different markets (Treynor, 1965; Sharpe, 1966; Jensen, 1968; Hendricks et al., 1993; Goetzmann and Ibbotson, 1994; Malkiel, 1995; Elton et al., 1996; Carhart, 1997; Blake and Morey, 2000). However, the focus on the performance at the mutual fund family level is limited only to a few (Tower and Zheng, 2008; Elton et al., 2007), possibly due to the complex nature of the analysis involved. This paper fills this gap by focusing on the relative performance of mutual fund families. This is an important area of study, as investors in mutual funds generally tend to invest in funds within the same mutual fund family rather than across a number of families. The reasons for investing within one mutual fund family include convenience in searching for investment opportunities and recordkeeping (Kempf and Ruehnl, 2008) and flexibility of switching funds without additional sales charges and restrictions imposed by the fund family (Elton et al., 2006, 2007).

We investigate the relative performance of large mutual fund families using a novel two-stage DEA model where the overall efficiency of a mutual fund family is decomposed into two components, namely, operational management efficiency and portfolio management efficiency: that is, we conceptualize fund family management as a two-stage process that consists of an operational management stage (stage 1) and a portfolio management stage (stage 2). Therefore, the overall efficiency of a fund family is a composition of operational management efficiency and portfolio management efficiency. The aim of decomposing the overall efficiency is to capture which of the two stages may have greater influence on the overall efficiency of the mutual fund family. Hence, our study is of interest to all stakeholders: the investors, the fund managers, and the management companies of mutual fund families.

Previous studies of mutual fund family performance consider overall return as the measure of performance. Kapur and Timmerman (2005) note that when the share market has performed very
well, the absolute return-based performance of mutual funds is an unreliable measure of managerial ability. They acknowledge that during a bullish market, it is more appropriate to remunerate fund managers based on relative performance rather than on absolute performance. Cooper et al. (2004) argue that DEA, which is a non-parametric method, is ideal for assessing relative performance.\(^1\) DEA models have the advantage of assessing performance in a multi-dimensional framework; that is, they can accommodate multiple inputs and multiple outputs.

For the efficiency decomposition, we propose a novel two-stage DEA model with \(i_2\) inputs to stage 1, \(D\) intermediate measures, \(i_2\) inputs to stage 2 (in addition to the intermediate measures), and \(s\) outputs from stage 2. The proposed model aligns with the network approach of Färe and Whittaker (1995). The novel aspect of our model stems from several methodological advances. Unlike previous work, we model efficiencies of both stages simultaneously and therefore our model adopts a non-standard approach. The proposed DEA model not only assesses the overall performance, it decomposes the overall efficiency into two components associated with the performance of each stage. Such decomposition of efficiency is not possible in the previous network approach of Färe and Whittaker (1995). Furthermore, our approach is not restrictive in terms of orientation as in Kao and Hwang’s (2008) two-stage model, which is valid only under the constant returns-to-scale (CRS) assumption. Our approach can be applied under CRS as well as variable returns-to-scale (VRS) situations. Holod and Lewis (2011) propose a two-stage DEA model to resolve the deposit dilemma. However, their model is not capable of obtaining separate efficiency estimates for each stage. In addition to the decomposition of overall efficiency, another major difference between our model and that of Holod and Lewis (2011) is that our model allows new variables in the second stage as inputs in addition to the intermediate variables that link stages 1 and 2.

The proposed two-stage DEA model is applied to investigate the relative performance of the mutual fund families as follows. In the first stage, we focus on the operational management efficiency of each mutual fund family by considering how efficiently they make use of inputs, such as marketing and distribution expenses and management fees, in producing the output, which is the net asset value. In the second stage, we focus on portfolio management efficiency by estimating how efficiently the mutual fund families make use of inputs, such as fund size, standard deviation of the returns, turnover ratio, expense ratio, and net asset value, in producing the output, which is the average return of the fund family; that is, the mutual fund family application presented in this paper is an illustration of the general two-stage DEA model with \(i_2 = 2, D = 1, i_2 = 4,\) and \(s = 1\). Although there is one output from the first stage and one output from the second stage in this particular application, the DEA model proposed in this paper works under multiple inputs, outputs, and intermediate measures.

In our formulation, we treat net asset value (NAV), which is the output variable of the first stage, as an input variable in the second stage; that is, net asset value is modeled as an intermediate variable that links stage 1 and stage 2. Holod and Lewis (2011) treat deposits in the same way in the two-stage DEA model they use in assessing bank performance. Brown et al. (2001) point out that even though relative performance appears to be the overriding concern of fund managers as well as their clients, considerably less attention is directed towards the equally important question of assessing the relative performance of portfolios. In this paper, we make a significant contribution to the literature on this issue by providing a methodology that is robust and flexible. Our modeling approach is general and hence can be applied to assess the performance of other financial institutions as well. For instance, it can be applied to assess the operational management efficiency and portfolio management efficiency of finance sector institutions, such as insurance companies, banks and credit unions.

The rest of the paper is organized as follows. Section 2 discusses the background of our study. Section 3 provides a general description of the proposed two-stage DEA model. Specific details regarding the formulation of the DEA model are provided in Appendix A, and the model used for frontier projection is given in Appendix B. Section 4 is devoted to a discussion of the data, sample selection, and calibrating the input–output variables used in the DEA model. The results are presented in Section 5, and Section 6 concludes the paper.

### 2. Background

The mutual fund industry in the US has shown significant growth over the last 10 years, having almost doubled in size since 1999. By the end of 2009 (Investment Company Institute, 2010), the number of firms constituting the US mutual fund industry was approximately 600, with 53% and 74% of the assets under management controlled by the top 10 and top 25 firms, respectively, up from 44% and 68% in 2000. The US$11.2 trillion worth of funds managed by the US fund industry accounts for 48% of the total managed funds worldwide.

The research on the performance evaluation of individual mutual funds is vast, but very little attention has been given to the performance evaluation of fund families. In a recent study, Tower and Zheng (2008) assess the relative performance of mutual fund families directly, and therefore their assessment may be viewed as being one-dimensional. We assess performance in a multi-dimensional framework. Tower and Zheng evaluate the performance of 51 US mutual fund families over the 11-year sample period 1994–2005 and rank them according to (i) a trading index constructed with 11 market indices, (ii) Wilshire 5000 index returns, and (iii) returns on historical portfolios by taking into consideration different classes of mutual funds. Tower and Zheng's sample of mutual fund families is restricted to the families that held 75% of their assets in diversified equities and no more than 5% of their assets in foreign stocks throughout their life span. They excluded sector funds, international funds, global funds, balanced funds, and bond funds, and their analysis at the family level is carried out with equally weighted returns of individual funds included in the fund family portfolio.

In 1996, Barron's introduced the first-ever ranking of mutual fund families based on their performance (Budgar, 1996). Barron's ranking takes into consideration five investment categories—(i) domestic equity, (ii) world equity, (iii) mixed equity (stocks and bonds), (iv) taxable bonds, and (v) tax-exempted funds—and investigates how each fund is ranked against the other funds in the same category. Barron's obtains each family's return in the five categories weighted on the size of the individual funds and then takes the average as the performance measure of the entire family (Strauss, 1985; Reinker and Tower, 2004). Barron's ranking is dominated by domestic equity, as it has the largest weighting of about 49%.

The above-mentioned studies of mutual fund family performance consider overall return as the basic measure of performance. Further, they restrict the analysis by focusing on investment categories to scale down the complexity of the problem. Our approach to ranking mutual fund families is different from previous approaches in at least two important aspects. First, we evaluate the overall performance of a mutual fund family relative to the other families in the sample by combining several fac-

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\(^1\) In the DEA literature, the terms “relative efficiency” and “relative performance” are used interchangeably.
tors, such as returns, fees and charges, investment risk, stock selection style, and portfolio management skills, into a single measure of performance. Second, the DEA model proposed in this study is based on a framework where the overall efficiency of a fund family is decomposed into two components, reflecting the efficiencies of two stages of the overall process, namely, operational management and portfolio management. It is to be noted here that as we do not restrict our analysis to specific investment categories as in the previous studies, the rankings of fund families obtained in our method is not comparable with the rankings of previous studies.

3. Development of the proposed two-stage DEA model

DEA is a methodology that is used to assess the relative efficiency of like entities referred to as decision-making units (DMUs). DEA is more appealing than conventional measures of performance, since DEA can assess performance in a multi-dimensional framework by accommodating multiple inputs and outputs. Cook et al. (2010) point out that in many instances, the underlying process of generating output from input may have a two-stage network structure with intermediate measures where outputs from the first stage become the inputs to the second stage. Chilingerian and Sherman (2004) describe a two-stage process used in measuring physician care. Their first stage is a manager-controlled process and the second stage is a physician-controlled process. Their inputs to the first-stage generate an intermediate measure as the output, which becomes the input to the second stage. Kao and Hwang (2008) consider the process of Taiwanese non-life-insurance companies as a two-stage process of premium acquisition and profit generation. The novelty in the two-stage DEA model proposed in this paper is its ability to obtain separate efficiency estimates for each stage of the two-stage process as well as the overall efficiency estimate for the entire process.

In our application, we assume that the activities of mutual fund families can be viewed as a two-stage process. As illustrated in Fig. 1, stage 1 represents the operational management process and stage 2 represents the portfolio management process. In other words, the overall efficiency of a mutual fund family is conceptualized as made up of two components: operational management efficiency (hereinafter referred to as operational efficiency) and portfolio management efficiency (hereinafter referred to as portfolio efficiency).

In stage 1, the fund family management makes an attempt to attract funds from investors, and therefore outgoings, such as management fees \(I_1\) and marketing and distribution expenses \(I_2\), that contribute directly towards generating funds are considered as input variables. In stage 1, we consider the net asset value labeled \(O_1\) in Fig. 1 as the output variable. Hence, a mutual fund family that can produce the highest net asset value with the least amount of management fees and marketing and distribution expenses will be operationally more efficient than the other families in the sample. Stage 2 is the portfolio management stage. Here we treat net asset value \(O_1\), fund size \(I_3\), net expense ratio \(I_4\), turnover ratio \(I_5\), and standard deviation of the returns of the family portfolio over the last 3 years \(I_6\) as the input variables and mean return of the family portfolio \(O_2\) as the output variable. Since net asset value \(O_1\), which is an output variable of stage 1, is also an input variable of stage 2 \(I_7\), it becomes an intermediate variable. \(I_7\) is not observable; it is obtained by adjusting \(O_1\), which is an observed variable. In stage 2, a fund family that can produce the highest average family portfolio return with the least amount of net asset value, fund size, net expense ratio, turnover ratio, and standard deviation is deemed more efficient compared with the other families in the sample.

A common approach to solving two-stage problems is to assume that the two stages operate independently and to apply a standard DEA model separately in each stage. Various problems can arise with this approach. In stage 1, for example, DMUs may attempt to maximize outputs in order to show their performance in the best possible light. As these outputs from stage 1 become inputs to the second stage, high output from stage 1 may lead to assessment of this DMU poorly in the second stage if the objective is of a maximization type. Kao and Hwang (2008) overcome this problem under the CRS assumption by assessing the overall efficiency of the two-stage process as the product of the efficiencies of the two stages. Chen et al. (2009) extend the Kao and Hwang (2008) approach by using additive efficiency decomposition under CRS and VRS. In this study, we use the VRS assumption, as some of the variables (e.g., returns) can be negative. The standard VRS DEA model has the translation invariance property so that a constant may be added to all values of the concerned variable to make them positive without altering the efficient frontier and the position of the DMUs relative to the efficient frontier (see Ali and Seiford, 1990).

The situation depicted in Fig. 1 is different from the two-stage process considered in Kao and Hwang (2008) and Chen et al. (2010). The proposed two-stage process allows new inputs to the second stage in addition to intermediate measures. The network DEA approach of Färe and Whittaker (1995) and Färe and Grosskopf (1996), the slack-based network DEA approach of Tone and Tsutsui (2009), and the dynamic effects in production networks of Chen (2009) are more general versions of the two-stage process described in Fig. 1, but they do not yield efficiencies at individual stages. We have overcome this problem in the DEA model proposed in this paper. For a review of the relevant recent literature on two-stage processes, see Cook et al. (2010). An application of the network DEA approach is available in Lewis and Sexton (2004). The existing approaches cannot be readily adopted to model the situation depicted in Fig. 1.

In order to understand the basic concepts behind the proposed two-stage DEA model, consider the simplified version presented in Fig. 2. Suppose we have one input \((x_1)\) to stage 1, one intermediate measure \((z)\), one additional input \((x_2)\) to stage 2, and one output \((y)\) from stage 2. To measure the overall efficiency of the two-stage process, first we calculate the expected (efficient) output \(y\) from input \(x_1\) indirectly and from input \(x_2\) directly with an intermediate

![Fig. 1. The proposed two-stage DEA model for evaluating the efficiency of mutual fund families. At stage 1, the operational management efficiency will be estimated, and at stage 2 the portfolio management efficiency will be estimated. The overall efficiency of the fund family is decomposed into the operational management efficiency (stage 1) and the portfolio management efficiency (stage 2). Variables \(I_1\) and \(I_2\) are the input variables and \(O_1\) is the output variable at stage 1 and \(I_3, I_4, I_5, I_6\) and \(I_7\) are the input variables and \(O_2\) is the output variable at stage 2. Net asset value is an intermediate variable and therefore \(I_7\) is the expected value of \(O_2\) estimated in stage 1.](image-url)
measure $z$. Assume that the DMU should have produced $z^*$ with input $x_1$ had it operated efficiently in stage 1 and should have produced $y^*$ with inputs $z$ and $x_2$ in stage 2. Then a measure of overall efficiency is $y / y^*$, a measure of stage 1 efficiency is $z / z^*$, and a measure of stage 2 efficiency is $(z^* + x_2) / (z + x_2)$.

When calculating the expected (efficient) output of stage 2, we require the intermediate measure to be the expected (efficient) output of stage 1. When this concept is generalized to the case with multiple intermediate measures, the "aggregate" value of intermediate measures must remain the same. According to Liang et al. (2008), such a modeling process treats the two stages as players in a cooperative game where both players "negotiate" on the expected value of intermediate measures. Such a modeling process does not fit into a standard DEA approach. Rather, it optimizes a joint efficiency of the two stages subject to the condition that the intermediate input to stage 2 is the expected output from stage 1. In that regard, the approach used in the two-stage DEA model proposed in this paper is very different from the iterative process used by Holod and Lewis (2011). Their two-stage process is based upon a non-oriented standard DEA model and is also not capable of computing separate efficiency estimates for each stage.

Next, we describe the DEA-based procedure used in this paper to model the relationship between the overall efficiency and the efficiencies at stage 1 and stage 2 in a single mathematical model under the VRS assumption.

Consider a general two-stage DEA network structure for DMU-$j$ with $i_1$ inputs to stage 1 denoted by $X_1^j = (x_{11}^j, x_{12}^j, \ldots, x_{1i_1}^j)$, $i_2$ inputs to stage 2 denoted by $X_2^j = (x_{21}^j, x_{22}^j, \ldots, x_{2i_2}^j)$, and $s$ outputs from stage 2 denoted by $y_j (r = 1, \ldots, s)$. With respect to our mutual fund family example in Fig. 1, $X^j$ has two input variables, $X^j$ has four input variables, $z$ has one variable, and $y$ has one variable. Following Banker et al. (1984), the VRS efficiency score of $DMU_j$ at the first and second stages can be calculated using models (1) and (2), respectively.

\[
\begin{align*}
\text{Max} & \sum_{i_1} u_{i_1} y_{0i} + u^1 \\
\text{s.t.} & \sum_{i_1} v_{i_1} x_{0i} + u^1 x_{0i} \leq 1, \quad j = 1, 2, \ldots, n \\
& v_{i_1}^1, u_{i_1} \geq \varepsilon; \quad u^1 \text{ free}
\end{align*}
\]

\[
\begin{align*}
\text{Max} & \sum_{i_2} w_{i_2} z_{2o} + u^2 \\
\text{s.t.} & \sum_{i_2} w_{i_2} x_{2i_2} + u^2 x_{2i_2} \leq 1, \quad j = 1, 2, \ldots, n \\
& w_{i_2}, u_{i_2} \geq \varepsilon; \quad u^2 \text{ free}
\end{align*}
\]

($v_{i_1}^1, u_{i_1}^1$) are decision variables (weights) to be optimized for the inputs to the first stage and the intermediate measures (outputs from the first stage), $u^1$ is the free variable associated with returns to scale (RTS) in DEA for stage 1.

\[
\begin{align*}
\text{Max} & \sum_{i_2} u_{i_2} y_{0i} + u^2 \\
\text{s.t.} & \sum_{i_2} v_{i_2} z_{2o} + \sum_{i_2} v_{i_2} y_{2i_2} \leq 1, \quad j = 1, 2, \ldots, n \\
& v_{i_2}, u_{i_2} \geq \varepsilon; \quad u^2 \text{ free}
\end{align*}
\]

($v_{i_2}^2, u_{i_2}^2$) are decision variables (weights) to be optimized for the inputs to the second stage, the intermediate measures, and outputs from the second stage. $u^2$ is the free variable associated with RTS in DEA for stage 2.

Note that if we assume $u^1 = u^2 = 0$, then the above models become the CRS models of Charnes et al. (1978), and therefore the following discussion is applicable to the CRS case as well. Similar to Kao and Hwang’s (2008) assumption and the centralized model in Liang et al. (2008), we assume that $\eta_{j1}^1 = \eta_{j2}^2 = \eta_{j} (d = 1, \ldots, D)$ in models (1) and (2). This assumption ensures that in both stages the same multipliers (weights) are applied to the intermediate measures. Then, as far as the intermediate variables are concerned, the expected outputs from stage 1 will be equal to the expected inputs to the second stage.

As in Chen et al. (2009), we compute the overall efficiency as a weighted average of the efficiency scores from stages 1 and 2 as

\[
w_1 \sum_{i_1} u_{i_1} y_{0i} + u^1 + w_2 \sum_{i_2} u_{i_2} y_{2i_2} + u^2 / (w_1 \sum_{i_1} v_{i_1} x_{0i} + u^1 + w_2 \sum_{i_2} v_{i_2} x_{2i_2} + u^2)
\]

where $w_1$ and $w_2$ are user-specified weights such that $w_1 + w_2 = 1$. If the geometric average as in Kao and Hwang (2008) is used, the product of $\sum_{i_1} v_{i_1} x_{0i} + u^1$ and $\sum_{i_2} v_{i_2} x_{2i_2} + u^2$ will not yield a linear objective function due to the fact that $\sum_{i_1} v_{i_1} x_{0i} + u^1$ cannot be cancelled. If we assume that $x_{2i_2}^1 = 1$ and $u^1 = 0$, the model would reduce to the CRS version and then the approach of Kao and Hwang (2008) can be applied.

Appendix A provides the details on how we develop our new DEA network model for a general two-stage network structure by converting (3) along with models (1) and (2) when $\eta_{j1}^1 = \eta_{j2}^2 = \eta_j$ ($d = 1, \ldots, D$). Appendix A also shows how to decompose the overall efficiency and develops a procedure for testing unique efficiency decomposition.

As in the conventional DEA models, the efficiency scores obtained for stages 1 and 2 provide information on how an inefficient unit can improve its performance. However, as noted in Chen et al. (2010), one needs to rely on the envelopment form of the DEA model to derive the DEA frontier for two-stage processes because the optimal (frontier projection) intermediate measures need to be determined. Note that our two-stage network structure is different from the one discussed in Chen et al. (2010), with added additional multiple inputs to the second stage. Therefore, in Appendix B we develop a new model for providing information on how to improve the DMUs’ performance under our newly developed two-stage DEA network model.

4. Data, sample selection, and variable construction

The data on US mutual funds are obtained from the Morningstar Direct database. The sample consists of 66 large mutual fund families with total funds under management in each family exceeding $1$ billion USD. The sample period is January 1993 to December 2008 (a total of 1056 family years). The 66 families comprise 1269 individual mutual funds, adding up to 20,304 fund years. For each of these individual funds, we compute monthly return and monthly standard deviation over the 16-year sample period.

Some funds have multiple share classes depending on the fee structure, and we consider them as separate mutual funds. Furthermore, we found that some families may offer the same fund with multiple share classes, and we consider them as separate mutual funds. For each of these individual funds, we identify if multiple additional inputs to the second stage. Therefore, in Appendix B we develop a new model for providing information on how to improve the DMUs’ performance under our newly developed two-stage DEA network model.
data that we require. Therefore, we consider only large mutual fund families with total fund under management in each family of at least $1 billion USD. Out of a total of 198 families reported in 2008, 101 families (51%) have a total fund size of at least $1 billion USD. Out of these 101 families, 35 families (34.7%) are dropped from the study due to non-availability of data on all the input and output variables given in Fig. 1. Therefore, our final sample contains 66 mutual fund families. Most of the families that we dropped from the study are small; that is, the fund size of 19 out of the 35 families dropped (54.3%) is less than $4 billion USD. The two largest families dropped from the study are PIMCO Funds (fund size of $217 billion USD with three mutual funds in it) and Dodge and Cox (fund size of $71 billion USD with three mutual funds in it). Total funds under management in each of the other 14 families dropped from the analysis are between $4 billion USD and $40 billion USD. In DEA, the efficiencies of mutual fund families are assessed relative to the other families in the sample, and therefore dropping large families from the sample may affect efficiency scores. However, as only a very small percentage of the funds dropped are large, their impact on the overall assessment is minimal. Even though the primary focus of this paper is to introduce a novel two-stage DEA model for efficiency decomposition, we make a significant effort to minimize the survivorship bias in the numerical example that we use here to demonstrate the applicability of the proposed model.

In mutual fund research, survivorship bias is an important issue. According to Carhart (1997), data used in mutual fund research may often be incomplete due to the following reasons. During the sample period, some funds may have ceased operations or some funds may not report data in poorly performing years. The availability of all the individual fund-level data for the 66 families in our sample throughout the entire survey period implies that all the funds in those selected families are healthy funds and none of them have ceased operations during the survey period.

Summary statistics for the 66 mutual fund families selected in our sample and sorted by total funds under management as of 2008 are presented in Table 1. American Funds is by far the largest in terms of funds under management ($1490 billion USD). Vanguard is the next largest, with $579 billion USD worth of funds under its control. In our sample, the fund family that offers the greatest number of individual mutual funds is Fidelity Investments, with 94 mutual funds worth $418 billion USD under its management. We consider each mutual fund family in the sample as a separate DMU.

The list of input and output variables used in the DEA models is given in Table 2. As illustrated in Fig. 1, stage 1 has two inputs and one output and stage 2 has five inputs and one output. These variables are selected following previous studies of mutual fund performance, such as Malhotra et al. (2007), Choi and Murthi (2001), Murthi et al. (1997), Nguyen-Thi-Thanh (2006) and Wilkins and Zhu (2005). For each family, the values of the input and output variables are calculated for each year from 1993 to 2008 using the data collected on the individual mutual funds in the family. A description of how the input and output variables used in stage 1 and stage 2 of the proposed model are calculated is given in Table 2.

5. Empirical application with US mutual fund families

First, we analyze the overall performance of mutual fund families based on their overall efficiency estimates obtained in the proposed two-stage DEA procedure described in Appendix A. Thereafter, we analyze the operational efficiency and the portfolio efficiency scores estimated in the proposed two-stage DEA model to gain insights on the source of the efficiency or inefficiency of fund families.

5.1. Overall efficiency estimated in the two-stage DEA model

Table 3 lists the 16 families that have performed consistently well overall over the most recent 3-year period from 2006 to 2008 based on the overall efficiency estimated in the two-stage DEA model. We judge the consistency of performance of a mutual fund family by the number of times a family has been ranked in the top 2, top 3, and so on up to top 10 during the 3-year period. Since the investigation period is 3 years, the maximum frequency possible under each category is 3. Table 3 reveals that Vanguard is clearly the best performer over the investigation period (ranked top 2 in all 3 years), followed by Fidelity Investments (ranked top 3 twice), Hartford Mutual Funds (ranked top 4 twice and top 5 three times), Allegiant (ranked top 4 twice and top 6 three times), and American Funds (ranked top 6 twice). It is not surprising that the Vanguard family of funds is the top performer over the most recent 3-year sample period, given its dominance with respect to the market share in terms of funds under passive management (Smith, 2010) and adherence to the fund family gospel that low-cost investments deliver the best returns (Dunstan, 2012). The Vanguard Group provides the necessary services to run the funds on an at-cost basis (Bogle, 2004). As a result, Vanguard has the reputation within the fund management industry as having the lowest operating expenses. In 2008, the Vanguard funds cost, on average, 0.27% of assets, or about 25% of the industry average (Morningstar, 2012). Vanguard is well known among investors for offering mutual funds with the lowest, or close to the lowest, annual operating expenses, and hence the high overall efficiency is not surprising. All the five fund families identified above (Vanguard, Fidelity, Hartford, Allegiant, and American) have substantial market share and a long history averaging over 80 years. Further, they received rankings in the top quartile for the 2007 fund family rankings released by Barron’s based on the performance in 2006.

On the other hand, American Century Investments and Neuberger Berman are ranked in the top 2 in one of the 3 years, and in the other 2 years both are ranked below 10, showing inconsistency in their performance from 2006 to 2008. The poor performance of Neuberger Berman after 2006 can be linked to the fallout of the global financial crisis.

Now we discuss consistency in the performance over a longer period, the 5 years from 2004 to 2008. Table 4 shows the best 17 mutual fund families based on overall performance over the 5-year period. As seen in Table 4, during this period Vanguard is always ranked in the top 2 and is clearly the best performer. Neuberger Berman is the next best, followed by Fidelity investments, Hartford Mutual Funds, and T. Rowe Price. Two out of these five families, Neuberger Berman and T. Rowe Price, do not feature in the list of the five best performers over the most recent 3-year period. The same 16 families reported in Table 3 also performed better than the other sampled families over the 5-year period from 2004 to 2008.

Similarly, we investigated the overall performance of the mutual fund families over the 10-year period from 1999 to 2008, and the results (not given here) reveal that Vanguard and Nuveen always rank within the top 10; and Aquila, Franklin Templeton Investments, Allegiant, and American Funds have been ranked under this category at least seven times.

One of the main contributions of the proposed two-stage DEA model compared with the conventional DEA models is the decomposition of efficiencies into two components, namely, operational efficiency and portfolio efficiency. In the next section, we discuss how the fund families have performed over the sample period with respect to operational and portfolio efficiency.
<table>
<thead>
<tr>
<th>Mutual fund family</th>
<th>Number of funds</th>
<th>Total funds (US $)</th>
<th>Average return</th>
<th>Average risk</th>
</tr>
</thead>
<tbody>
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<td>American Funds</td>
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<td>1490594275158.00</td>
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<td>Vanguard</td>
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<td>9.16</td>
<td>2.07</td>
</tr>
<tr>
<td>Fidelity Investments</td>
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</table>

This table illustrates the summary statistics of the 66 mutual fund families considered in the sample. The sample period is from January 1993 to December 2008. Return on individual mutual funds is obtained from the Morningstar Direct database. Average return is the average monthly return of all individual mutual funds that belong to the family. Average risk is the average of the standard deviations of monthly returns of individual mutual funds that belong to the family. The funds are sorted by total funds under its management at 2008.
5.2. Operational efficiency

Table 5 lists the 13 fund families that perform relatively better from 2006 to 2008 based on the operational efficiency scores estimated in the proposed two-stage DEA model. The operational efficiency score reflects how well a fund family has managed its resources in securing or generating funds for that family. Here, we observe that three families have been ranked top 2 in all 3 years of assessment: Vanguard, T. Rowe Price, and American Century Investments. Under the overall efficiency score rankings reported in Table 3, only Vanguard performs at this level. The next-best performer under operational efficiency is Neuberger Berman, with rankings of 3 or better in all 3 years, followed by American Funds and Fidelity Investments.

The top-performing families in terms of operational efficiency over the 5-year period 2004–2008 reported in Table 6 reveal that the same 13 families reported in Table 5 also performed better than the other sampled families over the 5-year period from 2004 to 2008. The top 5 performers from 2006 to 2008 are also the top 5 performers over the 5-year period.

These families have managed to realize high levels of net asset values given their levels of management and marketing fees. We were not able to obtain data on variables such as salaries and rent that may be relevant for operational performance assessment. If it were possible, one could easily include them in the model to further improve the discriminatory power of mutual fund families based on their operational performance.

5.3. Portfolio efficiency

Portfolio efficiency measures how well a mutual fund family manages its investment portfolio to realize returns subject to a chosen set of factors that may influence returns. Portfolio efficiency is important information not only for investors in making investment decisions, but also for fund family administrators in assessing the performance of their portfolio managers. The fund family adminis-
trators may be able to judge how well their fund managers have performed relative to their competitors. The benefits do not stop there. Information on relative performance at the portfolio management level is vital for recruiting agencies to identify the best-performing fund managers and those who are underperforming.

As in the previous cases, Tables 7 and 8 lists the fund families that have been ranked at or above different levels of ranking in the last 3- and 5-year periods based on their portfolio efficiency. According to Table 7, Hartford Mutual Funds, Vanguard, Nuveen, Aquila, Davis Funds, and Sun America have managed their portfo-
Top-performing mutual fund families in the 3-year period from 2006 to 2008 based on their portfolio efficiency estimated in the proposed two-stage DEA procedure.

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<td>2 (66.7%)</td>
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</table>

The table gives the number of times the family has been ranked in the top 2, top 3, etc., over the 3 years from 2006 to 2008. The entry in parentheses gives the percentage of times the family has been ranked under the corresponding category of rankings.
Table 9 provides the rankings of individual fund families each year from 1993 to 2008 based on the overall, operational, and portfolio efficiencies estimated in the two-stage DEA model. We report only the top 10 mutual fund families listed in Table 3 to conserve space. It is clear in Table 9 that the overall efficiency of mutual fund families may be affected by their portfolio and operational efficiencies being at varying degrees. For example, Vanguard is both operationally and portfolio efficient with a rank of 1 and hence overall efficient throughout the period 1994–2008. T. Rowe Price, on the other hand, is operationally efficient during the period 1994–2008, maintaining a rank of 1. However, T. Rowe Price is not portfolio efficient (except in 2003 and 2004), and therefore is not overall efficient in most of the years. More on the effect of portfolio and operational efficiencies on overall efficiency for a set of fund families is discussed and illustrated graphically in the next section.

5.4. Variation in efficiency across time and fund families

Selecting a few families as examples, we now illustrate graphically how operational efficiency and portfolio efficiency may affect the overall efficiency of fund families over time. Panels (a) and (b)
of Fig. 3 give the graphs for Allianz Funds and Morgan Stanley, respectively. Both these funds perform consistently poorly overall, due to consistent poor operational and portfolio performance. Panel (c) shows that Vanguard’s continual overall performance is due to the consistent in its portfolio management efficiency. Panels (a) and (b) of Fig. 4 show the corresponding graphs for Hartford Mutual Funds and Allegiant. These are examples of fund families that have improved their performance after 2003. The improvement of Hartford Mutual Funds family after 2003 is mainly due to the improvement in operational and portfolio efficiencies, and in the case of Allegiant more or less due to the improvement in portfolio efficiency. On the other hand, Putnam and Franklin Templeton Investments, whose graphs are shown in Panels (c) and (d), respectively, in Fig. 4, reveal that the poor portfolio efficiency appears to be the main contributor to their declining overall performance towards the end of the sample period.

Four families (Allianz Funds, Morgan Stanley, Hartford Mutual Fund, and Putnam), illustrated in Figs. 3 and 4, show relatively poor portfolio performance in 1993. We notice a similar behavior in several other mutual fund families in the sample as well. This is clear evidence of the effect of the 1991 currency crisis on the portfolios managed by some mutual fund families. The improvement shown in the relative rankings after 1994 suggests quick recovery from the crisis in 1991. Vanguard and Aquila have managed their mutual funds relatively efficiently during all financial crisis periods from 1993 to 2008.

During the last two quarters of 1990 and the first quarter of 1991, the US economy experienced a sustained period of negative growth. Other significant shocks to the market during the sample period include the collapse of Long-Term Capital Management in 1998, the dotcom bubble and the subsequent market crash in

![Graphs for Allianz Funds, Morgan Stanley, Vanguard, and Aquila fund families.](image-url)
March 2000, the market meltdown following the September 11 attacks in New York and the Enron debacle, and the recent global financial crisis (GFC) that impacted the markets post July 2007. The effects of the GFC continued well into the years that followed. In Figs. 3 and 4, we observe that the portfolio efficiency of Allianz Funds, Morgan Stanley, and Putnam families have been seriously affected (low portfolio efficiency ranking) by the recessions of 1990–1991 and 2000–2002 and the fallout from the GFC over the period 2007–2009. These three fund families have high exposure investment across domestic and international equity markets: Allianz Funds (94%), Morgan Stanley (79%), and Putnam (68%). In contrast, even though Hartford Mutual Fund has been affected by the downturn in market activity in 1991 and 2000 to an extent similar to that of the three aforementioned fund families, it has not been affected as much by the problems resulting from the GFC in 2007. The better showing of Hartford Mutual Fund in the later period may be attributed to improved operational and portfolio efficiencies in part driven by an appropriate fee structure. The performance of Allegiant has been affected by the 1998 Long-Term Capital Management collapse and the 2000 recession and has survived the impact of the 2007 crisis. The standout fund family within our sample, Vanguard, as far as operational, portfolio, and overall efficiencies are concerned, has been exceptional throughout the full sample period.

Aquila performs extremely well in terms of portfolio efficiency, but due to its poor operational efficiency its overall efficiency is also low. The Franklin Templeton Investments family has done extremely well in its portfolio management until 2006. As far as operational efficiency is concerned, it has not done well, with a rank of around 10. The operational and portfolio management performance of the Hartford Mutual Fund family is not relatively satisfactory up to 2003, but has shown tremendous improvement in these areas thereafter. The Allegiant family’s operational efficiency is relatively satisfactory over the sample period, but its portfolio efficiency is relatively weak. However, Allegiant’s overall performance shows an improvement after 2005. Allianz Funds and Mor-
gan Stanley show inferior overall performance due to their poor performance in both operational and portfolio management areas and show no sign of improvement over the sample period. The Putnam family’s operational management performance is relatively poor throughout the sample period. Its portfolio management performance has been relatively satisfactory until 2000 and has deteriorated thereafter. Overall, the above analysis clearly shows that the proposed DEA model is able to capture the dynamics of the operational and portfolio management efficiencies and overall efficiency of mutual fund families.

5.5. Frontier projection of DMUs

Another important feature of DEA is its ability to provide information to make inefficient DMUs efficient. In this subsection, we demonstrate this feature in a selected set of mutual fund families. Such information is very important for a fund family’s management decision making.

Following Chen et al. (2010), Appendix B develops a model for frontier projection of mutual fund families deemed inefficient according to the proposed two-stage DEA model. We apply the model in Appendix B for frontier projection of the mutual fund families with the values of the input, output, and intermediate variables corresponding to the year 2008. The input, output, and intermediate variable changes required for making the inefficient mutual fund families efficient are illustrated in Table 10 for a selected set of families. Under the column “NAV” (the intermediate measure), a positive percentage indicates that NAV should be increased, and a negative percentage indicates that NAV should be decreased in order to make the fund family efficient. Positive values with respect to the other input variables in Table 10 indicate that they should be decreased by the corresponding percentages.

According to Table 10, no changes are required for any of the input \((I_1, I_2, I_3, I_4, I_5, I_6)\) and output \((O_2)\) variables of Vanguard and Fidelity Investments, as they are operational, portfolio, and overall efficient in year 2008. This observation tallies with the 2008 ranking of these two families in Table 9, where they are ranked within the first three as far as overall, operational, and portfolio efficiencies are concerned. The percentage changes of the variables in the second stage for Davis Funds are all zero, indicating that it is portfolio efficient in 2008. This is evident in Tables 7 and 8, where this family has been ranked within the top 2 during the period 2004–2008. However, Davis Funds is operationally inefficient, and therefore it has to decrease the marketing and distribution fee and the management fee by 51% and 40% in the first stage, respectively, to become operationally efficient. This will make Davis Funds overall efficient as well. Evidence presented in Tables 3 and 4 supports this finding, as Davis Funds appears at the bottom of these tables as far as overall efficiency is concerned. On the other hand, American Century Investments is operationally efficient, but not efficient in managing the portfolio. This family needs to increase its return by 4% and decrease its inputs at stage 2—\(I_5\) by the following percentages: 34%, 34%, 52%, and 34%, respectively—in order to become portfolio efficient and thereby become overall efficient. According to the entries in Table 10, Morgan Stanley is a poor performer in 2008, with inefficient operational and portfolio management. This is evident in Fig. 3b, with the overall, operational, and portfolio rankings of this family lying in the range of 20–60. For Morgan Stanley to be overall efficient, it needs to reduce all its inputs at stage 1 and stage 2 by

<table>
<thead>
<tr>
<th>Mutual Fund Family</th>
<th>Marketing and distribution fees ((I_1)) (%)</th>
<th>Management fees ((I_2)) (%)</th>
<th>NAV ((I_3)) (%)</th>
<th>Fund size ((I_4)) (%)</th>
<th>Standard deviation ((I_5)) (%)</th>
<th>Net expense ratio ((I_6)) (%)</th>
<th>Turnover ((O_2))</th>
<th>Average return ((O_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Funds</td>
<td>72</td>
<td>8</td>
<td>37</td>
<td>74</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>Vanguard</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fidelity Investments Franklin Templeton Investments</td>
<td>29</td>
<td>29</td>
<td>24</td>
<td>72</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>14</td>
</tr>
<tr>
<td>Davis Funds</td>
<td>51</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hartford Mutual Funds</td>
<td>24</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>American Century Investments</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>52</td>
<td>4</td>
</tr>
<tr>
<td>Oppenheimer Funds</td>
<td>42</td>
<td>42</td>
<td>–80</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>23</td>
</tr>
<tr>
<td>Morgan Stanley</td>
<td>62</td>
<td>43</td>
<td>16</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>50</td>
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<td>40</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Principal Funds</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>Security Funds</td>
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<td>86</td>
<td>19</td>
<td>18</td>
<td>28</td>
<td>18</td>
<td>36</td>
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<tr>
<td>Baron Capital Group</td>
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<td>0</td>
</tr>
<tr>
<td>ING Funds</td>
<td>31</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RS Funds</td>
<td>46</td>
<td>46</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Merger</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pax World</td>
<td>53</td>
<td>53</td>
<td>38</td>
<td>88</td>
<td>15</td>
<td>15</td>
<td>24</td>
<td>15</td>
</tr>
<tr>
<td>Van Eck</td>
<td>51</td>
<td>51</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>US Global Investors</td>
<td>58</td>
<td>58</td>
<td>50</td>
<td>21</td>
<td>21</td>
<td>34</td>
<td>21</td>
<td>14</td>
</tr>
<tr>
<td>Eagle Funds</td>
<td>36</td>
<td>35</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Heartland</td>
<td>55</td>
<td>55</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

This table gives the percentage changes required in the inputs, outputs, and intermediate measure (NAV) of the two stages illustrated in Fig. 1 in order to make the inefficient mutual fund families efficient. For input variables, the percentages represent a decrease and for output variables they represent an increase. In order to improve the efficiency of stage 1 and stage 2, the input variables \((I_1–I_6)\) should be decreased by the percentages given, the intermediate measure (NAV) should be changed by the percentage given under the column labeled NAV, and the output should be increased by the percentage given under \((O_2)\).
the percentages given in Table 10 and increase its stage 1 output or the intermediate measure (NAV) by 16% and increase the return (output) by 19%. On the other hand, Oppenheimer Funds may decrease all its inputs at stages 1 and 2 (and NAV) by the percentages given in Table 10 and increase its return by 23% in order to become efficient. These two examples (Morgan Stanley with a positive change in NAV and Oppenheimer Funds with a negative change in NAV) demonstrate an interesting feature of the proposed DEA model; that is, the model treats the intermediate variable, NAV, as both an input as well as an output. In the proposed DEA model, the optimal NAV is determined by both stages through coordination in such a way that the performances of both stages are maximized. No other technique, such as a stochastic frontier approach, is capable of treating a variable as both an input and an output.

6. Conclusion

The mutual fund industry in the US is the largest such industry in the world, and numerous studies have investigated its fund performance at the individual mutual fund level. Studies at the individual fund level fail to reveal information on the performance of the fund family to which the individual fund belongs. This is important to investors who invest in funds within the same mutual fund family rather than across a number of families for various reasons, including practical convenience. For them, the information on how a given mutual fund family as a whole may have performed relative to other mutual fund families is crucial. Studies have paid little attention to this issue. In this study, we fill this gap by proposing a novel two-stage DEA model to analyze the performance of large US mutual fund families over the period 1993–2008. Unlike the traditional performance measures, the proposed DEA model allows a combination of several performance measures, such as returns, fees and charges, risk of investment, stock selection style and portfolio management skills, and operational management skills, into a single measure in evaluating the overall performance of a mutual fund family relative to other families in the sample.

The proposed two-stage DEA model provides greater insight into the performance of mutual fund families by decomposing the overall efficiency into two components: operational efficiency and portfolio efficiency. This is a significant contribution to the DEA literature, as the two-stage DEA model proposed in the past literature does not discuss decomposition of overall efficiency into different components. In addition to mutual fund families, the proposed DEA model can also be applied to the other financial institutions, such as banks, insurance companies and credit unions.

Numerical evaluation of the proposed DEA model over the period 1993–2008 reveals that the proposed model is able to highlight the mutual fund families that may have managed their portfolios well during financial crisis periods as well as which of the two components, operational management or portfolio management, may have been the contributory factor to their good or bad performance. This useful information can help investors make informed decisions and enables administrators of fund families to judge how well their portfolio managers have performed relative to their competitors.

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Appendix A. DEA model for two-stage network and efficiency decomposition

Since \( w_1 \) and \( w_2 \) in (3) are intended to reflect the relative importance or the contribution of the performance in the first and the second stage to the overall performance, a reasonable choice of weights is the proportion of total resources devoted to each stage. To be more specific, we define

\[
\begin{align*}
    w_1 &= \frac{\sum_i p_i^1 x_{1,i}^1}{\sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2} \quad \text{and} \\
    w_2 &= \frac{\sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2}{\sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2}.
\end{align*}
\]  

(A1)

where \( \sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2 \) represents the total amount of resources (inputs) consumed by the entire two-stage process and \( \sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2 \) represents the amount of resources consumed in the first and the second stage, respectively. These weights are functions of the optimization variables of models (1) and (2) and are not decision (optimization) variables.

Hence, under VRS, the overall efficiency score of DMU, in the two-stage process can be evaluated by solving the following fractional program (A2). The constraints in (A2) ensure that the efficiency scores of a DMU in both stages are non-negative and no greater than unity.

\[
\begin{align*}
    0^*_o &= \max \frac{\sum_i q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2 + u^1 + \sum_j h_j y_j + u^2}{\sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2 - \sum_p \eta_p^o x_p^o}, \\
    \text{s.t.} \quad \sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2 &\leq 1, j = 1, 2, \ldots, n, \\
    \sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2 &\leq 1, j = 1, 2, \ldots, n, \\
    1 &\geq w_1 = \frac{\sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2}{\sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2} = w_1^1, \\
    1 &\geq w_2 = \frac{\sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2}{\sum_i p_i^1 x_{1,i}^1 + \sum_j q_{j1} z_{j} + \sum_o p_i^2 x_{2,i}^2} = w_2^2, \\
    p_i^1, p_i^2, u^1, u^2 &\geq 0, u^1, u^2 \text{ free.}
\end{align*}
\]  

(A2)

It is to be noted here that the weights defined in (A1) are variables related to the decision variables in DEA models (1) and (2). Sensitivity analysis on the weights \( w_1 \) and \( w_2 \) can be performed by adding lower bounds \( w_1^1 \) and \( w_2^2 \) on \( w_1 \) and \( w_2 \). In this study, we assume 50% for both \( w_1^1 \) and \( w_2^2 \), assuming that operational management and portfolio management are equally important functions.

By applying the Charnes–Cooper transformation, the above fractional programming model (A2) can be transformed into the following linear programming model (A3).

\[
\begin{align*}
    \phi^{\text{eff}}_o &= \max \sum_d \pi_d z_{d,j} + \sum_r \mu_r y_{r,j} + u^1 + u^2, \\
    \text{st.} \quad \sum_d \pi_d z_{d,j} + u^1 &\leq \sum_i \omega_i^j x_{1,i}^1, \quad j = 1, 2, \ldots, n, \\
    \sum_r \mu_r y_{r,j} + u^2 &\leq \sum_d \pi_d z_{d,j} + \sum_i \omega_i^j x_{2,i}^2, \quad j = 1, 2, \ldots, n, \\
    \sum_i \omega_i^j x_{1,i}^1 + \sum_d \pi_d z_{d,j} + \sum_i \omega_i^j x_{2,i}^2 &\leq 1, \\
    1 &\geq \sum_i \omega_i^j x_{1,i}^1 \geq w_1^1, \\
    1 &\geq \sum_d \pi_d z_{d,j} + \sum_i \omega_i^j x_{2,i}^2 \geq w_2^2, \\
    \omega_i^j, \pi_d, \mu_r, \pi_d &\geq 0, u^1, u^2 \text{ free.}
\end{align*}
\]  

(A3)
A.1. Efficiency decomposition

Once we obtain an optimal solution to (A3), the efficiency scores for the two individual stages can be calculated as

$$
\theta_0^i = \frac{\sum_i \eta_i x_{i0} u^i}{\sum_i \eta_i x_{i0} u^i + u^i} \quad \text{and} \quad \theta_0^o = \frac{\sum_i \eta_i y_{i0} u^i}{\sum_i \eta_i y_{i0} u^i + u^i}.
$$

We can also obtain a set of weights as

$$
w_i^0 = \sum_i \eta_i x_{i0} u^i + u^i \quad \text{and} \quad w_o^0 = \sum_i \eta_i y_{i0} u^i + u^i.
$$

However, since model (A3) can have alternative optimal solutions, the $\theta_0^i$ and $\theta_0^o$ components of overall efficiency may not be unique. Therefore, we follow the procedure adopted by Kao and Hwang (2008) and Chen (2009) to find a set of multipliers that would produce the highest first- or second-stage efficiency score while maintaining the overall efficiency score of the entire process.

Denote the overall efficiency score of $DMU_o$ obtained by model (A3) as $\theta_0^o$. We maximize the first-stage efficiency score first while maintaining the overall efficiency score at $\theta_0^o$ and the weighted first- or second-stage efficiency scores at no greater than $\theta_0^i$ as

$$
\theta_0^i = \max \left\{ \sum_i \eta_i x_{i0} u^i + u^i \right\}
$$

subject to

$$
\sum_i \eta_i x_{i0} u^i + u^i \leq 1, \quad j = 1, 2, \ldots, n
$$

and

$$
The \text{second-stage efficiency score for } DMU_o \text{ is calculated as } \theta_0^o = \frac{\sum_i \eta_i y_{i0} u^i}{\sum_i \eta_i y_{i0} u^i + u^i}.\!
$$

Note that (\(\cdot\)) is used in $\theta_0^o$ to indicate that the first-stage efficiency score is optimized first. In this case, the resulting efficiency score for the second stage is denoted by $\theta_0^o$ (without (\(\cdot\))).

Similarly, the following linear program can be formulated to maximize the second-stage efficiency score while maintaining the overall efficiency score at $\theta_0^o$ and the weighted first- or second-stage efficiency scores at no greater than $\theta_0^i$ as

$$
\theta_0^o = \max \sum_i \mu_i y_{i0} + u^i
$$

subject to

$$
\sum_i \eta_i x_{i0} u^i + u^i \leq 1, \quad j = 1, 2, \ldots, n
$$

and

$$
The \text{weighted first- or second-stage efficiency scores at no greater than } \theta_0^i \text{ as } \theta_0^i = \frac{\sum_i \eta_i x_{i0} u^i}{\sum_i \eta_i x_{i0} u^i + u^i}.\!
$$

In model (A4), the constraints (A4a) and (A4b) ensure that the efficiency scores of all DMUs at both stages are no greater than unity and the constraint (A4c) maintains the overall efficiency score at $\theta_0^o$. Model (A4) can be equivalently converted into the following linear program (A5),

$$
\theta_0^i = \max \sum_i \eta_i x_{i0} u^i + u^i
$$

subject to

$$
\sum_i \eta_i x_{i0} u^i + u^i \leq 1, \quad j = 1, 2, \ldots, n
$$

and

$$
The \text{first-stage efficiency score is calculated as } \theta_0^i = \frac{\sum_i \eta_i x_{i0} u^i}{\sum_i \eta_i x_{i0} u^i + u^i}.\!
$$

Appendix B. Frontier projection

Model (A3) does not yield information on optimal intermediate measures. Therefore, following Chen et al. (2010), we develop a model for frontier projection of the DMUs as follows:

$$
\min \left\{ \sum_i \lambda_i x_i^1 + \sum_i \lambda_i x_i^2 + \sum_i \lambda_i s_i^3 \right\}
$$

subject to

$$
\sum_i \lambda_i s_i^1 = \sum_i \lambda_i s_i^2 = \sum_i \lambda_i s_i^3 = 0, \quad i_1 = 1, 2, \ldots, D
$$

and

$$
\sum_i \lambda_i s_i^1 + \sum_i \lambda_i s_i^2 + \sum_i \lambda_i s_i^3 \geq \lambda_i, \quad i_1 = 1, 2, \ldots, D
$$

and

$$
\lambda_i, \lambda_i s_i^1, \lambda_i s_i^2, \lambda_i s_i^3 \geq 0.
$$

Let $\lambda_i^1, \lambda_i^2, \lambda_i^3, \mu_i, \pi_i, u^i, u^i, u^i$ represent the optimal values of $\lambda_i^1, \lambda_i^2, \lambda_i^3, \mu_i, \pi_i, u^i, u^i, u^i$ in model (A5). Then the first-stage efficiency score is $\theta_0^1 = \sum_i \eta_i x_{i0} u^i + u^i$ and the optimal weights for the two stages are $w_1 = \frac{1}{\sum_i \eta_i x_{i0} u^i + u^i}$ and $w_2 = 1 - w_1$, respectively. The second-stage efficiency score for $DMU_o$ is calculated as $\theta_0^o = \frac{\sum_i \eta_i y_{i0} u^i}{\sum_i \eta_i y_{i0} u^i + u^i}$. Note that (\(\cdot\)) is used in $\theta_0^o$ to indicate that the first-stage efficiency score is optimized first. In this case, the resulting efficiency score for the second stage is denoted by $\theta_0^o$ (without (\(\cdot\))).
where \( w_i, w_j \) are obtained from the two-stage network DEA model developed in Appendix A.

The above model is based on the production possibility set with 
\[ \sum_{j=1}^{n} \lambda_j = 1 \] and 
\[ \sum_{j=1}^{n} \eta_j = 1 \] indicating that both stages exhibit VRS, as in the standard DEA model. \[ \sum_{j=1}^{n} \lambda_j \tilde{y}_j = \sum_{j=1}^{n} \eta_j \tilde{y}_j, \] \( d = 1, 2, \ldots, D \) ensures that both stages determine the optimal (or frontier projection of) intermediate measures. Because model (A.3) does not yield information on the optimal intermediate measures, the above model is needed to determine the projection point for inefficient DMUs.

If we fix \( \alpha \) and \( \beta \) in the above model as \( \alpha^0, \beta^0 \) obtained from our two-stage model, the above model adopts the principle of the “second-stage” model for calculating DEA slacks (Cooper et al., 2004). In this case, the model becomes

\[
\max \sum_{i} s_i + \sum_{j} \tilde{r}_i \\
\text{st.} \sum_{j} \tilde{y}_j + s_i = \tilde{x}_i, i = 1, 2, \ldots, n \\
\sum_{j=1}^{n} \lambda_j = 1 \\
\sum_{j=1}^{n} \eta_j = 1 \\
\sum_{j=1}^{n} \lambda_j \tilde{y}_j = \sum_{j=1}^{n} \eta_j \tilde{y}_j \\
\lambda_j, \eta_j, s_i, \tilde{r}_i \geq 0
\]

Both stages determine the best projection levels for the intermediate measures as \( \sum_{j=1}^{n} \lambda_j \tilde{y}_j = \sum_{j=1}^{n} \eta_j \tilde{y}_j \). The frontier projection point is given by \( (\tilde{x}_i^0, \tilde{y}_j^0 - s_i^0, \sum_{j=1}^{n} \lambda_j \tilde{y}_j, \tilde{x}_i - s_i^0) \).

**References**


