Journal of the Operational Research Society (1998) 49, 1060–1071 0160-5682/98 $12.00

http://www.stockton-press.co.uk/jor

Sensitivity analysis of DEA models for simultaneous changes in all the data

LM Seiford[1] and J Zhu[2]

[1]*University of Massachusetts and* [2]*Worcester Polytechnic Institute, USA*

In data envelopment analysis (DEA) efficient decision making units (DMUs) are of primary importance as they define the efficient frontier. The current paper develops a new sensitivity analysis approach for the basic DEA models, such as, those proposed by Charnes, Cooper and Rhodes (CCR), Banker, Charnes and Cooper (BCC) and additive models, when variations in the data are simultaneously considered for all DMUs. By means of modified DEA models, in which the specific DMU under examination is excluded from the reference set, we are able to determine what perturbations of the data can be tolerated before efficient DMUs become inefficient. Our approach generalises the usual sensitivity analysis approach developed in which perturbations of the data are only applied to the test DMU while all the remaining DMUs remain fixed. In our framework data are allowed to vary simultaneously for all DMUs across different subsets of inputs and outputs. We study the relations of the infeasibility of modified DEA models employed and the robustness of DEA models. It is revealed that the infeasibility means stability. The empirical applications demonstrate that DEA efficiency classifications are robust with respect to possible data errors, particularly in the convex DEA case.

Keywords: data envelopment analysis (DEA); efficiency; sensitivity analysis

Introduction

As a data-based method, the stability of data envelopment analysis (DEA), has often been questioned. Since data can be contaminated by statistical noise, a frequently asked question is 'To what extent can perturbations in the data observations be tolerated before the DEA efficiency is changed?' The issue of the robustness of DEA efficiency models has been the subject of an extensive research effort which has resulted in a number of sensitivity analysis papers.

The first DEA sensitivity analysis paper of Charnes *et al*[1] examined change in a single output. This was followed by a series of sensitivity analysis articles by Charnes and Neralic[2] in which they determine sufficient conditions, for a simultaneous change in all outputs and (or) all inputs of an efficient decision making unit (DMU), which preserve efficiency. All of these methods examine data changes via updating the inverse of the basis matrix associated with a specific efficient DMU.

As noted by Charnes *et al*[3], the sufficient conditions obtained from the above sensitivity analysis methods tend to be too restrictive. Charnes *et al*[3] therefore provide a 1-norm and ∞-norm to compute stability regions for efficiency classifications under the additive model. Their method, in fact, is based on a modification of DEA models in which the test DMU is excluded from the reference set. (see also Charnes *et al*[4] for a different version).

More recently, Zhu[5] employed the modified CCR models to determine necessary and sufficient conditions for preserving efficiency of the efficient DMUs under the CCR ratio model (Charnes *et al*[6]) when inputs and (or) outputs of the test efficient DMU are changed. The method is generalised in Seiford and Zhu[7] to yield the entire (largest) stability region which encompasses that of Charnes *et al.*[3]

The above sensitivity analysis methods are based on a basis matrix or modified DEA models and were developed for situations where variations in the data are only applied to the test DMU. However, in reality, possible data errors may occur for any DMU. Thompson *et al*[8] use the strong complementary slackness condition (SCSC) multipliers to analyse the stability of the CCR model when the data for all efficient and all inefficient DMUs are simultaneously changed in opposite directions. Nevertheless, their method is dependent upon the particular SCSC solution employed. Thompson *et al*[9] propose an approach to select a SCSC solution when using their method, but the criteria are not completely satisfying and the resulting analysis is inexact.

In this paper, we utilise the modified DEA models to study the stability of DEA scores when all the data (including the efficient DMU under consideration and others) are changed simultaneously. We consider a worst-case analysis where the efficiency of the test DMU is

Correspondence: Dr LM Seiford, Department of Mechanical and Industrial Engineering, University of Massachusetts at Amherst, Amherst, MA 01003, USA.
E-mail: SEIFORD@ECS.UMASS.EDU

deteriorating while the efficiencies of the other DMUs are improving. For each efficient DMU being analyzed for efficiency stability, we calculate a range of stability such that all possible perturbations of the test DMU and the remaining DMUs within the range preserve the dominance of the test DMU, namely, the DMU under evaluation remains a DEA frontier point. We may have different ranges of stability for perturbations of different subsets of outputs and (or) inputs. It is shown that our sensitivity analysis approach generalises the methods of Charnes *et al*[3] and Zhu[5] to the situation where data variations are made for all DMUs for any subset of inputs and outputs. It is also shown that our sensitivity analysis method yields more exact robust sensitivity analysis results as compared to those obtained by using the Thompson *et al*[8] SCSC approach.

As firstly noted by Zhu,[5] we may encounter infeasibility in modified DEA models when used to determine the stability of efficiency classifications. Seiford and Zhu[7] show that infeasibility means that the Charnes, Cooper and Rhodes (CCR) efficiency[6] of the test DMU remains stable to data changes. Here we show that the result also holds true under other DEA models (for example see the Banker, Charnes and Cooper (BCC) model of Banker *et al*[10]) when all DMUs data change simultaneously.

The current article proceeds as follows: In the next section, we review several sensitivity analysis approaches, based on modified DEA models when only the efficient DMU under analysis changes its data. We then develop our sensitivity analysis method for simultaneous data changes in all DMUs. The sensitivity of the DEA CCR model[6] and of the DEA convex models (namely BCC model,[10] and additive model of Charnes *et al*[11]) are investigated. We discuss the sensitivity to simultaneous data variations of all DMUs under two cases—percentage and absolute change cases. We also explore the infeasibility of the modified DEA models. Finally, we apply the new sensitivity method to two structurally different real world data sets. One is the Chinese cities data set (Charnes *et al*[12]) in which the CCR efficiency scores are almost the same as the corresponding BCC efficiency scores. The other is the Chinese textiles data set (Zhu[13]) in which larger discrepancies between the CCR and the BCC efficiency scores are discovered.

Precursors

Suppose we have a set of n DMUs. Each DMU$_j$ ($j = 1, 2, \ldots, n$), produces s different outputs y_{rj} ($r = 1, 2, \ldots, s$) utilizing m different inputs x_{ij} ($i = 1, 2, \ldots, m$). Consider a specific efficient DMU$_o$ among them (here the 'efficient' means that the radial optimal value is equal to one). Charnes *et al*[3] provided the following linear programming problem, a modified DEA model, to study the sensitivity of efficiency classifications in the additive model

$$\theta^* = \min \theta$$

subject to

$$\begin{aligned}
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} - \theta \leqslant x_{io} \qquad i = 1, 2, \ldots, m;\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} + \theta \geqslant y_{ro} \qquad r = 1, 2, \ldots, s;\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j = 1;\\
&\theta,\ \lambda_j (j \neq o) \geqslant 0.
\end{aligned} \tag{1}$$

The optimal value θ^* is called the radius of stability under the ∞-norm. The absolute increase of inputs and absolute decrease of outputs are considered only for DMU$_o$. The constraint of $\sum_{j=1,\, j\neq o}^{n} \lambda_j = 1$ was excluded in Charnes *et al*[4] and the model was modified to the case of percentage change. If for each i and r, we use different θ_i and θ_r and minimise $\sum_{i=1}^{m} \theta_i + \sum_{r=1}^{s} \theta_r$, then the optimal value is the radius of stability under the 1-norm.

Zhu[5] provided the following two linear programming formulations to study the robustness of efficient DMUs under the CCR model.

$$\beta_k^* = \min \beta_k \quad \text{for each } k = 1, \ldots, m$$

subject to

$$\begin{aligned}
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{kj} \leqslant \beta_k x_{ko}\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} \qquad i \neq k\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} \qquad r = 1, 2, \ldots, s\\
&\beta_k, \lambda_j (j \neq o) \geqslant 0.
\end{aligned} \tag{2}$$

and

$$\alpha_l^* = \max \alpha_l \quad \text{for each } l = 1, 2, \ldots, s$$

subject to

$$\begin{aligned}
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{lj} \geqslant \alpha_l y_{lo}\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro};\ r \neq l\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} \qquad i = 1, \ldots, m\\
&\alpha_l, \lambda_j (j \neq o) \geqslant 0.
\end{aligned} \tag{3}$$

Using the optimal values of β_k^* ($k = 1, 2, \ldots, m$), and α_l^* ($l = 1, 2, \ldots, s$), one can compute the upper and lower boundaries of proportionate changes of inputs and outputs while preserving the efficiency of DMU$_o$. Seiford and Zhu[7] generalised the method to determine the exact stability region for DMU_o.

It can be seen that a unique feature of each of the above methods is that DEA-like formulations in which the DMU under evaluation is not included in the reference set are employed to analyse the sensitivity and stability of the efficiency classifications. It can also be seen that these methods only consider DMU_os data perturbations and assume that all other DMUs data remain unchanged.

Simultaneous change in all the data

An increase of any output or a decrease of any input can not worsen the efficiency of DMU_o. Therefore we restrict our attention to decrease in outputs and increase in inputs for DMU_o. In order to simultaneously consider the data changes for the other DMUs, we suppose increased output and decreased input for all other DMUs. That is, our discussion is based on a worst-case scenario in which efficiency of DMU_o declines and the efficiencies of all other DMU_j $(j \neq o)$ improve.

Let I and O denote respectively the input and output subsets in which we are interested. Then the simultaneous changes of input/output of all DMU_j $(j \neq o)$ and DMU_o can be classified into two cases:

Percentage change case

For DMU_o

$$\begin{cases} \hat{x}_{io} = \delta x_{io} & \delta \geqslant 1, i \in I \\ \hat{x}_{io} = x_{io} & i \notin I \end{cases}$$

or, equivalently,

$$\begin{cases} \hat{x}_{io} = x_{io} + (\delta - 1)x_{io} & \delta \geqslant 1, i \in I \\ \hat{x}_{io} = x_{io} & i \notin I \end{cases}$$

$$\begin{cases} \hat{y}_{ro} = \tau y_{ro} & 0 < \tau \leqslant 1, r \in O \\ \hat{y}_{ro} = y_{ro} & r \notin O \end{cases}$$

or, equivalently,

$$\begin{cases} \hat{y}_{ro} = y_{ro} - (1 - \tau)y_{ro} & 0 < \tau \leqslant 1, r \in O \\ \hat{y}_{ro} = y_{ro} & r \notin O \end{cases}$$

For DMU_j $(j \neq o)$

$$\begin{cases} \hat{x}_{ij} = x_{ij}/\delta & \delta \geqslant 1, i \in I \\ \hat{x}_{ij} = x_{ij} & i \notin I \end{cases}$$

or, equivalently,

$$\begin{cases} \hat{x}_{ij} = x_{ij} - \dfrac{\delta - 1}{\delta} x_{ij} & \delta \geqslant 1, i \in I \\ \hat{x}_{ij} = x_{ij} & i \notin I \end{cases}$$

$$\begin{cases} \hat{y}_{rj} = y_{rj}/\tau & 0\tau \leqslant 1, r \in O \\ \hat{y}_{rj} = y_{rj} & r \notin O \end{cases}$$

or, equivalently,

$$\begin{cases} \hat{y}_{rj} = y_{rj} + \dfrac{1 - \tau}{\tau} y_{rj} & 0 < \tau \leqslant 1, r \in O \\ \hat{y}_{rj} = y_{rj} & r \notin O \end{cases}$$

Absolute change case

For DMU_o

$$\begin{cases} \hat{x}_{io} = x_{io} + \rho & \rho \geqslant 0, i \in I \\ \hat{x}_{io} = x_{io} & i \notin I \end{cases}$$

and

$$\begin{cases} \hat{y}_{ro} = y_{ro} - \varphi & \varphi \geqslant 0, r \in O \\ \hat{y}_{ro} = y_{ro} & r \notin O \end{cases}$$

For DMU_j $(j \neq o)$

$$\begin{cases} \hat{x}_{ij} = x_{ij} - \rho & \rho \geqslant 0, i \in I \\ \hat{x}_{ij} = x_{ij} & i \notin I \end{cases}$$

and

$$\begin{cases} \hat{y}_{rj} = y_{rj} + \varphi & \varphi \geqslant 0, r \in O \\ \hat{y}_{rj} = y_{rj} & r \notin O \end{cases}$$

where ($\hat{}$) represents adjusted data.

On the basis of these two cases, we will study the sensitivity of the CCR, BCC and additive models. We first assume that the modified DEA models employed are feasible.

CCR ratio model

In this case we assume that DMU_o is CCR efficient, namely the CCR radial efficiency score is equal to one. We will calculate the upper-bound of δ and the lower-bound of τ, namely the ranges for δ and τ such that all the input and output perturbations within these ranges preserve the efficiency of DMU_o.

Consider the following (input-based) modification of the CCR DEA model

$$\beta^* = \min \beta$$

subject to

$$\begin{aligned} &\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij} \leqslant \beta x_{io} && i \in I \\ &\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} && i \notin I \\ &\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} && r = 1, 2, \ldots, s \\ &\beta, \lambda_j (j \neq o) \geqslant 0. \end{aligned} \tag{4}$$

Theorem 1 *For the percentage change case, if* $1 \leqslant \delta \leqslant \sqrt{\beta^*}$, *then* DMU$_o$ *remains efficient, where* β^* *is the optimal value to* (4).

Proof Since DMU$_o$ is efficient (and since we assume (4) is feasible), then by Zhu[5] we know that $\beta^* \geqslant 1$, and therefore $\sqrt{\beta^*} \geqslant 1$. Now suppose $1 \leqslant \delta_o \leqslant \sqrt{\beta^*}$ and DMU_o is inefficient when $\hat{x}_{io} = \delta_o x_{io}$ and $\hat{x}_{ij} = x_{ij}/\delta_o, i \in I$. Then there exists $\lambda_j\ (j \neq o) \geqslant 0$ and $\theta < 1$ such that

$$\begin{aligned} &\sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j \frac{x_{ij}}{\delta_o} \leqslant \theta\delta_o x_{io} && i \in I \\ &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} && i \notin I \\ &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} && r = 1, 2, \ldots, s. \end{aligned}$$

This means that $\lambda_j\ (j\ \neq o) \geqslant 0$ and $\theta\delta_o{}^2$ is a feasible solution to (4). But $\theta\delta_o{}^2 < \beta^*$ violating the optimality of β^*.

For the special case $I = \{k\}$, $k \in \{1, 2, \ldots, m\}$ then (4) becomes (2). By Theorem 1, we have:

Corollary 1 *For the case of a change in only one input* (*for example, the* kth *input*), *if* $1 \leqslant \delta \leqslant \sqrt{\beta_k^*}$, *then* DMU$_o$ *remains efficient, where* β_k^* *is the optimal value to* (2).

If $I = \{1, 2, \ldots, m\}$ then (4) is the input-based modified CCR model. The square root of the optimal value now gives the maximum proportional increase of all inputs for DMU$_o$ and the maximum proportional decrease of all inputs for all other DMU$_j$ ($j \neq o$) while preserving the efficiency of DMU$_o$.

For change in outputs, we consider the following (output-based) modification of the CCR DEA model

$$\alpha^* = \max \alpha$$

subject to

$$\begin{aligned} &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant \alpha y_{ro} && r \in O \\ &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} && r \notin O \\ &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} && i = 1, \ldots, m \\ &\alpha, \lambda_j (j \neq o) \geqslant 0. \end{aligned} \tag{5}$$

Similar to Theorem 1, we have

Theorem 2 *For the percentage change case, if* $\sqrt{\alpha^*} \leqslant \tau \leqslant 1$, *then* DMU$_o$ *remains efficient, where* α^* *is the optimal value to* (5).

For the special case $O = \{l\}$, where $l \in \{1, 2, \ldots, r\}$, we have:

Corollary 2 *For the case of a change in only one output* (*for example, the lth input*), *if* $\sqrt{\alpha_l^*} \leqslant \tau \leqslant 1$, *then* DMU$_o$ *remains efficient, where* α_l^* *is the optimal value to* (3).

When $O = \{1, 2, \ldots, s\}$, then (5) is the output-based modified CCR model. The square root of the optimal value gives the lower bound for τ when all outputs of all DMUs change proportionally.

Theorem 3 $\beta^* = 1/\alpha^*$ *when* $I = \{1, 2, \ldots, m\}$ and $O = \{1, 2, \ldots, s\}$.

Proof We can immediately obtain $\beta^* = 1/\alpha^*$ by the relationship between the input-based and the output-based CCR models.

The above theorem indicates that in the case of simultaneous proportionate change of all inputs or all outputs, we can know the possible input (proportional) perturbations from the possible output (proportional) perturbations, and vice versa. Either $\sqrt{\beta^*}$ or $\sqrt{\alpha^*}$ will provide the information on the bounds of δ and τ, that is, we have

$$1 \leqslant \delta \leqslant \sqrt{\beta^*}\left(= \frac{1}{\sqrt{\alpha^*}}\right) \quad \text{and} \quad (\sqrt{\alpha^*} =)\frac{1}{\sqrt{\beta^*}} \leqslant \tau \leqslant 1.$$

It can be seen that β_k^* (β^*) and α_l^* (α^*) are respectively the upper bound for input perturbations and the lower bound for output perturbations of DMU$_o$ when the remaining DMU$_j$ ($j \neq o$) are fixed. Therefore the maximum proportional perturbations of inputs and outputs under the simultaneous changes of all DMUs are the square roots of the corresponding maximum proportional perturbations under the situations when only the test efficient DMUs data is perturbed (Zhu[5]).

Next we consider the following modified DEA model for simultaneous variations of inputs and outputs

$$\Gamma^* = \min \Gamma$$

subject to

$$\begin{aligned} &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant (1+\Gamma) x_{io} && i \in I \\ &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} && i \notin I \\ &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant (1-\Gamma) y_{ro} && r \in O \\ &\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} && r \notin O \\ &\lambda_j (j \neq o) \geqslant 0, \Gamma \text{ unrestricted.} \end{aligned} \tag{6}$$

If $I = \{1, 2, \ldots, m\}$ and $O = \{1, 2, \ldots, s\}$, then (6) is identical to the model of Chames *et al.*[4]

Theorem 4 *For the percentage case with simultaneous changes of inputs and outputs, if* $1 \leqslant \delta \leq \sqrt{1+\Gamma^*}$ *and* $\sqrt{1-\Gamma^*} \leqslant \tau \leqslant 1$, *then* DMU_o *remains efficient, where* Γ^* *is the optimal value to* (6).

Proof The proof is similar to that of Theorem 1 and is omitted.

Note that we are unable to discuss absolute changes directly through the modified CCR and DEA models. However after obtaining the percentage changes, we can transform them into the absolute changes DMU by DMU.

Convex DEA model

In this situation, for the percentage change case, Theorems 1, 2, 3 and Corollaries 1, 2 hold for BCC efficient DMUs where we add the additional constraint of $\sum_{j=1, j\neq o}^{n} \lambda_j = 1$ respectively into (2) – (6).

Next consider absolute change; in this case the sensitivity analysis results are also suitable to the additive model. Consider the following linear programming problem which generalises model (1).

$$u^* = \min u$$

subject to

$$\begin{aligned}
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} + u \qquad i \in I\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} \qquad i \notin I\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} - u \qquad r \in O\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} \qquad r \notin O\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j = 1\\
&u, \lambda_j (j \neq o) \geqslant 0.
\end{aligned} \tag{7}$$

Theorem 5 *For the absolute change case, if* $0 \leqslant \rho, \varphi \leqslant \mathrm{u}^*/2$, *then* DMU_o *remains efficient, where* u^* *is the optimal value to* (7).

Proof The proof is similar to that of Theorem 1.

If $O = \emptyset$ then (7) only considers absolute changes in input. If $I = \emptyset$ then (7) only considers absolute changes in output. Let u_I^* and u_O^* respectively denote the optimal values to (7) under $O = \emptyset$ and $I = \emptyset$, then we have

Theorem 6

(i) For the absolute change of input case, if $0 \leqslant \rho \leqslant u_I^*/2$, *then* DMU_o *remains efficient;*
(ii) *For the absolute change of output case, if* $0 \leqslant \varphi \leqslant u_o^*/2$, *then* DMU_o *remains efficient*.

For different choices of subsets I and O, we can determine the sensitivity of DMU_o to the absolute changes of different sets of inputs or (and) outputs where DMU_os efficiency is deteriorating and DMU_js $(j \neq o)$ efficiencies are improving.

If for each $i \in I$ and each $r \in O$, we use separate u_i and u_r, and change the objective function of (7) to minimise $\sum_{i\in I} u_i + \sum_{r\in O} u_r$, then we obtain the generalised model under the 1–norm

$$\min \sum_{i\in I} u_i + \sum_{r\in O} u_r$$

subject to

$$\begin{aligned}
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} + u_i \qquad i \in I\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leqslant x_{io} \qquad i \notin I\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} - u_r \qquad r \in O\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geqslant y_{ro} \qquad r \notin O\\
&\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j = 1\\
&u_i, u_r, \lambda_j (j \neq o) \geqslant 0.
\end{aligned} \tag{8}$$

The optimal values of $u_i^*/2$ $(i \in O)$ and $u_r^*/2$ $(r \in O)$ also give the range of possible different input and output perturbations of DMU_o and DMU_j $(j \neq o)$. If we let $\Gamma\ x_{io} = u_i$ $(i \in I)$ and $\Gamma\ y_{ro} = u_r$ $(r \in O)$, then (8) is equivalent to (6) except for the constraint $\sum_{j=1, j\neq o}^{n} \lambda_j = 1$, that is the absolute change can be transformed into the percentage change case, and vice versa.

Infeasibility and stability

In the previous developments, we assumed that the modified DEA models employed, say (4), (5) and (7), in

Table 1 Data for the example

DMU	*1*	*2*	*3*
Output 1	1	0.25	0.25
Input 1	1	0.25	1
Input 2	1	1	0.5

Table 2 Robustness of the efficiency of DMU_1

Percentage change	*Ratio model*	*Convex model*	*Absolute change* (u^*)	*Convex model*
β_1^*	Infeasibility	Infeasibility	$I=\{1\}$, $O=\emptyset$	Infeasibility
β_2^*	4	Infeasibility	$I=\{2\}$, $O=\emptyset$	Infeasibility
β^*, $I=\{1, 2\}$	14/5	Infeasibility	$I=\{1, 2\}$, $O=\emptyset$	Infeasibility
$\alpha^* = \alpha_1^*$	5/14	1/4	$I=\emptyset$, $O=\{1\}$	3/4
Γ^*, $I=\{1\}$, $O=\{1\}$	9/16	3/4	$I=\{1\}$, $O=\{1\}$	3/4
Γ^*, $I=\{2\}$, $O=\{1\}$	9/17	3/4	$I=\{2\}$, $O=\{1\}$	3/4
Γ^*, $I=\{1, 2\}$, $O=\{1\}$	9/19	3/4	$I=\{1, 2\}$, $O=\{1\}$	3/4

sensitivity analysis were always feasible. However, this assumption is not necessarily true, particularly in the convex DEA case.

Consider the example from Charnes *et al*[3] given in Table 1, with three DMUs, a single output, and two inputs.

DMU_1 is an efficient DMU. Applying different modified DEA models for DMU_1 yields the results shown in Table 2. Seven infeasibility cases are detected. Of which six are for the convex DEA situation. To interpret Table 2, we say that, for example, the '4' in the third row under the ratio model indicates that in the event of change to the second input only, the input of DMU_1 can be increased from 1 to 2 units while that of DMU_2 and DMU_3 can be decreased by 2 units while maintaining the efficiency of DMU_1 under the CCR ratio model. (Note that $\sqrt{4}=2$ and therefore $1 \leqslant \delta \leqslant 2$ when $I=\{2\}$.)

Similarly, the '3/4' in the last row of the third column means that the two inputs and the single output of DMU_1 can respectively be increased and decreased by $\sqrt{3}/2$ and meanwhile the corresponding inputs and output of DMU_2 and DMU_3 can respectively be decreased and increased by $\sqrt{3}/2$ while keeping the efficiency of DMU_1 under the BCC model or additive model. Also one can obtain the absolute change case by dividing the related numbers of 3/4 by 2, that is, 3/8.

At first glance, it appears that we are unable to derive the sensitivity information from the infeasibilities in Table 2. Recall that Seiford and Zhu[7] showed that in the case of change to the test efficient DMUs data only, the infeasibility means that the test efficient DMU can infinitely increase (decrease) its corresponding inputs (outputs) and still preserve its efficiency under the CCR ratio model. As a matter of fact, this result can be generalised to the situation where all DMUs change their data for the ratio as well as the convex DEA models.

We first consider absolute change, namely, formulation (7).

Theorem 7 (Absolute change case) *For any nonnegative* ρ *and* φ, DMU_o *remains efficient after the simultaneous absolute data changes of* DMU_o *and the remaining* DMU_j ($j \neq o$) *if and only if* (7) *is infeasible.*

Proof The proof follows from the results of Seiford and Zhu[7] and Theorem 1.

Therefore, the infeasibility cases under the absolute changes in Table 2 means that the efficiency of DMU_1 is robust with respect to input changes of DMUs 1, 2 and 3. The above result is also true for the percentage change case since absolute changes can be transformed into percentage changes. In fact, similar to Theorem 7, we have:

Theorem 8 Percentage change case *For any* $\delta \geqslant 1$ *and* $0 < \tau \leqslant 1$, DMU_o *remains efficient after the simultaneous percentage data changes for* DMU_o *and the other* DMU_j ($j \geqslant o$) *if and only if* (6) *is infeasible.*

Furthermore, we have:

Corollary 3

(i) *In the input percentage change case, for any* $\delta \geqslant 1$, DMU_o *remains efficient after the simultaneous data changes for* DMU_o *and the remaining* DMU_j ($j \neq o$) *if and only if* (4) *is infeasible.*
(ii) *In the output percentage change case, for any* $0 < \tau \leqslant 1$, DMU_o *remains efficient after the simultaneous data changes of* DMU_o *and the remaining* DMU_j ($j \neq o$) *if and only if* (5) *is infeasible.*

One can conclude that infeasibility of the modified DEA models can be interpreted as stability of the efficiency classification of DMU_o with respect to the changes of corresponding inputs and (or) outputs in all DMUs.

In addition, note that since the modified convex DEA models have the constraint $\sum_{j=1, j\neq o}^{n} \lambda_j = 1$, any solution to a specific modified convex DEA model is also a feasible solution to the associated modified ratio DEA model. Therefore if the modified ratio DEA model (say, modified CCR model) is infeasible, then the corresponding modified convex DEA model (say, modified BCC model) must be infeasible. Therefore we have:

Theorem 9 *If infeasibility occurs in the modified ratio* - DEA *model, then it must also occur in the corresponding modified convex* DEA *model.*

Therefore the DEA efficiency of efficient DMUs for the BCC model is more stable than that in the CCR model.

Finally we should note that if $I = \{1, 2, \ldots m\}$ and $O = \{1, 2, \ldots s\}$, then (6) and (7) are always feasible. Note also that when the DEA data domain contains some zero data, it is better to use the modified DEA models under the absolute change case, say (7) or (8).

Illustrative applications

The newly developed sensitivity analysis method is applied to two real world data sets: Chinese cities (Charnes *et al*[12]) and Chinese textiles (Zhu[13]). The sensitivity analysis is carried out as follows: (i) We consider the percentage changes of each individual input, each individual output, all inputs and all outputs respectively; (ii) We determine the upper-bound levels of g_o $(=\delta - 1)$ and g $(=(\delta - 1)/\delta)$ for input changes and h_o $(= 1 - \tau)$ and $h(=(1 - \tau)/\tau)$ for output changes as described in the percentage change case. Upper bounds of, for example, g_o and g can respectively be obtained as $\sqrt{\beta^*} - 1$ and $(\sqrt{\beta^*} - 1)/\sqrt{\beta^*}$ for $i \in I$. Here we interpret (g_o, g) and (h_o, h) as a sensitivity index in which g_o and h_o describe the change of each test efficient DMU and g and h describe the change of remaining DMUs; (iii) Using the sensitivity index, we study the sensitivity of both the CCR ratio model and the BCC convex model.

Chinese cities data set

The raw data for 28 Chinese cities in the year 1984 are given in Table 3. The data consists of three inputs of labor, working funds (WF), and investment, and three outputs of gross industrial output value (GIOV), profit & taxes (P&T), and retail sales (RS). From the last two columns of Table 3, we know that this data set has the same CCR and BCC efficient DMUs except for the DMU27. As a result, the CCR inefficiency scores are almost equal to the BCC inefficiency scores.

Tables 4 and 5 respectively report the sensitivity analysis results for the CCR and BCC models where the symbol '+' indicates that the modified DEA models employed are infeasible. The sensitivity index in each cell gives the upper-bound levels of (g_o, g) and (h_o, h). For instance, consider the DMU1 in Table 4. In the labor row, (4.14%, 3.97%) means that labor input of DMU1 can be increased by 4.14% and meanwhile the labor input of all other DMUs can be decreased by 3.97%. In the P&T row, (24.92%, 33.19%) means that the simultaneous 24.92% decrement of

Table 3 Data for the Chinese cities and DEA efficiency scores

	Outputs			*Inputs*			*Efficiency scores*[a]	
DMU No.	*GIOV*	*P & T*	*RS*	*Labor*	*WF*	*INV.*	*CCR*	*BCC*
1	7443700	1692100	1323800	487.44	1594040	718953	1	1
2	2817200	589600	1016600	375.42	955124	522032	0.64449	0.80210
3	2514900	443600	566200	273.69	782585	371314	0.62255	0.65157
4	1337100	162700	410300	208.82	489676	138017	0.50059	0.53053
5	1377600	232900	382900	199.99	519686	142688	0.51265	0.52715
6	1333600	209700	614500	181.90	480392	259092	0.68822	0.70422
7	758947	102893	298563	150.93	410727	95775	0.42528	0.43452
8	1157572	166978	412991	188.23	470104	134031	1	1
9	973951	166056	294927	126.48	296534	130325	0.61436	0.61915
10	668107	83735	248680	122.70	335329	105474	0.44484	0.46797
11	834600	128455	856868	133.13	335605	103431	1	1
12	540428	86288	298182	109.31	277571	65906	0.55594	0.59475
13	541923	87296	145379	94.45	204998	130349	0.44374	0.48127
14	918508	169458	290519	113.16	309779	115381	0.63190	0.63200
15	849956	128665	274982	87.54	209197	64903	0.78541	0.78623
16	540857	130638	141709	73.70	250558	86045	0.52156	0.59803
17	546065	82058	192074	77.64	181067	52858	1	1
18	547122	140122	120739	74.19	158850	52715	0.59853	0.64068
19	686383	201297	152881	89.69	203720	76580	0.60856	0.60867
20	450743	89031	171123	75.52	184676	78686	0.52922	0.60335
21	922572	61174	255685	70.73	136273	13424	1	1
22	1008736	137108	298717	68.10	235282	12365	1	1
23	664434	62610	217554	58.58	95712	7454	1	1
24	1093882	97857	214078	69.27	174731	13906	1	1
25	709278	69343	150142	47.97	111573	10502	1	1
26	649295	38004	256040	67.77	105075	10317	1	1
27	162454	12841	29041	20.07	55384	1847	0.98673	1
28	359956	38869	201795	72.37	133826	4322	1	1

[a]The efficiency scores are obtained from the input-based DEA model.

Table 4 Sensitivity analysis results of the Chinese cities for the CCR ratio model

Efficient DMUs	*1*	*8*	*11*	*17*	*21*	*22*
Labor	4.14%, 3.97%	+	+	+	3.65%, 3.52%	+
W.F.	+	+	+	+	4.57%, 4.37%	+
Investment	+	6.40%, 6.02%	+	+	+	+
All inputs	**4.14%, 3.97%**	**6.40%, 6.02%**	**25.48%, 20.30%**	**126.06%, 55.76%**	**1.17%, 1.16%**	**11.07%, 9.96%**
GIOV	4.86%, 5.11%	+	+	+	2.09%, 2.13%	+
P & T	24.92%, 33.19%	14.33%, 16.72%	+	55.76%, 126.06%	+	+
Retail sales	+	+	20.30%, 25.48%	+	+	+
All outputs	**3.98%, 4.14%**	**6.02%, 6.40%**	**20.30%, 25.48%**	**55.76%, 126.06%**	**1.16%, 1.17%**	**9.96%, 11.07%**

continued

Table 4 (continued)

Efficient DMUs	23	*24*	*25*	*26*	*28*
Labor	+	10.89%, 9.82%	1.00%, 1.00%	+	+
WF	+	+	0.30%, 0.30%	3.14%, 3.05%	+
Investment	+	+	+	+	25.41%, 20.26%
All inputs	**12.42%, 11.05%**	**4.57%, 4.37%**	**0.22%, 0.22%**	**2.98%, 2.89%**	**23.70%, 19.16%**
GIOV	+	4.34%, 4.57%	0.20%, 0.20%	+	+
PRT	+	+	3.00%, 3.00%	+	+
Retail sales	+	+	+	2.97%, 3.06%	19.16%, 23.70%
All outputs	**11.05%, 12.42%**	**4.37%, 4.57%%**	**0.22%, 0.22%**	**2.89%, 2.98%**	**19.16%, 23.70%**

Table 5 Sensitivity analysis results of the Chinese cities for the BCC convex model

Efficient DMUs	*1*	*8*	*11*	*17*	*21*	*22*
Labor	+	+	+	+	+	+
W.F.	+	+	+	+	9.37%, 8.57%	+
Investment	+	84.73%, 45.87%	+	+	+	+
All inputs	+	**84.73%, 45.87%**	**85.85%, 46.19%**	**144.42%, 59.09%**	**6.74%, 6.31%**	**22.08%, 18.09%**
GIOV	+	+	+	+	4.76%, 5.00%	+
P & T	+	+	+	55.91%, 126.80%	+	+
Retail sales	+	+	29.49%, 41.83%	+	+	+
All outputs	**40.19%, 67.20%**	**14.05%, 16.35%**	**29.49%, 41.83%**	**55.85%, 126.51%**	**2.90%, 2.99%**	**11.56%, 13.07%**

continued

Table 5 Continued.

Efficient DMUs	*23*	*24*	*25*	*26*	*27*	*28*
Labor	+	+	8.34%, 7.70%	+	+	+
W.F.	+	+	+	20.15%, 16.77%	+	+
Investment	+	+	+	+	+	27.13%, 21.34%
All inputs	**13.72%, 12.07%**	**22.76%, 18.54%**	**4.00%, 3.85%**	**8.59%, 7.91%**	**80.02%, 44.45%**	**27.13%, 21.34%**
GIOV	+	13.13%, 15.11%	7.00%, 7.52%	+	+	+
P & T	+	+	+	+	+	+
Retail sales	+	+	+	4.11%, 4.29%	+	25.54%, 34.29%
All outputs	**13.98%, 16.25%**	**6.58%, 7.04%**	**5.23%, 5.51%**	**3.67%, 3.81%**	+	**25.42%, 34.08%**

P&T in DMU1 and 33.19% increments of PT in the remaining 27 DMUs can not change the current efficiency classification of DMU1.

In both the CCR and the BCC cases, DMU25 is the most sensitive unit to possible data errors, since DMU25 has the lowest values for the sensitivity index in all cases of input and output variations. DMUs 11, 17, 22, 23, 27 and 28 are relatively insensitive to the possible data errors, since these DMUs have larger values of sensitivity index.

Recall that the infeasibility means stability, that is, the upper-bound levels of g_o, h and g, h_o are respectively $+\infty$ and (100 - ε)%, where ε is a small enough positive number. Therefore the symbol '+' means that (i) the input of the test DMU can infinitely be increased and simultaneously the corresponding inputs of the remaining DMUs can be reduced to any positive numbers; (ii) the output of the test DMU can be reduced to any positive numbers while the corresponding outputs of the remaining DMUs can infinitely be increased. There are more infeasibility cases under the BCC model than these under the CCR model.

Chinese textiles data set

This data set consists of 30 DMUs in the year 1989 with three inputs (circulating fund, investment, and labor) and three outputs (revenue from selling the products, profit & taxes, and net industrial output value). Table 6 contains the raw data for which 9 DMUs are CCR efficient and 13 DMUs are BCC efficient. The discussion to follow is similar to the above one for the Chinese cities. However, the structure of this textiles data set is very different from the previous one in the sense that larger discrepancies are found between the CCR and the BCC scores in this data set.

Tables 7 and 8 respectively show the 72 and 104 sensitivity analysis results for all 9 CCR efficient DMUs and all 13 BCC efficient DMUs when data variations are made for 8 different subsets of inputs and outputs. Of the 72 CCR sensitivity analysis results, 37 result in infeasibility and of the 104 BCC sensitivity analysis results, 68 are infeasible. Therefore, under both DEA models, over 50% of the sensitivity analysis results demonstrate that the DEA models are stable to particular data variations. Note that under the CCR case, 32% of the sensitivity indices are less than (20%, 20%); while under the BCC case, 19% of the sensitivity indices are less than (20%, 20%). Therefore the DEA efficiency is more robust in the convex DEA model.

Conclusions

The current paper develops a new approach for the sensitivity analysis of DEA models including the CCR, BCC and additive models. By the additional constraint on $\sum_{j=1,j\neq o}^{n} \lambda_j$, the approach can easily be modified to study

Table 6 Data for the Chinese textiles and DEA efficiency scores

	Outputs			*Inputs*			*Efficiency scores*[a]	
DMU No.	*Revenue*	*P & T*	*NIOV*	*Fund*	*INV*	*Labor*	*CCR*	*BCC*
1	193588	17870	5682	46509	66635	4063	1	1
2	19574	1287	330	6385	4054	481	0.95014	1
3	127168	11070	2235	26066	21548	4762	0.93383	1
4	49050	2805	886	10832	14419	1365	0.90252	0.91041
5	19520	663	667	6487	2590	1267	0.67273	0.94744
6	18388	400	684	6271	5735	1342	0.54438	0.60063
7	26040	1335	780	5542	10552	1185	0.78048	0.86823
8	5318	414	190	2389	1003	534	0.48470	0.60973
9	54783	270	834	10346	17037	1083	1	1
10	42313	1106	503	8731	10721	837	1	1
11	11539	1425	488	3560	4879	843	0.66911	0.73149
12	19922	392	541	9415	7292	1594	0.42530	0.42621
13	35568	4798	1612	10529	6213	880	1	1
14	12856	810	571	2789	2558	475	1	1
15	10211	691	425	3096	3671	558	0.68387	0.72863
16	9055	703	313	2233	339	454	1	1
17	12454	994	346	2319	1965	476	0.96261	0.99144
18	6841	32	145	2026	873	395	0.74003	0.78739
19	6748	469	264	1502	1616	453	0.78594	0.83353
20	5042	157	87	765	524	252	0.98903	1
21	2534	120	20	1102	311	213	0.54650	1
22	10874	303	366	2713	2362	538	0.73934	0.77487
23	16341	506	339	1447	236	706	1	1
24	6930	736	176	2292	1618	390	0.65934	0.81873
25	9430	1790	307	4383	337	593	0.76405	0.80145
26	4087	717	240	2072	1568	448	0.57913	0.73479
27	6178	1018	300	1332	1134	381	1	1
28	2331	1514	521	5548	3584	983	0.87582	0.90380
29	12374	649	550	4744	5168	1426	0.51327	0.56229
30	12115	899	227	1750	1812	498	1	1

[a]The efficiency scores are obtained from the input-based DEA model.

Table 7 Sensitivity analysis results of the Chinese textiles for the CCR ratio model

Efficient DMUs	*1*	*9*	*10*	*13*	*14*
Fund	+	+	+	+	8.80%, 8.09%
Investment	+	+	+	+	+
Labor	12.77%, 10.04%	10.32%, 8.48%	10.05%, 9.13	48.56%, 32.69%	18.08%, 15.31%
All inputs	**4.23%, 4.05%**	**4.07%, 3.91%**	**3.53%, 3.42%**	**34.33%, 25.55%**	**6.08%, 5.73%**
Revenue	5.49%, 5.81%	4.40%, 4.60%	3.42%, 3.53%	+	+
P & T	+	+	+	+	+
NIOV	+	+	+	+	7.30%, 7.87%
All outputs	**4.05%, 4.23%**	**3.91%, 4.07%**	**3.42%, 3.53%**	**25.55%, 34.33%**	**5.73%, 6.08%**

continued

Table 7 Continued

Efficient DMUs	*16*	*23*	*27*	*30*
Fund	+	31.00%, 23.67%	35.80%, 26.36%	6.39%, 6.00%
Investment	96.14%, 49.02%	+	+	+
Labor	+	+	+	6.77%, 6.34%
All inputs	**94.98%, 48.71%**	**29.82%, 22.97%**	**25.67%, 20.42%**	**2.22%, 2.17%**
Revenue	+	+	+	3.2%, 3.31%
P & T	+	+	+	7.92%, 8.61%
NIOV	+	+	+	+
All outputs	**48.71%, 94.98%**	**22.97%, 29.82%**	**20.42%, 25.67%**	**2.17%, 2.22%**

Table 8 Sensitivity analysis results of the Chinese textiles for the BCC convex model

Efficient DMUs	*1*	*2*	*3*	*9*	*10*	*13*	*14*
Fund	+	+	+	+	+	+	10.52%, 9.52%
Investment	+	+	+	+	+	+	+
Labor	+	2.90%, 2.82%	+	32.98%, 24.80%	10.98%, 9.89%	+	+
All inputs	+	**2.87%, 2.79%**	**38.34%, 27.71%**	**6.02%, 5.68%**	**3.81%, 3.67%**	**53.83%, 35.00%**	**7.12%, 6.65%**
Revenue	+	4.98%, 5.52%	+	4.96%, 5.22%	3.90%, 4.06%	+	+
P & T	+	+	+	+	+	+	+
NIOV	+	+	+	+	+	+	10.64%, 11.90%
All outputs	**38.88%, 63.60%**	**4.23%, 4.42%**	**22.86%, 29.63%**	**4.97%, 5.23%**	**3.90%, 4.06%**	**29.52%, 41.89%**	**5.89%, 6.27%**

continued

Table 8 Continued

Efficient DMUs	*16*	*20*	*21*	*23*	*27*	*30*
Fund	+	+	+	41.59%, 29.37%	+	8.49%, 7.83%
Investment;	96.23%, 49.04%	+	+	+	+	+
Labor	+	+	+	+	+	+
All inputs	**95.27%, 48.79%**	**23.73%, 19.18%**	**22.13%, 18.12%**	**37.60%, 27.33%**	**27.30%, 21.45%**	**2.87%, 2.79%**
Revenue	+	+	+	+	+	4.42%, 4.62%
P & T	+	+	+	+	+	13.19%, 15.19%
NIOV	+	+	+	+	+	+
All outputs	**69.29%, 225.60%**	+	+	**23.35%, 30.46%**	**26.66%, 36.34%**	**3.25%, 3.36%**

the sensitivity of other DEA models that satisfy different returns to scale (Seiford and Thrall[14]). Compared to the existing DEA sensitivity analysis methods, our approach (i) simultaneously considers the changes of all DMUs, namely, the change of the test DMU and the changes of the remaining DMUs; (ii) all remaining DMUs work at improving their efficiencies against the deteriorating efficiency of the test efficient DMU; (iii) relies solely on DEA-type technique so that the extraneous assumptions used by sensitivity analysis in linear programming are not needed;

and (iv) yields more exact sensitivity analysis results (see Appendix).

The paper has focused on sufficient conditions for a test DMU to preserve its efficiency when the data of other DMUs are also changed. As a matter of fact, we could employ the technique in Seiford and Zhu[7] to determine necessary conditions when there exists evidence that we need to explore a larger efficiency stability region for the test DMU. In addition, although the paper identifies a maximum (same) percentage (lower or upper bound) change in the data, the data change rates for a test DMU and others are not necessarily the same within the variation ranges determined by the maximum percentage.

The application here is an illustrative one. The results have shown that the DEA efficiency models, particularly the convex DEA model (BCC model), overall, are robust. It can be seen that some DMUs are extremely insensitive to the potential data errors with sensitivity index values greater than (50%, 50%) while some DMUs are extremely sensitive to the potential data errors with sensitivity index values less than (5%, 5%). The performance of these efficient DMUs is worth further study. In our opinion, the newly developed sensitivity analysis approach could also be used for further analysis of managerial performance, particularly for those DMUs having extremely large or small values of the sensitivity index. This may in turn require interaction with management of these DMUs. Use of this newly developed sensitivity analysis approach as a performance management tool is a subject for future research.

Appendix

This Appendix will compare our sensitivity analysis approach to that of Thompson *et al*[8] for the example used in their paper in which DMUs 1, 2 and 3 are CCR efficient (see Table 9).

Table 9 Six DMUs with two inputs and one output

DMU	*1*	*2*	*3*	*4*	*5*	*6*
Input 1	4	2	1	2	3	4
Input 2	1	2	4	3	2	4
Output	1	1	1	1	1	1

Table 10 Comparison of sensitivity analysis results

Efficient DMUs	*1*	*2*	*3*
(g_0, g)	41.42%, 29.29%	11.80%, 10.56%	41.42%, 29.29%
SCSC-1	20%	14%	20%
SCSC-2	32%	9.1%	32%

Table 11 SCSC solutions

	SCSC-1	*SCSC-2*
	μ^*, v_1^*, v_1^*	$\mu^*, v_1{}^*\ v_2{}^*$
DMU 1 :	$\left(1, \frac{1}{10}, \frac{3}{5}\right)$	$\left(1, \frac{1}{85}, \frac{81}{85}\right)$
DMU 2 :	$\left(1, \frac{14}{43}, \frac{15}{86}\right)$	$\left(1, \frac{1}{5}, \frac{3}{10}\right)$
DMU 3 :	$\left(1, \frac{3}{5}, \frac{1}{10}\right)$	$\left(1, \frac{81}{85}, \frac{1}{85}\right)$

Note: μ^* stands for output multiplier and $v_1{}^*$, $v_2{}^*$ respectively stand for input multipliers in the dual from of the CCR model.

Table 10 reports the sensitivity analysis results under the situation when all the two inputs change in percentage. Row 2 represents the results obtained by using our approach. For example, the two inputs of DMU2 can be increased by 11.80% and simultaneously the two inputs of DMUs 1, 3, 4, 5, and 6 can be decreased by 10.56% while DMU2 is still CCR efficient. Rows 3 and 4 show the results by using two different sets of SCSC solutions given in Table 11. The '20%' in the second and the last columns means that the two inputs of DMUs 1, 2 and 3 can be increased by 20% and the two inputs of DMUs 4, 5 and 6 can be decreased by 20% while DMU1 and DMU2 are still CCR efficient. Here the percentage numbers mean that the variation under which the test efficient DMU remains CCR efficient, rather than the switch point under which the test efficient DMU is replaced (see Thompson *et al*[8] for a detailed illustration for the switch).

Note that under different SCSC solutions, we have different upper-bound levels of input variations. Consider, for example, DMU2, under SCSC-1, we have 14% which is greater than 11.80% and 10.56%, while under SCSC-2, we have 9.1% which is less than 11.80% and 10.56%. Thus our approach gives more robust and accurate results. Note that our condition in data variations is even more restrictive than that in Thompson *et al*[8] Since our sensitivity analysis approach considers the situation of all other DMUs improving their efficiency against *the test* efficient DMU, while Thompson *et al*[8] considered the situation of all inefficient DMUs improving their efficiency against *all* efficient DMUs.

References

1 Charnes A *et al* (1985). Sensitivity and stability analysis in DEA. *Annals of Opns Res* **2**: 139–156.

2 Charnes A and Neralic L (1990). Sensitivity analysis of the additive model in data envelopment analysis. *Eur J Opl Res* **48**: 332–341.

3 Charnes A, Haag S, Jaska P and Semple J (1992). Sensitivity of efficiency classifications in the additive model of data envelopment analysis. *Int J Systems Sci* **23**: 789–798.
4 Charnes A, Rousseau J and Semple J (1996). Sensitivity and stability of efficiency classifications in data envelopment analysis. *J Prod Anal* **7**: 5–18.
5 Zhu J (1996). Robustness of the efficient DMUs in data envelopment analysis. *Eur J Opl Res* **90**: 451–460.
6 Charnes A, Cooper WW and Rhodes E (1978). Measuring the efficiency of decision making units. *Eur J Opns Res* **2**: 429–444.
7 Seiford LM and Zhu J (1998). Stability regions for maintaining efficiency in data envelopment analysis. *Eur J Opns Res* **108**: 127–139.
8 Thompson RG, Dharmapala PS and Thrall RM (1994). DEA sensitivity analysis of efficiency models with applications to Kansas farming and Illinois coal mining. In: Charnes A, Cooper WW, Lewin AY and Seiford LM (eds). *Data Envelopment Analysis*: *Theory, Methodology and Applications*. Klewer Academic Publishers, Boston, USA, 393–422.
9 Thompson RG *et al* (1996). DEA multiplier analytic center sensitivity with an illustrative application to independent oil companies. *Annals of Opns Res* **66**: 163–177.
10 Banker RD, Charnes A and Cooper WW (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Mgmt Sci* **30**: 1078–1092.
11 Charnes A *et al* (1985). Foundation of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. *J of Econ* **30**: 91–107.
12 Charnes A Cooper WW and Li S (1989). Using DEA to evaluate relative efficiencies in the economic performance of Chinese cities. *Socio-Econ Plan Sci* **23**: 325–344.
13 Zhu J (1996). DEA/AR analysis of the 1988–1989 performance of Nanjing Textile Corporation. *Annals of Opns Res* **66**: 311–335.
14 Seiford LM and Thrall RM (1990). Recent developments in DEA: The mathematical programming approach to frontier analysis. *J of Econ* **46**: 7–38.
15 Charnes A, Cooper WW, Lewin AY and Seiford LM (1994). *Data Envelopment Analysis: Theory, Methodology and Applications*. Klewer Academic Publishers, Boston, USA.

Received September 1996;
accepted May 1998 after one revision