A modified super-efficiency DEA model for infeasibility

WD Cook¹, L Liang², Y Zha² and J Zhu³*

¹York University, Toronto, Ontario, Canada; ²University of Science and Technology of China, He Fei, An Hui Province, PR China; and ³Worcester Polytechnic Institute, Worcester, MA, USA

The super-efficiency data envelopment analysis (DEA) model is obtained when a decision making unit (DMU) under evaluation is excluded from the reference set. This model provides for a measure of stability of the "efficient" status for frontier DMUs. Under the assumption of variable returns to scale (VRS), the super efficiency model can be infeasible for some efficient DMUs, specifically those at the extremities of the frontier. The current study develops an approach to overcome infeasibility issues. It is shown that when the model is feasible, our approach yields super-efficiency scores that are equivalent to those arising from the original model. For efficient DMUs that are infeasible under the super-efficiency model, our approach yields optimal solutions and scores that characterize the extent of super-efficiency in both inputs and outputs. The newly developed approach is illustrated with two real world data sets.

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1. Introduction

The data envelopment analysis (DEA) model of Charnes et al. (1978) provides a methodology for evaluating a relative efficiency score for each member of a set of peer decision-making units (DMUs); this constant returns (CRS) model was extended by Banker et al. (1984) to include variable returns to scale (VRS). An important problem in the DEA literature is that of ranking those DMUs deemed efficient by the DEA model, all of which have a score of unity. One approach to the ranking problem is that provided by the super efficiency model of Andersen and Petersen (1993). See also Banker et al. (1989). The super efficiency model involves executing the standard DEA models (CRS or VRS), but under the assumption that the DMU being evaluated is excluded from the reference set. Specifically, the super efficiency score in the input-oriented model, provides a measure of the proportional increase in the inputs for a DMU that could take place without destroying the 'efficient' status of that DMU relative to the frontier created by the remaining DMUs. The super efficiency score can also be thought of as a measure of stability. That is, if input data for instance, is subject to error or change over time, the super efficiency score provides a means of evaluating the extent to which such changes could occur without violating that DMU's status as an efficient unit. Hence, the score yields a measure of stability.

In addition to being a tool for ranking, the super-efficiency concept has been used in other situations, for example, two-person ratio efficiency games (Rousseau and Semple, 1995), and acceptance decision rules (Seiford and Zhu, 1998a), among others.

It is well known that under certain conditions, the super-efficiency DEA model may not have feasible solutions for efficient DMUs (see, eg, Zhu, 1996; Dulá and Hickman, 1997; Seiford and Zhu, 1998a, b, 1999). Given the wide use of the super-efficiency concept, it is thus worthwhile to develop approaches that can overcome this infeasibility problem. This is the scope of the current paper. As shown in Seiford and Zhu (1999), infeasibility must occur in the case of the VRS super-efficiency model. Although infeasibility implies a form of stability in DEA sensitivity analysis (Seiford and Zhu, 1998b), limited efforts have been made to provide numerical super-efficiency scores for those efficient DMUs for which feasible solutions are unavailable in the VRS super-efficiency model. Lovell and Rouse (2003) developed a standard DEA approach to the super-efficiency model by scaling up the inputs (scaling down the outputs) of a DMU under evaluation. As a result, a feasible solution can be found for efficient DMUs that do not have such (feasible) solutions in the standard VRS super-efficiency model. The super-efficiency scores for all efficient DMUs without feasible solutions are then equal to the user-defined scaling factor. Chen (2004, 2005) suggests using both the input- and output-oriented VRS super-efficiency models to quantify the super-efficiency when infeasibility occurs. However, Chen's approach will fail if both the input- and output-oriented VRS super-efficiency models are infeasible.
The current paper proposes an alternative approach to solve the infeasibility problem in the VRS super-efficiency model. Our approach provides VRS super-efficiency scores that are equivalent to those arising from the VRS super-efficiency model when feasibility is present. When the VRS super-efficiency model is infeasible, our approach determines a (virtual) referent DMU formed by the remaining DMUs and yields a score that characterizes the super-efficiency in inputs and outputs. Since the super-efficiency and standard DEA models always have feasible solutions and yield equivalent results for inefficient DMUs, the current study assumes that a DMU under evaluation is efficient (i.e. on the DEA frontier).

The rest of the paper is organized as follows. Section 2 presents the existing VRS super-efficiency models. Section 3 develops our new approach. The relationship between our approach and that of Lovell and Rouse (2003) is discussed in Section 4. Section 5 applies the newly developed approach to data on the 20 largest Japanese companies and 15 US cities that are used in Chen (2004). Conclusions are presented in Section 6.

2. Super-efficiency DEA

Suppose we have a set of $n$ DMUs $\{DMU_j : j = 1, 2, \ldots, n\}$. Each $DMU_j$ has a set of $s$ outputs, $y_{rj}$ ($r = 1, 2, \ldots, s$), and a set of $m$ inputs, $x_{ij}$ ($i = 1, 2, \ldots, m$). Based upon the VRS DEA model (Banker et al., 1984), the input-oriented VRS super-efficiency model can be expressed as

$$
\min \theta \\
\text{s.t. } \sum_{j \neq k} \lambda_j x_{ij} \leq \theta x_{ik}, \quad i = 1, 2, \ldots, m \\
\sum_{j \neq k} \lambda_j y_{rj} \geq \theta y_{rk}, \quad r = 1, 2, \ldots, s \\
\sum_{j \neq k} \lambda_j = 1 \\
\theta \geq 0 \\
\lambda_j \geq 0, \quad j \neq k
$$

(1)

where the $DMU_k$ under evaluation is excluded from the reference set.

Note that model (1) is an input-oriented super-efficiency DEA model. When $DMU_k$ is efficient and model (1) is feasible, $\theta^* > 1$, indicating that $DMU_k$’s inputs are increased to reach the frontier formed by the rest of the DMUs. That is, super-efficiency is expressed in terms of input increases. We can use $\theta^*$ as the super-efficiency score for $DMU_k$.

The output-oriented VRS super-efficiency model can be expressed as

$$
\max \phi \\
\text{s.t. } \sum_{j \neq k} \lambda_j x_{ij} - \lambda_j^* x_{ik}, \quad i = 1, 2, \ldots, m \\
\sum_{j \neq k} \lambda_j y_{rj} - \lambda_j^* y_{rk}, \quad r = 1, 2, \ldots, s \\
\sum_{j \neq k} \lambda_j = 1 \\
\phi \geq 0 \\
\lambda_j \geq 0, \quad j \neq k
$$

(2)

When $DMU_k$ is efficient and model (2) is feasible, $\phi^* \leq 1$, indicating that $DMU_k$’s outputs are decreased to reach the frontier formed by the rest of the DMUs, that is, super-efficiency is represented by output reductions, and the degree of output-oriented super-efficiency is given by $1/\phi^* \geq 1$.

These models can be infeasible in certain situations. Seiford and Zhu (1999) provide the necessary and sufficient conditions for the infeasibility of these models. It is shown that in the case that a VRS efficient DEA model has the highest output(s) (regardless of the input values), model (1) must be infeasible; if it has the highest input(s) (regardless of the output values), model (2) must be infeasible. Note that any data set always contains such efficient DMUs. Thus, models (1) and (2) must be infeasible for these efficient DMUs.

Note that when model (1) (or model (2)) is infeasible, $\theta^* > 1$, no numerical value can be assigned to $\theta^*$ (or $1/\phi^*$). Next section will develop an approach to provide a numerical value to the input-oriented (or output-oriented) super-efficiency score.

3. New model

Infeasibility of model (1) (or (2)) occurs when a VRS efficient DMU under evaluation cannot reach the frontier formed by the rest of DMUs via increasing the inputs (or decreasing the outputs). Unlike the standard super efficiency models (1) and (2), each of which has a specific orientation (input or output), our model proposes moving to the frontier by way of projection in both directions. In practical terms, rather than asking, for a given efficient DMU, either how much increase in inputs is possible, or how much reduction in outputs is possible, while still retaining its efficient status, our model describes the minimum movement in both directions needed to reach the frontier generated by the remaining DMUs. Viewed another way, in the case of infeasibility, our model derives the minimum change needed to project a data point, classified as an extremity, to a non-extreme position.
Consider the following model for $DMU_k$

$$\text{Min } \tau + M \times \beta$$

s.t. $\sum_{j=1}^{n} \lambda_{ij} x_{ij} \leq (1 + \tau) x_{ik}, \quad i = 1, 2, \ldots, m$

$\sum_{j=1}^{n} \lambda_{ij} y_{ij} \geq (1 - \beta) y_{ik}, \quad r = 1, 2, \ldots, s$

$\sum_{j=1}^{n} \lambda_{ij} = 1$

$\beta \geq 0, \lambda_{ij} \geq 0, \quad j = 1, 2, \ldots, n; j \neq k$ (3)

where $M$ is a user-defined large positive number. (In our application, $M$ is set equal to $10^5$.)

**Theorem 1** Model (1) is infeasible if and only if $\beta^* > 0$, where $\beta^*$ is the optimal solution in model (3).

**Proof** Note that $\beta > 0$. Suppose model (1) is infeasible. If $\beta^* = 0$, this means that model (1) is feasible, which is a contradiction. Therefore, $\beta^* > 0$. Now, suppose $\beta^* > 0$, and suppose model (1) is feasible. This means that $\beta^* = 0$ is a feasible solution to model (3), in contradiction to the fact that $\beta^* > 0$ is optimal. Therefore, model (1) is infeasible. This completes the proof. □

Theorem 1 indicates that model (1) is feasible if and only if $\beta^* = 0$. This further indicates that $1 + \tau^* = 0^*$, where $(*)$ denotes the optimal values in models (1) and (3). In other words, when model (1) is feasible, model (3) is equivalent to model (1), in the sense that the objective function values of the two models are identical.

**Theorem 2** $1 > \beta^* \geq 0$ and $\tau^* > -1$ in model (3).

**Proof** Note that in model (3), $\sum_{j=1}^{n} \lambda_{ij} y_{ij} > 0, r = 1, 2, \ldots, s$. Thus, there must exist $\beta \in [0, 1]$ such that $\sum_{j=1}^{n} \lambda_{ij} y_{ij} > (1 - \beta) y_{ik}, \quad r = 1, 2, \ldots, s$. Note also that model (3) minimizes $\beta$. Therefore, $1 > \beta^* \geq 0$. Similarly, we can prove that $\tau^* > -1$. □

From Theorem 2 it follows that when model (1) is infeasible, $1/(1 - \beta^*) > 1$ and $1 + \tau^* > 0$, that is, in order to have a feasible solution, $DMU_k$ must decrease its outputs. Further, we define the super-efficiency score as $1 + \tau^* + 1/(1 - \beta^*)$ (>1). This super-efficiency score consists of a component for input super-efficiency, namely $1 + \tau^* > 0$, and a component for output super-efficiency, namely $1/(1 - \beta^*) > 1$.

There are two cases associated with this new super-efficiency score for DMUs not having feasible solutions in model (1).

**Case 1:** $\tau^* > 0$ (or $1 + \tau^* > 1$). In this case $DMU_k$ must increase its inputs and decrease its outputs to reach the frontier formed by the rest of the DMUs. This means that $DMU_k$ exhibits super-efficiency in both inputs and outputs.

**Case 2:** $\tau^* < 0$ (or $1 + \tau^* < 1$). This indicates that $DMU_k$ must decrease its inputs and outputs to reach the frontier formed by the rest of the DMUs, meaning that $DMU_k$’s super-efficiency is reflected by the outputs only.

In summary, our new model (3) yields a score equivalent to the original VRS super-efficiency score if model (1) is feasible. (When $\beta = 0$, the actual value of the super-efficiency score based upon model (3) is $1 + \tau^* + 1 = 0^* + 1$, that is, the input super-efficiency component of $1 + \tau^*$ is the original VRS super-efficiency score $0^*$.) When model (1) is infeasible, our model (3) determines an optimal solution and yields a numerical super-efficiency score that captures the super-efficiency in both inputs and outputs.

The above discussion is based upon an input-orientation and model (1). For the output-orientation, we have the equivalent model

$$\text{Min } \gamma + M \times \delta$$

s.t. $\sum_{j=1}^{n} \lambda_{ij} x_{ij} \leq (1 + \delta) x_{ik}, \quad i = 1, 2, \ldots, m$

$\sum_{j=1}^{n} \lambda_{ij} y_{ij} \geq (1 - \gamma) y_{ik}, \quad r = 1, 2, \ldots, s$

$\sum_{j=1}^{n} \lambda_{ij} = 1$

$\gamma \geq 0, \lambda_{ij} \geq 0, \quad j = 1, 2, \ldots, n; j \neq k$ (4)

where $M$ is a user-defined large positive number.

Similar to Theorems 1 and 2, we have

**Theorem 3** Model (2) is infeasible if and only if $\delta^* > 0$.

**Theorem 4** $\gamma^* < 1$ and $\delta^* > 0$ in model (4).

Theorem 3 indicates that when model (2) is feasible, model (4) is equivalent to model (2) in that $1 - \gamma^* = \phi^*$. When model (2) is infeasible, Theorem 4 indicates that $1/(1 - \gamma^*) > 0$, meaning that in order to have a feasible solution, $DMU_k$ must increase its inputs. Further, we define the super-efficiency score for DMUs not having feasible solutions under model (2) as $1 + \delta^* + 1/(1 - \gamma^*)$ (>1).

When $1 > \gamma^* > 0$, $DMU_k$ must decrease its outputs and increase its inputs to reach the frontier formed by the rest of the DMUs, indicating that $DMU_k$ exhibits super-efficiency in both inputs and outputs. When $\gamma^* < 0$, $DMU_k$ must increase both inputs and outputs to reach the frontier formed by the rest of the DMUs, indicating that $DMU_k$ exhibits super-efficiency in inputs only.
4. Discussion

In this section, we examine the relation between our approach and that of Lovell and Rouse (2003). While the discussion is based upon input-oriented models, similar arguments apply to output-oriented models.

The Lovell and Rouse (2003) model is defined as follows:

$$\text{Min } \theta$$

s.t. \[ \sum_{j=1}^{n} \lambda_{j} x_{ij} + \lambda_{k} (x \times x_{ik}) \leq \theta (x \times x_{ik}), \quad i = 1, 2, \ldots, m \]

\[ \sum_{j=1}^{n} \lambda_{j} y_{kj} + \lambda_{k} y_{rk} \geq y_{rk}, \quad r = 1, 2, \ldots, s \]

\[ \sum_{j=1}^{n} \lambda_{j} = 1 \]

\[ \lambda_{j} \geq 0, \quad j = 1, 2, \ldots, n \] (5)

where \( \lambda > 1 \) is a user-defined scaling factor. Lovell and Rouse (2003) suggest using \( \lambda = \text{Max} (\text{Max } x_{ij}/\text{Min } x_{ij}) + 1 \). The super-efficiency is then defined as \( \theta \times \lambda \).

Model (5) is equivalent to the following model

$$\text{Min } \theta$$

s.t. \[ \sum_{j=1}^{n} \lambda_{j} x_{ij} \leq (\theta - \lambda_{k}) x_{ik}, \quad i = 1, 2, \ldots, m \]

\[ \sum_{j=1}^{n} \lambda_{j} y_{kj} \geq (1 - \lambda_{k}) y_{rk}, \quad r = 1, 2, \ldots, s \]

\[ \sum_{j=1}^{n} \lambda_{j} = 1 - \lambda_{k} \]

\[ \lambda_{j} \geq 0, \quad j = 1, 2, \ldots, n \] (6)

Now, let \( (\theta - \lambda_{k})x = 1 + \tau \), where \( \tau \) is a variable that is free in sign. Then, we have \( \theta = (1 + \tau + \lambda_{k})/\lambda \) and the super-efficiency score can be expressed as \( \lambda \theta = 1 + \tau + \lambda_{k} \).

Because \( \lambda \) is a user-defined constant, model (6) is equivalent to the following model

$$\text{Min } \tau + \lambda_{k}$$

s.t. \[ \sum_{j=1}^{n} \lambda_{j} x_{ij} \leq (1 + \tau) x_{ik}, \quad i = 1, 2, \ldots, m \]

\[ \sum_{j=1}^{n} \lambda_{j} y_{kj} \geq (1 - \lambda_{k}) y_{rk}, \quad r = 1, 2, \ldots, s \]

\[ \sum_{j=1}^{n} \lambda_{j} = 1 - \lambda_{k} \]

\[ \lambda_{j} \geq 0, \quad j = 1, 2, \ldots, n \] (7)

Theorem 5 In model (7), there does not exist any optimal solution such that \( 1 > \lambda_{k} > 0 \).

Proof Suppose there exists an optimal solution in model (7) such that \( 1 > \lambda_{k} > 0 \). This indicates that the following model has a feasible solution

$$\text{Min } \tau$$

s.t. \[ \sum_{j=1}^{n} \lambda_{j} x_{ij} \leq \frac{1 + \tau}{1 - \lambda_{k}} x_{ik}, \quad i = 1, 2, \ldots, m \]

\[ \sum_{j=1}^{n} \lambda_{j} y_{kj} \geq y_{rk}, \quad r = 1, 2, \ldots, s \]

\[ \sum_{j=1}^{n} \lambda_{j} = 1 \]

\[ \lambda_{j} \geq 0, \quad j = 1, 2, \ldots, n \]

This further indicates that model (1) is always feasible, in contradiction. This proves the theorem. \( \square \)

Theorem 5 indicates that either \( \lambda_{k} = 0 \) or \( \lambda_{k} = 1 \). If \( \lambda_{k} = 0 \), this means that model (1) is feasible. As a result, model (6) is equivalent to model (3), that is, both Lovell and Rouse (2003) and our approaches yield the identical results to the original super-efficiency model (1). If \( \lambda_{k} = 1 \), Theorem 5 also indicates that model (1) is infeasible. In this case, model (7) yields the optimal solutions of \( \lambda_{k} = 0 \), \( j \neq k \) and \( \tau = -1 \). As a result, Lovell and Rouse’s (2003) super-efficiency is equal to the user-defined \( \lambda \) for all the efficient DMUs not having feasible solutions in model (1).

We finally note that the value of \( \lambda \) has nothing to do with the constraints of model (7) when model (1) is infeasible for DMUs. In fact, if we change \( \sum_{j=1}^{n} \lambda_{j} = 1 - \lambda_{k} \) to \( \sum_{j=1}^{n} \lambda_{j} = 1 \), and set \( \lambda \) equal to a large enough value, model (6) becomes model (3).

5. Application

We apply models (3) and (4) to two data sets used in Chen (2004). One consists of the 20 largest Japanese companies in 1999 (see Table 1). The other consists of 15 of Fortune’s top US cities in 1996 (see Table 2).

5.1. Japanese companies

The DEA inputs are assets (million $), equity (million $) and number of employees and the DEA output is revenue (million $). Either model (1) or model (2) indicates that five of them are VRS-efficient (see last two columns in Table 1). DMU1 is infeasible under model (1) and DMU18 is infeasible under model (2).

Table 3 reports the results from models (3) and (4) for the five efficient DMUs. Model (3) yields a super-efficiency score of 2.0161 for DMU1 with \( \tau = 0.0104 \) and \( \beta = 0.0057 \), indicating that DMU1 has super-efficiency in both inputs and outputs. Model (4) yields a super-efficiency score of 3.22798 for DMU18 with \( \delta = 1.8999 \) and \( \gamma = -2.048 \), indicating that DMU18 has super-efficiency in inputs only.
The bold numerals enable easy identification of items discussed in the paper.

### Table 1  Japanese companies

<table>
<thead>
<tr>
<th>DMU Company</th>
<th>Asset</th>
<th>Equity</th>
<th>Employee</th>
<th>Revenue</th>
<th>Model (1) θ₀</th>
<th>Model (2) 1/θ₀²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MITSUI &amp; CO.</td>
<td>50905.3</td>
<td>5137.9</td>
<td>40000</td>
<td>106793.2</td>
<td>infeasible</td>
<td>1.01523</td>
</tr>
<tr>
<td>2 ITOCHU CORP.</td>
<td>51432.5</td>
<td>2333.8</td>
<td>5775</td>
<td>106184.1</td>
<td>6.69295</td>
<td>1.3177</td>
</tr>
<tr>
<td>3 MITSUBISHI CORP.</td>
<td>67553.2</td>
<td>7253.2</td>
<td>36000</td>
<td>104656.3</td>
<td>0.74248</td>
<td>0.98068</td>
</tr>
<tr>
<td>4 TOYOTA MOTOR CORP.</td>
<td>112698.1</td>
<td>47177.4</td>
<td>183879</td>
<td>973876.7</td>
<td>0.4108</td>
<td>0.91191</td>
</tr>
<tr>
<td>5 MARUBENI CORP.</td>
<td>49742.9</td>
<td>2704.3</td>
<td>5844</td>
<td>91361.7</td>
<td>0.91739</td>
<td>0.89127</td>
</tr>
<tr>
<td>6 SUMITOMO CORP.</td>
<td>41168.4</td>
<td>4351.5</td>
<td>30700</td>
<td>86921</td>
<td>infeasible</td>
<td>1.02091</td>
</tr>
<tr>
<td>7 NIPPON TELEGRAPH &amp; TEL.</td>
<td>133008.8</td>
<td>47467.1</td>
<td>138150</td>
<td>74323.4</td>
<td>0.26865</td>
<td>0.69594</td>
</tr>
<tr>
<td>8 NISSHO IWI CORP.</td>
<td>35581.9</td>
<td>1274.4</td>
<td>19461</td>
<td>66144</td>
<td>1.14580</td>
<td>1.14784</td>
</tr>
<tr>
<td>9 HITACHI LTD.</td>
<td>73917</td>
<td>21941.2</td>
<td>328351</td>
<td>60937.9</td>
<td>0.40528</td>
<td>0.57061</td>
</tr>
<tr>
<td>10 MATSUSHITA ELECTRIC INDL.</td>
<td>60639</td>
<td>26988.4</td>
<td>282153</td>
<td>58361.6</td>
<td>0.47569</td>
<td>0.54648</td>
</tr>
<tr>
<td>11 SONY CORP.</td>
<td>48117.4</td>
<td>13930.7</td>
<td>177000</td>
<td>51903</td>
<td>0.54156</td>
<td>0.51337</td>
</tr>
<tr>
<td>12 NISSAN MOTOR</td>
<td>52842.1</td>
<td>9583.6</td>
<td>39467</td>
<td>50263.5</td>
<td>0.47975</td>
<td>0.4707</td>
</tr>
<tr>
<td>13 HONDA MOTOR</td>
<td>38455.8</td>
<td>13473.8</td>
<td>112200</td>
<td>47597.9</td>
<td>0.62931</td>
<td>0.59028</td>
</tr>
<tr>
<td>14 TOSHIBA CORP.</td>
<td>46013</td>
<td>8023.3</td>
<td>198000</td>
<td>40492.7</td>
<td>0.45933</td>
<td>0.41827</td>
</tr>
<tr>
<td>15 FUJITSU LTD.</td>
<td>39052.2</td>
<td>8602.4</td>
<td>188000</td>
<td>40050.3</td>
<td>0.53631</td>
<td>0.48833</td>
</tr>
<tr>
<td>16 TOKYO ELECTRIC POWER</td>
<td>110055.8</td>
<td>12157.7</td>
<td>50558</td>
<td>38869.5</td>
<td>0.18567</td>
<td>0.36397</td>
</tr>
<tr>
<td>17 NEC CORP.</td>
<td>38015</td>
<td>6517.4</td>
<td>157773</td>
<td>36356.4</td>
<td>0.50901</td>
<td>0.45666</td>
</tr>
<tr>
<td>18 TOMEN CORP.</td>
<td>16896</td>
<td>676.1</td>
<td>3654</td>
<td>30205.3</td>
<td>2.89988</td>
<td>infeasible</td>
</tr>
<tr>
<td>19 JAPAN TOBACCO</td>
<td>17023.6</td>
<td>10816.6</td>
<td>31000</td>
<td>29612.2</td>
<td>0.98076</td>
<td>0.9563</td>
</tr>
<tr>
<td>20 MITSUBISHI ELECTRIC CORP.</td>
<td>31997</td>
<td>4129.6</td>
<td>116479</td>
<td>28982.2</td>
<td>0.5218</td>
<td>0.44136</td>
</tr>
</tbody>
</table>

The bold numerals enable easy identification of items discussed in the paper.

### Table 2  US cities

<table>
<thead>
<tr>
<th>DMU</th>
<th>City</th>
<th>Houseprice</th>
<th>Rental</th>
<th>Violent</th>
<th>Income</th>
<th>B. Degree</th>
<th>Doctor</th>
<th>Model (1) θ₀</th>
<th>Model (2) 1/θ₀²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Seattle</td>
<td>586</td>
<td>581</td>
<td>1193.06</td>
<td>46928</td>
<td>0.6534</td>
<td>9.878</td>
<td>1.44335</td>
<td>1.0934</td>
</tr>
<tr>
<td>2</td>
<td>Denver</td>
<td>475</td>
<td>558</td>
<td>1131.64</td>
<td>42879</td>
<td>0.5529</td>
<td>5.301</td>
<td>1.01593</td>
<td>1.0527</td>
</tr>
<tr>
<td>3</td>
<td>Philadelphia</td>
<td>201</td>
<td>600</td>
<td>3468</td>
<td>43576</td>
<td>1.135</td>
<td>18.2</td>
<td>infeasible</td>
<td>infeasible</td>
</tr>
<tr>
<td>4</td>
<td>Minneapolis</td>
<td>299</td>
<td>609</td>
<td>1340.35</td>
<td>45673</td>
<td>0.729</td>
<td>7.209</td>
<td>1.22752</td>
<td>1.086</td>
</tr>
<tr>
<td>5</td>
<td>Raleigh</td>
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<tr>
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<td>41984</td>
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<td>0.92652</td>
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<tr>
<td>11</td>
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<td>43249</td>
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<td>0.77243</td>
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<td>775</td>
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<td>0.80117</td>
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</table>

The bold numerals enable easy identification of items discussed in the paper.

### Table 3  Results for Japanese companies

<table>
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<tr>
<th>DMU Company</th>
<th>Input-oriented</th>
<th>Output-oriented</th>
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</thead>
<tbody>
<tr>
<td>Super-efficiency</td>
<td>τ⁺</td>
<td>β⁺</td>
</tr>
<tr>
<td>1 MITSUI &amp; CO.</td>
<td>2.0161⁺</td>
<td>0.0104</td>
</tr>
<tr>
<td>2 ITOCHU CORP.</td>
<td>6.693</td>
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<tr>
<td>3 MITSUBISHI CORP.</td>
<td>1.0209</td>
<td>0</td>
</tr>
<tr>
<td>4 TOYOTA MOTOR CORP.</td>
<td>1.1458</td>
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</tr>
<tr>
<td>5 MARUBENI CORP.</td>
<td>2.8999</td>
<td>0</td>
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</tbody>
</table>

*This indicates that the original super-efficiency model is infeasible.*
Table 4  Results for US cities

<table>
<thead>
<tr>
<th>DMU</th>
<th>City</th>
<th>Input-oriented</th>
<th>Output-oriented</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Super-efficiency</td>
<td>γ*</td>
</tr>
<tr>
<td>3</td>
<td>Philadelphia</td>
<td>2.893∗</td>
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</tr>
<tr>
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<td>Raleigh</td>
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<td>0.1677</td>
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<tr>
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<td>StLouis</td>
<td>1.5163</td>
<td>0.5163</td>
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<tr>
<td>9</td>
<td>Pittsburgh</td>
<td>1.0453</td>
<td>0.0453</td>
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<tr>
<td>13</td>
<td>Boston</td>
<td>2.5908∗</td>
<td>0.5653</td>
</tr>
</tbody>
</table>

This indicates that the original super-efficiency model is infeasible.

The current study, therefore, offers the possibility to extend the Malmquist productivity index and DEA benchmarking models into situations where the non-CRS assumption of the DEA frontier is required.

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References

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