DEA Models for Two-Stage Processes: Game Approach and Efficiency Decomposition

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Abstract: Data envelopment analysis (DEA) is a method for measuring the efficiency of peer decision making units (DMUs). This tool has been utilized by a number of authors to examine two-stage processes, where all the outputs from the first stage are the only inputs to the second stage. The current article examines and extends these models using game theory concepts. The resulting models are linear, and imply an efficiency decomposition where the overall efficiency of the two-stage process is a product of the efficiencies of the two individual stages. When there is only one intermediate measure connecting the two stages, both the noncooperative and centralized models yield the same results as applying the standard DEA model to the two stages separately. As a result, the efficiency decomposition is unique. While the noncooperative approach yields a unique efficiency decomposition under multiple intermediate measures, the centralized approach is likely to yield multiple decompositions. Models are developed to test whether the efficiency decomposition arising from the centralized approach is unique. The relations among the noncooperative, centralized, and standard DEA approaches are investigated. Two real world data sets and a randomly generated data set are used to demonstrate the models and verify our findings. Naval Research Logistics 55: 643–653, 2008

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1. INTRODUCTION

Data envelopment analysis (DEA), introduced by Charnes et al [2], is an approach for identifying best practices of peer decision making units (DMUs), in the presence of multiple inputs and outputs. In many cases, DMUs may also have intermediate measures. For example, Seiford and Zhu [8] use a two-stage process to measure the profitability and marketability of US commercial banks. In their study, profitability is measured using labor and assets as inputs, and the outputs are profits and revenue. In the second stage for marketability, the profits and revenue are then used as inputs, while market value, returns, and earnings per share are used as outputs. Chilingerian and Sherman [5] describe another two-stage process in measuring physician care. Their first stage is a manager-controlled process with inputs including registered nurses, medical supplies, and capital and fixed costs. These inputs generate the outputs or intermediate measures (inputs to the second stage), including patient days, quality of treatment, drug dispensed, among others. The outputs of the second (physician controlled) stage include research grants, quality of patients, and quantity of individuals trained, by specialty.

Seiford and Zhu [8] use the standard DEA approach, which does not address potential conflicts between the two stages arising from the intermediate measures. For example, the second stage may have to reduce its inputs (intermediate measures) to achieve an efficient status. Such an action would, however, imply a reduction in the first stage outputs, thereby reducing the efficiency of that stage. To address that conflict issue, Chen and Zhu [3] and Chen et al. [4] present a linear DEA type model where the intermediate measures are set as decision variables. However, their individual stage efficiency scores do not provide information on the overall performance and best-practice of the two-stage process.

The current study seeks alternative ways to (i) address the conflict between the two stages caused by the intermediate measures, and (ii) provide efficiency scores for both

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individual stages and the overall process. We model the two-stage processes via concepts adopted from noncooperative and cooperative games. For example, suppose a DMU consists of a manufacturer and a retailer. In such a setting, traditionally the manufacturer holds manipulative power and acts as a leader, and the retailer is treated as a follower in modeling noncooperative supply chains [6]. In a similar manner, our noncooperative approach assumes that one of the stages is the leader that seeks to maximize its DEA efficiency. Then the efficiency of the other stage (the follower) is calculated subject to the leader-stage maintaining its DEA efficiency. The leader stage can be viewed as being more important than the other stage(s) in improving its efficiency.

In a more cooperative environment, the manufacturer and retailer may wish to work together in determining price, order quantity, and other factors to achieve maximum savings and/or profit for the manufacturer-retailer chain. Our specific approach herein assumes that initially both stages' efficiency scores are maximized simultaneously, while determining a set of optimal (common) weights assigned to the intermediate measures. It is pointed out that this approach is not specifically in line with conventional cooperative game theory logic, where players would jointly decide upon a multiplier space that is acceptable. We refer to our approach as "centralized", in that it is the combined stages that are of interest (see, e.g., Cachon [3]). We then apply a second order model (see Appendix) to arrive at a "cooperative" efficiency decomposition that is fair to both players. In this latter sense our combined centralized/cooperative approach is in the spirit of cooperative games.

It is shown that both the non-cooperative and centralized approaches yield an efficiency decomposition, where the overall efficiency of the two-stage process is a product of those of the two individual stages. Note that such an efficiency decomposition is not available in the standard DEA approach of Seiford and Zhu [8], and the multi-stage approaches of Chen and Zhu [3].

The current study further shows that when there is only one intermediate measure, both the noncooperative and centralized approaches yield the same results, and unique efficiency decomposition occurs, as is the case in applying the standard DEA model to each stage separately. Although the noncooperative approach yields a unique efficiency decomposition under multiple intermediate measures, the centralized approach may yield multiple efficiency decompositions. Models are developed to test whether the centralized approach to efficiency decomposition is unique.

The rest of the article is organized as follows. Section 2 presents the generic two-stage process. We then present in Sections 3 and 4 our noncooperative (or leader-follower) model, and the centralized model. It is shown how to test for uniqueness of efficiency decomposition. Section 5 discusses the relations among the standard DEA model and the noncooperative and centralized approaches. The issue of unique efficiency decomposition is also studied. In Section 6 our models are then applied to three data sets to verify our findings. One data set is from Wang et al. [9], and has only one intermediate measure. The models are then applied to the data set of Seiford and Zhu [8] with two intermediate measures. It is shown that the efficiency decomposition is unique. Finally, to further examine differences that can occur between outcomes from the noncooperative and centralized approaches, a randomly generated data set is examined. Conclusions follow in Section 7.

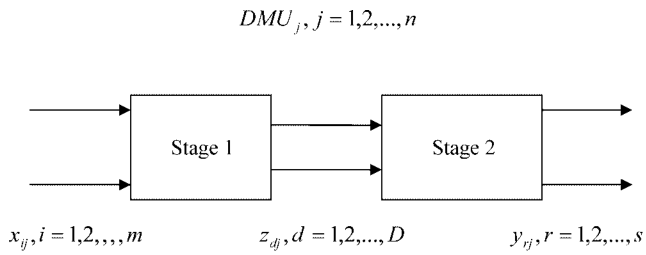

Figure 1. Two-stage process.

2. TWO-STAGE PROCESSES

Consider a generic two-stage process as shown in Fig. 1, for each of a set of n DMUs. Using the notions in Chen and Zhu [3], we assume each DMU_j $(j = 1, 2, \ldots, n)$ has m inputs x_{ij}, $(i = 1, 2, \ldots, m)$ to the first stage, and D outputs z_{dj}, $(d = 1, 2, \ldots, D)$ from that stage. These D outputs then become the inputs to the second stage and will be referred to as intermediate measures. The outputs from the second stage are y_{rj}, $(r = 1, 2, \ldots, s)$.

For DMU_j we denote the efficiency for the first stage as e_j^1 and the second as e_j^2. On the basis of the radial (CRS) DEA model of Charnes et al. [2], we define

$$e_j^1 = \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \quad \text{and} \quad e_j^2 = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \tilde{w}_d z_{dj}} \tag{1}$$

where v_i, w_d, $\tilde{w}_d$, and u_r are unknown non-negative weights. It is noted that w_d can be set equal to $\tilde{w}_d$, and in many if not most situations this would be an appropriate course of action. In the case examined herein we make the assumption that the "worth" or value accorded to the intermediate variables is the same regardless of whether they are being viewed as inputs or outputs.

Clearly, one can apply two separate DEA analyses to the two stages as in Seiford and Zhu [8]. One criticism of such an approach is the inherent conflict that arises between these two analyses. For example, suppose the first stage is DEA efficient and the second stage is not. When the second stage improves its performance (by reducing the inputs z_{dj} via an input-oriented DEA model), the reduced z_{dj} may render the first stage inefficient. This indicates a need for a DEA approach that provides for coordination between the two stages.

Before presenting our models, it is useful to point out that given the individual efficiency measures e_j^1 and e_j^2, it is reasonable to define the efficiency of the overall two-stage process either as $\frac{1}{2}(e_j^1 + e_j^2)$ or $e_j^1 \bullet e_j^2$. If the input-oriented DEA model is used, then we should have $e_j^1 \le 1$ and $e_j^2 \le 1$. The above definition ensures that the two-stage process is efficient if and only if $e_j^1 = e_j^2 = 1$.

Finally, if we define $e_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$ as the two-stage overall efficiency, our models imply $e_j = e_j^1 \bullet e_j^2$ at optimality.

3. NONCOOPERATIVE MODEL

One form of a noncooperative game is characterized by the leader-follower assumption. (The term noncooperative game is used to characterize either leader-follower situations, or normal form/ simultaneous game situations. In the game theory literature, the leader-follower paradigm is also referred to as the Stackelberg model, borrowed from the notion of Stackelberg games). For example, consider a case of noncooperative advertising between a manufacturer (leader) and a retailer (follower). The manufacturer, if assumed to be the leader, determines its optimal brand name investment and local advertising allowance, based on an estimation of the local advertising that will be undertaken by the retailer to maximize its profit. The retailer, as a follower on the other hand, based on the information from the manufacturer, determines the optimal local advertising cost, to maximize its profit [7].

In a similar manner, if we assume that the first stage is the leader, then the first stage performance is more important, and the efficiency of the second stage (follower) is computed, subject to the requirement that the leader's efficiency stays fixed.

Adopting the convention that the first stage is the leader, and the second stage, the follower, we calculate the efficiency for the first stage, using the CCR model [2]. That is, we solve for a specific DMU$_o$ the liner programming model

$$\begin{aligned} e_o^{1*} = \text{Max} & \sum_{d=1}^{D} w_d z_{do} \\ \text{s.t.} & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & w_d \ge 0, d = 1,2,\ldots,D; v_i \ge 0, i = 1,2,\ldots,m. \end{aligned} \tag{2}$$

Note that since model (2) is the standard (CCR) DEA model, then e_o^{1*} is the regular DEA efficiency score.

Once we obtain the efficiency for the first stage, the second stage will only consider those variables w_d that maintain $e_o^1 = e_o^{1*}$. Or, in other words, the second stage now treats $\sum_{d=1}^{D} w_d z_{dj}$ as the "single" input subject to the restriction that the efficiency score of the first stage remains at e_o^{1*}. The model for computing e_o^2, the second stage's efficiency, can be expressed as

$$\begin{aligned} e_o^{2*} = \text{Max} & \frac{\sum_{r=1}^{s} U_r y_{ro}}{Q \sum_{d=1}^{D} w_d z_{do}} \\ \text{s.t.} & \frac{\sum_{r=1}^{s} U_r y_{rj}}{Q \sum_{d=1}^{D} w_d z_{dj}} \le 1 \quad j = 1,2,\ldots,n \\ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{d=1}^{D} w_d z_{do} = e_o^{1*} \\ & U_r, Q, w_d, v_i \ge 0, \\ & r = 1,2,\ldots,s; d = 1,2,\ldots,D; i = 1,2,\ldots,m \end{aligned} \tag{3}$$

Note that in model (3), the efficiency of the first stage is set equal to e_o^{1*}. Let $u_r = \frac{U_r}{Q}, r = 1,2,\ldots,s$. Model (3) is then equivalent to the following linear model

$$\begin{aligned} e_o^{2*} = \text{Max} & \left(\sum_{r=1}^{s} u_r y_{ro}\right) / e_o^{1*} \\ \text{s.t.} & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{d=1}^{D} w_d z_{do} = e_o^{1*} \\ & w_d \ge 0, d = 1,2,\ldots,D; v_i \ge 0, \\ & i = 1,2,\ldots,m; u_r \ge 0, r = 1,2,\ldots,s \end{aligned} \tag{4}$$

In a similar manner, if we assume the second stage to be the leader, we first calculate the regular DEA efficiency ($e_o^{2^o}$) for that stage, using the appropriate CCR model. Then, one solves the first stage (follower) model, with the restriction that the second stage score, having already been determined, cannot be decreased from that value.

We finally note that in (4), $e_o^{1*} \bullet e_o^{2*} = \sum_{r=1}^{s} u_r^* y_{ro}$ at optimality, with $\sum_{i=1}^{m} v_i^* x_{io} = 1$. That is, $e_o^{1*} \bullet e_o^{2*} = \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{i=1}^{m} v_i^* x_{io}}$. (A similar result is true, if the leader/follower definition is reversed). This indicates that our non-cooperative approach implies an efficiency decomposition for the two-stage DEA analysis. That is, the overall efficiency is equal to the product of the efficiencies of individual stages. Further, note that in the first-stage leader case, e_o^{1*} and e_o^{2*} are optimal values to linear programs. Therefore, such an efficiency decomposition is unique. The same is true of the decomposition following from the second-stage leader case. It is pointed out, however, that these two decompositions may not be the same.

4. CENTRALIZED MODEL

An alternative approach to measuring the efficiency of the two stage process is to view them from a centralized perspective, and determine a set of optimal weights on the intermediate factors that maximizes the aggregate or global efficiency score (as would be true where the manufacturer and retailer jointly determine the price, order quantity, etc. to achieve maximum profit [7]). In other words, the centralized approach is characterized by letting $w_d = \tilde{w}_d$ in (1), and the efficiencies of both stages are evaluated simultaneously.[1] Generally, the model for maximizing the average of e_o^1 and e_o^2 is a non-linear program. We note, however, that because of the assumption of $w_d = \tilde{w}_d$ in (1), $e_o^1 \bullet e_o^2$ becomes $\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$. Therefore, instead of maximizing the average of e_o^1 and e_o^2, we have

$$e_o^{\text{centralized}} = \text{Max } e_o^1 \bullet e_o^2 = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$
$$\text{s.t. } e_j^1 \le 1 \text{ and } e_j^2 \le 1 \text{ and } w_d = \tilde{w}_d. \quad (5)$$

Model (5) can be converted into the following linear program

$$\begin{aligned} e_o^{\text{centralized}} &= \text{Max } \sum_{r=1}^{s} u_r y_{ro} \\ \text{s.t. } & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & w_d \ge 0, d = 1,2,\ldots,D; v_i \ge, \\ & i = 1,2,\ldots,m; u_r \ge, r = 1,2,\ldots,s \end{aligned} \quad (6)$$

Model (6) gives the overall efficiency of the two-stage process. Assume the above model (6) yields a unique solution. We then obtain the efficiencies for the first and second stages, namely

$$e_o^{1,\text{Centralized}} = \frac{\sum_{d=1}^{D} w_d^* z_{do}}{\sum_{i=1}^{m} v_i^* x_{io}} = \sum_{d=1}^{D} w_d^* z_{do} \text{ and}$$
$$e_o^{2,\text{Centralized}} = \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{d=1}^{D} w_d^* z_{do}}. \quad (7)$$

If we denote the optimal value to model (6) as $e_o^{\text{centralized}}$, then we have $e_o^{\text{centralized}} = e_o^{1,\text{Centralized}} \bullet e_o^{2,\text{Centralized}}$. Note that optimal multipliers from model (6) may not be unique, meaning that $e_o^{1,\text{Centralized}}$ and $e_o^{2,\text{Centralized}}$ may not be unique. To test for uniqueness, we can first determine the maximum achievable value of $e_o^{1,\text{Centralized}}$ via

$$\begin{aligned} e_o^{1+} &= \text{Max } \sum_{d=1}^{D} w_d z_{do} \\ \text{s.t. } & \sum_{r=1}^{s} u_r y_{ro} = e_o^{\text{centralized}} \\ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1,2,\ldots,n \\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & w_d \ge 0, d = 1,2,\ldots,D; v_i \ge 0, \\ & i = 1,2,\ldots,m; u_r \ge 0, r = 1,2,\ldots,s \end{aligned} \quad (8)$$

It then follows that the minimum of $e_o^{2,\text{Centralized}}$ is given by $e_o^{2-} = \frac{e_o^{\text{centralized}}}{e_o^{1+}}$.

The maximum of $e_o^{2,\text{Centralized}}$, which we denote by e_o^{2+}, can be calculated in a manner similar to the above, and the minimum of $e_o^{1,\text{Centralized}}$ is then calculated as $e_k^{1-} = e_o^{\text{centralized}}/e_o^{2+}$. Note that $e_o^{1-} = e_o^{1+}$ if and only of $e_o^{2-} = e_o^{2+}$. Note also that if $e_o^{1-} = e_o^{1+}$ or $e_o^{2-} = e_o^{2+}$, then $e_o^{1,\text{Centralized}}$ and $e_o^{2,\text{Centralized}}$ are uniquely determined via model (6). If $e_o^{1-} \ne e_o^{1+}$ or $e_o^{2-} \ne e_o^{2+}$, then presumably some flexibility exists in setting values for $e_o^{1,\text{Centralized}}$ and $e_o^{2,\text{Centralized}}$. A legitimate reason for taking advantage of such flexibility is one of cooperation and

[1] Note that in the end, a common set of weights is assigned to both stages in our non-cooperative game approach. However, in that approach, e_o^1 and e_o^2 are not optimized simultaneously.

fairness. That is, in the spirit of cooperative games, once the optimal value for the centralized score is determined, it is reasonable to search for a decomposition that is as fair as possible to both parties. The Appendix provides a procedure to obtain an alternative decomposition of $e_o^{1,\text{Centralized}}$ and $e_o^{2,\text{Centralized}}$.

5. RELATIONS AND UNIQUE EFFICIENCY DECOMPOSITION

In this section, we discuss the relationships among the above developed noncooperative and centralized models, and the standard DEA approach. We show that under the condition of one intermediate measure, the noncooperative, centralized and regular DEA approaches yield the same results.

Let θ_o^1 and θ_o^2 be the (CCR) efficiency scores for the two stages. That is, for a specific DMU$_o$, (i) θ_o^1 is the DEA efficiency based upon inputs of x_{io} and outputs of z_{do} and (ii) θ_o^2 is the DEA efficiency based upon inputs of z_{do} and outputs of y_{ro}.

We first consider a special case of one intermediate measure. We have

THEOREM 1: If there is only one intermediate measure, then $e_o^{1*} = \theta_o^1$ and $e_o^{2*} = \theta_o^2$ regardless of the assumption of whether the first stage is a leader or follower, where e_o^{1*} and e_o^{2*} are obtained via our noncooperative approach.

PROOF: Suppose the first stage is a leader. Recall that model (1) is the standard DEA model for the first stage. Therefore, $e_o^{1*} = \theta_o^1$. We next prove $e_o^{2*} = \theta_o^2$.

Consider model (3), which now becomes

$$
\begin{aligned}
e_o^{2*} = \max\ & \frac{\sum_{r=1}^{s} U_r y_{ro}}{Q w z_o} \\
\text{s.t. } & \frac{\sum_{r=1}^{s} U_r y_{rj}}{Q w z_j} \le 1 \quad j = 1,2,\ldots,n \\
& w z_j - \sum_{i=1}^{m} v_i x_{ij} \le 0 \, j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& w z_o = e_o^1 \\
& U_r, Q, w, v_i \ge 0, r = 1,2,\ldots,s; i = 1,2,\ldots,m \quad (9)
\end{aligned}
$$

Note that $w = \frac{e_o^{1*}}{z_o}$. Letting $Q' = Q(e_o^{1*}/z_o)$, model (9) is equivalent to

$$
\begin{aligned}
e_o^{2*} = \max\ & \frac{\sum_{r=1}^{s} U_r y_{ro}}{Q' z_o} \\
\text{s.t. } & \frac{\sum_{r=1}^{s} U_r y_{rj}}{Q' z_j} \le 1 \quad j = 1,2,\ldots,n \\
& (e_o^{1*}/z_o) z_j - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& U_r, Q', v_i \ge 0, r = 1,2,\ldots,s; i = 1,2,\ldots,m \quad (10)
\end{aligned}
$$

Note that values of v_i do not affect the optimal value to model (10), indicating that $(e_o^{1*}/z_o)z_j - \sum_{i=1}^{m} v_i x_{ij} \le 0$ and $\sum_{i=1}^{m} v_i x_{io} = 1$ are redundant. As a result, model (10) becomes the standard DEA model for the second stage, hence $e_o^{2*} = \theta_o^2$.

Similarly, it can be shown that the theorem is true when the first stage is a follower. □

Thus, Theorem 1 indicates that when there is only one intermediate measure, the non-cooperative approach yields the same result as applying the standard DEA model to each stage.

Under the condition of multiple intermediate measures, we note that the feasible region of model (6) contains the feasible region of model (4). Thus, the optimal value to model (6) must be greater than or equal to $e_o^{1*} \bullet e_o^{2*}$ arising from model (4). This can be summarized as

THEOREM 2: For a specific DMU$_o$, $e_o^{\text{centralized}} \ge e_o^{1*} \bullet e_o^{2*}$, where $e_o^{\text{centralized}}$ is the optimal value to model (6), and e_o^{1*} and e_o^{2*} are obtained via the noncooperative (leader-follower) approach.

In the presence of a single intermediate measure, Theorem 1 shows that e_o^{1*} and e_o^{2*} are respectively their DEA efficiency scores, hence are the maximum achievable efficiencies. Therefore, based upon Theorems 1 and 2, we must have

THEOREM 3: In the presence of a single intermediate measure, $e_o^{\text{centralized}} = \theta_o^1 \bullet \theta_o^2$, with $\theta_o^1 = e_o^{1,\text{Centralized}}$ and $\theta_o^2 = e_o^{2,\text{Centralized}}$, where θ_o^1 and θ_o^2 are the (CCR) efficiency scores for the two stages, respectively, and $e_o^{1,\text{Centralized}}$ and $e_o^{2,\text{Centralized}}$ are defined in (7).

When there is a single intermediate measure, Theorem 3 indicates that

1. non-cooperative and centralized approaches yield the same result as applying the standard DEA model to each stage, and
2. the efficiency decomposition under the model (6) is unique.

We finally note the following is true with respect to the relations between the non-cooperative and centralized approaches.

Table 1. IT data set.

DMU	Fixed assets ($ billion)	IT budget ($ billion)	No. of employees (thousand)	Deposits ($ billion)	Profit ($ billion)	Fraction of loans recovered
1	0.713	0.15	13.3	14.478	0.232	0.986
2	1.071	0.17	16.9	19.502	0.34	0.986
3	1.224	0.235	24	20.952	0.363	0.986
4	0.363	0.211	15.6	13.902	0.211	0.982
5	0.409	0.133	18.485	15.206	0.237	0.984
6	5.846	0.497	56.42	81.186	1.103	0.955
7	0.918	0.06	56.42	81.186	1.103	0.986
8	1.235	0.071	12	11.441	0.199	0.985
9	18.12	1.5	89.51	124.072	1.858	0.972
10	1.821	0.12	19.8	17.425	0.274	0.983
11	1.915	0.12	19.8	17.425	0.274	0.983
12	0.874	0.05	13.1	14.342	0.177	0.985
13	6.918	0.37	12.5	32.491	0.648	0.945
14	4.432	0.44	41.9	47.653	0.639	0.979
15	4.504	0.431	41.1	52.63	0.741	0.981
16	1.241	0.11	14.4	17.493	0.243	0.988
17	0.45	0.053	7.6	9.512	0.067	0.98
18	5.892	0.345	15.5	42.469	1.002	0.948
19	0.973	0.128	12.6	18.987	0.243	0.985
20	0.444	0.055	5.9	7.546	0.153	0.987
21	0.508	0.057	5.7	7.595	0.123	0.987
22	0.37	0.098	14.1	16.906	0.233	0.981
23	0.395	0.104	14.6	17.264	0.263	0.983
24	2.68	0.206	19.6	36.43	0.601	0.982
25	0.781	0.067	10.5	11.581	0.12	0.987
26	0.872	0.1	12.1	22.207	0.248	0.972
27	1.757	0.0106	12.7	20.67	0.253	0.988

THEOREM 4:

1. $e_o^{1,\text{Centralized}} \geq e_0^{1*}$ when the second stage is the leader,
2. $e_o^{2,\text{Centralized}} \geq e_o^{2*}$ when the first stage is the leader.
3. θ_o^2 $(= e_o^{2*}) \geq e_o^{2,\text{Centralized}}$, and θ_o^1 $(= e_o^{1*}) \geq e_o^{1,\text{Centralized}}$ always hold, regardless of which stage is the leader.

6. APPLICATION

In this section, we consider three data sets. The first data set has a single intermediate measure that was first used in Wang et al. [9], and then in Chen and Zhu [3] in examining the IT impact on productivity. The second data set consists of 30 top US commercial banks and has two intermediate measures [8]. The final data set is randomly generated using the RAND() function in Excel.

6.1. Information Technology

Table 1 presents the data set, which consists of 27 observations on firms in the banking industry. The inputs for the first stage are fixed assets, numbers of employees, and IT investment. The intermediate measure is the deposits generated. The second stage outputs are profits and the fraction of loans recovered.[2]

Since there is only one intermediate measure, both the noncooperative (whichever is the leader), and centralized results are identical, with a unique efficiency decomposition. Table 2 reports the results. In this case, the scores in columns 2 and 3 are also the DEA efficiencies for stage 1 and stage 2, respectively. Column 4 displays the centralized score obtained in (6), which is equal to the product of the related scores in columns 2 and 3. These results verify Theorems 1 and 3.

The last column under the heading θ_o reports the DEA efficiency when the intermediate measures are ignored, i.e., θ_o is a DEA score with inputs being fixed assets, IT budget and employees, and outputs being profit and fraction of loans recovered. It can be seen that $\theta_o = 1$ for DMU4, while both stages are (DEA) inefficient. This points to the fallacy of applying the DEA model directly, and ignoring the intermediate measures.

[2] For detailed discussion on the data, the reader is referred to Wang et al. [9].

Table 2. Results for IT data.

DMU	e_o^1	e_o^2	$e_o^1 \bullet e_o^2$	θ_o
1	0.6388	0.7459	0.4764	0.7371
2	0.6507	0.7819	0.5087	0.8026
3	0.5179	0.7730	0.4003	0.6415
4	0.5986	0.7142	0.4275	1
5	0.5556	0.7236	0.4020	0.9125
6	0.7599	0.5758	0.4376	0.6436
7	1	0.5758	0.5758	1.0000
8	0.5352	0.8250	0.4415	0.6827
9	0.6249	0.6347	0.3966	0.4870
10	0.4963	0.7188	0.3567	0.5359
11	0.4945	0.7188	0.3555	0.5329
12	0.6685	0.5949	0.3977	0.8780
13	0.9487	0.8582	0.8141	0.9133
14	0.5880	0.5783	0.3400	0.4997
15	0.6582	0.6035	0.3972	0.5809
16	0.6646	0.6434	0.4276	0.6226
17	0.7177	0.7877	0.5653	1
18	1	1.0000	1.0000	1
19	0.8144	0.5926	0.4826	0.7260
20	0.6934	1.0000	0.6934	1
21	0.7067	0.9936	0.7022	1
22	0.7942	0.6408	0.5089	1
23	0.7802	0.6993	0.5456	1
24	0.9300	0.7135	0.6636	0.8934
25	0.6270	0.6516	0.4085	0.7424
26	1	0.5152	0.5152	0.7895
27	1	0.5644	0.5644	1

6.2. Top US Commercial Banks

Seiford and Zhu [8] examine the performance of the US commercial banks in 1995 via a two-stage production process defined in terms of profitability and marketability. The inputs to the first stage are numbers of employees, assets ($millions) and equity ($million). The intermediate measures are profit ($millions) and revenue ($millions). Outputs from the second stage are market value ($millions), earnings per share ($) and returns to the investors (%). Table 3 displays the data for the top 30 banks, and Table 4 reports the results of the application of model (6). The last column shows, for each DMU, the DEA score for the overall process when employees, assets and equity are used as the inputs and profit and revenue are used as the outputs. Conceptually, such a DEA score is similar to the $e_o^{1*} \bullet e_o^{2*}$ in the non-cooperative approach or $e_o^{\text{centralized}}$ in the centralized approach of model (6). However, the relation $\theta_o = \theta_o^1 \bullet \theta_o^2$ does not always hold. This can be seen by using the scores reported in columns 2 and column 6. Column 2 represents the DEA scores for the first stage and column 6 represents the DEA scores for the second stage.

To test the uniqueness of our efficiency decomposition under the centralized approach, we also calculate e_o^{1+} (model (8)) and e_o^{2+}. Our results indicate that $e_o^{1-} = e_o^{1+}$ and $e_o^{2-} = e_o^{2+}$ for all the DMUs. Therefore, $e_o^{1,\text{Centralized}}$ and $e_o^{2,\text{Centralized}}$ defined in (7), are uniquely determined via model (6) in our case.

Finally note that the results in Table 4 also verify our Theorems 2 and 4. Note also that $e_o^{\text{centralized}} = e_o^{1*} \bullet e_o^{2*}$ holds for all the banks, where e_o^{1*} and e_o^{2*} represent the efficiency scores for the two stages when the first stage is treated as the leader. This may indicate that the first stage or the profitability stage is more important.

6.3. Randomly Generated Data Set

One is tempted to conclude from the previous example that there might be a direct connection between the centralized optimal score and that arising from the results of treating stage 1 as the leader. To gain further insights, a randomly generated set of data was created as displayed in Table 5. The outcomes from the various analyses appear in Table 6. Among the 27 DMUs, 15 show centralized scores that exceed the corresponding aggregate scores in both the stage 1 leader and stage 2 leader cases. For the remaining 12 DMUs, the following are the outcomes:

1. For three of the DMUs (8, 10, and 12), the stage 1 leader scores and decomposition match those of the centralized analysis, but differ from the stage 2 leader results;
2. For six of the DMUs (1, 3, 7, 19, 20, and 27), the stage 2 leader results match those of the centralized analysis, but differ from those of the stage 1 leader;
3. For the three DMUs 14, 16, and 23, all three analyses produce the same results.

This latter set of outcomes appears to point to the general unpredictability of any connection between the results from the centralized approach and those from the two approaches involving a leader and a follower.

7. CONCLUSIONS

In many DEA situations, DMUs may take the form of multiple stages with intermediate measures. It has been recognized that the existing DEA approaches, including the standard DEA models, do not appropriately address such multi-stage structures. This paper presents alternative ways to address the conflict between stages caused by the intermediate measures, and at the same time provide efficiency scores for both individual stages and the overall process. Our noncooperative and centralized approaches show that the overall efficiency of the two-stage process is the product of efficiencies of the two stages.

Table 3. US commercial bank.

	Bank	Employees	Assets	Equity	Revenue	Profits	Market value	Earnings	Returns
1	Citicorp	85, 300	256, 853	19, 581	31, 690	3, 464	33221.7	7.21	66.1
2	BankAmerica Corp.	95, 288	232, 446	20, 222	20, 386	2, 664	27148.6	6.49	69.4
3	NationsBank Corp.	58, 322	187, 298	12, 801	16, 298	1, 950	20295.9	7.13	59.7
4	Chemical Banking Corp.	39, 078	182, 926	11, 912	14, 884	1, 805	16971.3	6.73	70.5
5	J.P. Morgan & Co.	15, 600	184, 879	10, 451	13, 838	1, 296	15003.5	6.42	49.4
6	Chase Manhattan Corp.	33, 365	121, 173	9, 134	11, 336	1, 165	12616.4	5.76	82.4
7	First Chicago NBD Corp.	35, 328	122, 002	8, 450	10, 681	1, 150	12351.1	3.45	50
8	First Union Corp.	44, 536	131879.9	9043.1	10582.9	1430.2	16, 815	5.04	39.9
9	Banc One Corp.	46, 900	90, 454	8197.5	8970.9	1277.9	14807.4	2.91	54.9
10	Bankers Trust New York Corp.	14, 000	104, 000	5, 000	8, 600	215	5252.4	2.03	28.3
11	Fleet Financial Group	30, 800	84432.2	6364.8	7919.4	610	10428.7	1.57	31.8
12	Norwest Corp.	45, 404	72134.4	5312.1	7582.3	956	12268.6	2.76	45.5
13	PNC Bank Corp.	26, 757	73, 404	5, 768	6389.5	408.1	9938.2	1.19	61.4
14	KeyCorp	28, 905	66339.1	5152.5	6, 054	825	8671.2	3.45	51.6
15	Bank of Boston Corp.	17, 881	47, 397	3, 751	5410.6	541	5310.1	4.55	84.7
16	Wells Fargo & Co.	19, 700	50, 316	4, 055	5, 409	1, 032	11342.5	20.37	52.8
17	Bank of New York Co.	15, 850	53, 685	5, 223	5, 327	914	10101.5	4.57	69.9
18	First Interstate Bancorp	27, 200	58, 071	4, 154	4827.5	885.1	12, 138	11.02	108.5
19	Mellon Bank Corp.	24, 300	40, 129	4, 106	4, 514	691	7476.7	4.5	83.8
20	Wachovia Corp.	15, 996	44981.3	3773.8	3755.4	602.5	7623.6	3.5	46.9
21	SunTrust Banks	19, 415	46471.5	4269.6	3740.3	565.5	7922.5	4.94	46.9
22	Barnett Banks	20, 175	41553.5	3272.2	3, 680	533.3	5774.9	5.3	59
23	National City Corp.	20, 767	36, 199	2, 921	3449.9	465.1	4912.2	3.03	33.9
24	First Bank System	13, 231	33, 874	2, 725	3328.3	568.1	8, 304	4.19	54.3
25	Comerica	13, 500	35469.9	2607.7	3112.6	413.4	4, 537	3.54	71.7
26	Boatmen's Bancshares	17, 023	33703.8	2928.1	2996.1	418.8	4, 997	3.25	57.3
27	U.S. Bancorp	14, 081	31794.3	2, 617	2897.3	329	4865.1	2.09	66.8
28	CoreStates Financial Corp.	13, 598	29620.6	2379.4	2868	452.2	5, 788	3.22	52
29	Republic New York Corp.	4, 900	43881.6	3007.8	2859.6	288.6	3, 218	4.66	41.1
30	MBNA	11, 171	13228.9	1265.1	2565.4	353.1	6543.3	1.54	60.7

Table 4. US commercial bank results.

	Stage 1 as the Leader			Stage 2 as the Leader			Centralized			DEA
Bank	$\theta_o^1\ (= e_o^{1*})$	e_o^{2*}	$e_o^{1*} \bullet e_o^{2*}$	$e_o^{1^O}$	$\theta_o^2\ (= e_o^{2^O})$	$e_o^{1^O} \bullet e_o^{2^O}$	$e_o^{1,\text{Centralized}}$	$e_o^{2,\text{Centralized}}$	$e_o^{\text{Centralized}}$	θ_o
1	1	0.4487	0.4487	0.8381	0.4859	0.4073	1	0.4487	0.4487	0.5991
2	0.6823	0.5326	0.3634	0.5935	0.5442	0.3230	0.6821	0.5327	0.3634	0.4522
3	0.7946	0.5305	0.4216	0.6858	0.5669	0.3888	0.7946	0.5305	0.4216	0.5320
4	0.8729	0.4896	0.4274	0.8171	0.5221	0.4267	0.8463	0.5050	0.4274	0.6146
5	1	0.6061	0.6061	1	0.6061	0.6061	1	0.6061	0.6061	1
6	0.8333	0.5016	0.4180	0.6898	0.5881	0.4056	0.8180	0.5110	0.4180	0.5675
7	0.7885	0.4997	0.3940	0.6624	0.5546	0.3674	0.7816	0.5042	0.3940	0.5252
8	0.7451	0.6371	0.4747	0.6634	0.6470	0.4292	0.7451	0.6371	0.4747	0.5850
9	0.7022	0.6388	0.4486	0.6654	0.6471	0.4306	0.7021	0.6389	0.4486	0.5159
10	1	0.3393	0.3393	0.3056	1	0.3056	0.4884	0.6946	0.3393	0.5017
11	0.7414	0.5608	0.4158	0.4731	0.7601	0.3596	0.6619	0.6282	0.4158	0.5285
12	0.7089	0.6406	0.4541	0.6648	0.6760	0.4494	0.6906	0.6576	0.4541	0.4593
13	0.6809	0.6557	0.4464	0.4062	1	0.4062	0.5843	0.7641	0.4464	0.5797
14	0.7139	0.5845	0.4173	0.6514	0.6002	0.3910	0.7131	0.5852	0.4173	0.4836
15	0.8808	0.7290	0.6421	0.5747	0.8608	0.4947	0.8469	0.7582	0.6421	0.8051
16	1	1	1	1	1	1	1	1	1	1
17	1	0.7144	0.7144	0.8542	0.7469	0.6381	1.0000	0.7144	0.7144	0.9700
18	0.8041	0.9917	0.7974	0.7974	1.0000	0.7974	0.7974	1.0000	0.7974	0.8813
19	0.7484	0.7804	0.5841	0.7080	0.8009	0.5670	0.7478	0.7811	0.5841	0.6623
20	0.7542	0.7844	0.5916	0.6594	0.7997	0.5273	0.7542	0.7844	0.5916	0.7471
21	0.6550	0.8661	0.5673	0.6063	0.8830	0.5354	0.6550	0.8661	0.5673	0.6623

Table 4. (continued)

	Stage 1 as the Leader			Stage 2 as the Leader			Centralized			DEA
Bank	$\theta_o^1\ (=e_o^{1*})$	e_o^{2*}	$e_o^{1*}\bullet e_o^{2*}$	$e_o^{1^O}$	$\theta_o^2\ (=e_o^{2^O})$	$e_o^{1^O}\bullet e_o^{2^O}$	$e_o^{1,\text{Centralized}}$	$e_o^{2,\text{Centralized}}$	$e_o^{\text{Centralized}}$	θ_o
22	0.6732	0.7718	0.5196	0.6308	0.8054	0.5081	0.6491	0.8005	0.5196	0.6114
23	0.6430	0.6182	0.3975	0.6115	0.6479	0.3962	0.6280	0.6330	0.3975	0.4034
24	0.8711	0.9478	0.8257	0.7476	0.9838	0.7355	0.8711	0.9478	0.8257	1
25	0.7403	1.0000	0.7403	0.7403	1	0.7403	0.7403	1	0.7403	0.9128
26	0.6345	0.8363	0.5306	0.6158	0.8368	0.5153	0.6345	0.8363	0.5306	0.6265
27	0.6573	0.9963	0.6549	0.6549	1	0.6549	0.6549	1.0000	0.6549	0.8121
28	0.7736	0.8011	0.6198	0.7399	0.8033	0.5943	0.7736	0.8011	0.6198	0.7171
29	0.8230	0.9834	0.8093	0.8093	1.0000	0.8093	0.8093	1	0.8093	1
30	1	1	1	1	1	1	1	1	1	1

Table 5. Randomly generated data set.[a]

DMU	x1	x2	x3	z1	z2	z3	y1	y2
1	0.160533	0.461705	0.801963	0.406688	0.206037	0.439965	0.949366	0.916808
2	0.856448	0.609654	0.621833	0.157097	0.584966	0.539785	0.549246	0.631047
3	0.053146	0.509803	0.591925	0.814871	0.893295	0.196896	0.404961	0.586558
4	0.415682	0.729281	0.654994	0.280068	0.815275	0.626362	0.711695	0.482451
5	0.780034	0.228201	0.774622	0.641715	0.260856	0.368447	0.217347	0.626201
6	0.788877	0.89585	0.311125	0.709338	0.070128	0.321896	0.990062	0.2855
7	0.145599	0.496386	0.597429	0.313176	0.098118	0.372811	0.911683	0.507515
8	0.636588	0.060005	0.489479	0.901919	0.049167	0.530128	0.624024	0.279208
9	0.799036	0.130151	0.652046	0.34687	0.916859	0.780976	0.703579	0.817204
10	0.35652	0.051187	0.740171	0.01868	0.598429	0.132531	0.322513	0.418686
11	0.96692	0.783424	0.576451	0.934552	0.111523	0.524625	0.067739	0.04896
12	0.348914	0.32516	0.021103	0.87194	0.618316	0.069364	0.143042	0.658826
13	0.588182	0.802032	0.871989	0.076066	0.930698	0.423215	0.661049	0.390431
14	0.71487	0.001791	0.373654	0.597038	0.302935	0.918436	0.420139	0.009398
15	0.390041	0.991319	0.860601	0.213514	0.792131	0.664094	0.337214	0.539449
16	0.224625	0.873669	0.240125	0.759566	0.95913	0.65947	0.513231	0.022899
17	0.364733	0.985311	0.335471	0.971992	0.261091	0.317436	0.757609	0.715948
18	0.148532	0.792447	0.442971	0.692554	0.406036	0.401411	0.636992	0.598899
19	0.030093	0.298586	0.950891	0.181835	0.580099	0.152668	0.546692	0.61798
20	0.048855	0.430107	0.965306	0.537189	0.568998	0.072174	0.87554	0.70092
21	0.394998	0.453573	0.390144	0.208714	0.802642	0.594124	0.794968	0.242338
22	0.04861	0.598112	0.916754	0.713031	0.386939	0.309058	0.195411	0.072147
23	0.252245	0.331712	0.648642	0.334391	0.868483	0.60101	0.670012	0.304392
24	0.44217	0.879742	0.997697	0.829225	0.451481	0.507086	0.25146	0.092647
25	0.17037	0.210851	0.073873	0.0113	0.269678	0.305566	0.074505	0.481819
26	0.959086	0.801094	0.977649	0.328863	0.549756	0.324605	0.827717	0.605161
27	0.558997	0.844132	0.867018	0.083575	0.820318	0.317238	0.967138	0.366415

[a]This data set was generated using the RAND() function in Excel.

Table 6. Results for randomly generated data set.

	Stage 1 as the Leader			Stage 2 as the Leader			Centralized		
DMU	$\theta_o^1\ (=e_o^{1*})$	e_o^{2*}	$e_o^{1*}\bullet e_o^{2*}$	$e_o^{1^O}$	$\theta_o^2\ (=e_o^{2^O})$	$e_o^{1^O}\bullet e_o^{2^O}$	$e_o^{1,\text{Centralized}}$	$e_o^{2,\text{Centralized}}$	$e_o^{\text{Centralized}}$
1	0.92987	0.21457	0.199523	0.647491	1.00000	0.647491	0.647491	1	0.647491
2	0.45941	0.35332	0.162318	0.201775	0.77230	0.155831	0.344674	0.616706	0.212562
3	1.00000	0.490794	0.490794	1	0.49079	0.490794	1	0.490794	0.490794
4	0.66141	0.228374	0.15105	0.343526	0.53783	0.184758	0.539179	0.484125	0.26103
5	0.53410	0.392592	0.209685	0.308399	0.72931	0.224919	0.453264	0.590856	0.267814
6	0.41674	0.397161	0.165514	0.284908	1.00000	0.284908	0.341368	0.840245	0.286833
7	0.84549	0.201584	0.170438	0.538628	1.00000	0.538628	0.538628	1	0.538628
8	1.00000	0.311359	0.311359	0.112017	1.00000	0.112017	1	0.311359	0.311359

Table 6. (continued)

	Stage 1 as the Leader			Stage 2 as the Leader			Centralized		
DMU	θ_o^1 $(= e_o^{1*})$	e_o^{2*}	$e_o^{1*} \bullet e_o^{2*}$	$e_o^{1^O}$	θ_o^2 $(= e_o^{2^O})$	$e_o^{1^O} \bullet e_o^{2^O}$	$e_o^{1,\text{Centralized}}$	$e_o^{2,\text{Centralized}}$	$e_o^{\text{Centralized}}$
9	1.00000	0.366425	0.366425	0.584143	0.57335	0.334915	0.756948	0.50378	0.381335
10	1.00000	0.45104	0.45104	0.424668	1.00000	0.424668	1	0.45104	0.45104
11	0.50955	0.020148	0.010267	0.065641	0.08381	0.005502	0.428929	0.04091	0.017547
12	1.00000	0.952256	0.952256	0.794639	0.97802	0.777175	1	0.952256	0.952256
13	0.54011	0.148573	0.080245	0.068894	0.72645	0.050048	0.333706	0.560818	0.187148
14	1.00000	0.227884	0.227884	1	0.22788	0.227884	1	0.227884	0.227884
15	0.63741	0.083643	0.053315	0.252866	0.43715	0.11054	0.503621	0.387645	0.195226
16	1.00000	0.235413	0.235413	1	0.23541	0.235413	1	0.235413	0.235413
17	0.77963	0.079939	0.062323	0.295451	0.92127	0.27219	0.57082	0.560192	0.319769
18	0.86242	0.288998	0.249238	0.606217	0.56459	0.342265	0.705415	0.529013	0.373174
19	1.00000	0.904554	0.904554	0.913218	1.00000	0.913218	0.913218	1	0.913218
20	0.77559	0.0944	0.073216	0.710352	1.00000	0.710352	0.710352	1	0.710352
21	0.94837	0.183191	0.173732	0.343412	0.62127	0.213352	0.728933	0.528974	0.385586
22	1.00000	0.143574	0.143574	0.894135	0.15801	0.141285	0.97248	0.148567	0.144479
23	1.00000	0.403742	0.403742	1	0.40374	0.403742	1	0.403742	0.403742
24	0.58428	0.10096	0.058988	0.44589	0.14732	0.06569	0.516951	0.142868	0.073856
25	1.00000	0.763745	0.763745	0.716089	1.00000	0.716089	0.886921	0.865347	0.767495
26	0.28302	0.453736	0.128414	0.231684	0.83290	0.19297	0.235925	0.822569	0.194064
27	0.46713	0.197038	0.092042	0.274536	1.00000	0.274536	0.274536	1	0.274536

APPENDIX

In case of multiple optimal solutions that lead to nonunique of e_o^1 and e_o^2 in the centralized approach, we develop the following procedure to achieve a fair and alternative distribution of e_o^1 and e_o^2 between the two stages.

Suppose there exists a λ, such that

$$\left[\lambda e_o^{1-} + (1-\lambda)e_o^{1+}\right] \times \left[\lambda e_o^{2-} + (1-\lambda)e_o^{2+}\right] = e_o = e_o^{\text{Centralized}} \quad (11)$$

Let $a = e_o^{1-}e_o^{2-} + e_o^{1+}e_o^{2+} - 2e_o$, $b = 2(e_o - e_o^{1+}e_o^{2+})$ and $c = e_o^{1+}e^{2+} - e_o$, then (11) becomes $a\lambda^2 + b\lambda + c = 0$. On the basis of the related solution λ^*, we can obtain a fair distribution of $e_o^1 = \lambda^* e_o^{1-} + (1-\lambda^*)e_o^{1+}$ and $e_o^2 = \lambda^* e_o^{2-} + (1-\lambda^*)e_o^{2+}$.

We next need to test whether there exist a set of weights that are related to the above efficiency distribution. If not, we then find a set of weights and efficiency distribution that is close to the above distribution.

Consider the following model

$$\begin{aligned}
&\text{Max } \frac{\sum_{d=1}^{D} W_d z_{d0}}{\sum_{i=1}^{m} V_i x_{i0}} \\
&\text{s.t. } \frac{\sum_{r=1}^{s} U_r y_{r0}}{\sum_{i=1}^{m} V_i x_{i0}} = e_o^{\text{Centralized}} \\
&\frac{\sum_{r=1}^{s} U_r y_{r0}}{\sum_{d=1}^{D} W_d z_{d0}} \geq \lambda^* e_o^{2-} + (1-\lambda^*)e_o^{2+} \\
&\frac{\sum_{d=1}^{D} W_d z_{dj}}{\sum_{i=1}^{m} V_i x_{ij}} \leq 1, \quad j = 1,2,\ldots,n \\
&\frac{\sum_{r=1}^{s} U_r y_{rj}}{\sum_{d=1}^{D} W_p z_{pj}} \leq 1, \quad j = 1,2,\ldots,n \\
&W_d \geq 0, d = 1,2,\ldots,D; V_i \geq 0, \\
&i = 1,2,\ldots,m; U_r \geq 0, r = 1,2,\ldots,s
\end{aligned} \quad (12)$$

which is equivalent to

$$\begin{aligned}
&\text{Max } \sum_{d=1}^{D} w_d z_{do} \\
&\text{s.t. } \sum_{r=1}^{s} u_r y_{ro} = e_o^{\text{Centralized}} \\
&\left[\lambda^* e_o^{2-} + (1-\lambda^*)e_o^{2+}\right] \sum_{d=1}^{D} w_d z_{do} - e_o^{\text{Centralized}} \leq 0 \\
&\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1,2,\ldots,n \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0 \quad j = 1,2,\ldots,n \\
&\sum_{i=1}^{m} v_i x_{io} = 1 \\
&w_d \geq 0, d = 1,2,\ldots,D; v_i \geq 0, \\
&i = 1,2,\ldots,m; u_r \geq 0, r = 1,2,\ldots,s
\end{aligned} \quad (13)$$

Let the optimal solution be $w'_{do}, v'_{io}, u'_{ro}$. Then $e_o^1 = \sum_{d=1}^{D} w'_{do} z_{do}$, and $e_o^2 = E_o^* / \sum_{d=1}^{D} w'_{do} z_{do}$. Now, consider

$$\begin{aligned}
&\text{Max } \frac{\sum_{r=1}^{s} U_r y_{ro}}{\sum_{d=1}^{D} W_d z_{do}} \\
&\text{s.t. } \frac{\sum_{r=1}^{s} U_r y_{ro}}{\sum_{i=1}^{m} V_i x_{io}} = e_o^{\text{Centralized}} \\
&\frac{\sum_{d=1}^{D} W_d z_{do}}{\sum_{i=1}^{m} V_i x_{io}} \geq \left[\lambda^* e_o^{1-*} + (1-\lambda^*)e_o^{1+*}\right] \\
&\frac{\sum_{d=1}^{D} W_d z_{dj}}{\sum_{i=1}^{m} V_i x_{ij}} \leq 1, j = 1,2,\ldots,n
\end{aligned}$$

$$\frac{\sum_{r=1}^{s} U_r y_{rj}}{\sum_{d=1}^{D} W_d z_{dj}} \leq 1, j = 1,2,\ldots,n$$
$$W_d \geq 0, d = 1,2,\ldots,D; V_i \geq 0,$$
$$i = 1,2,\ldots,m; U_r \geq 0, r = 1,2,\ldots,s, \tag{14}$$

which is equivalent to

$$\begin{aligned}
\text{Max } & \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t. } & \sum_{r=1}^{s} u_r y_{ro} - e_o^{\text{Centralized}} \times \sum_{i=1}^{m} v_i x_{io} = 0 \\
& \left[\lambda^* e_o^{1-} + (1-\lambda^*) e_o^{1+}\right] \sum_{i=1}^{m} v_i x_{io} - 1 \leq 0 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0 \quad j = 1,2,\ldots,n \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1,2,\ldots,n \\
& \sum_{d=1}^{D} w_d z_{do} = 1 \\
& w_d \geq 0, d = 1,2,\ldots,D; v_i \geq 0, i = 1,2,\ldots,m; \\
& u_r \geq 0, r = 1,2,\ldots,s
\end{aligned} \tag{15}$$

Let the optimal solution be $w''_{do}, v''_{io}, u''_{ro}$, then $e_o^1 = e_o^{\text{Centralized}} / \sum_{r=1}^{s} u''_{ro} y_{ro}$ and $e_o^2 = \sum_{r=1}^{s} u''_{ro} y_{ro}$.

Let

$$d' = \left(\sum_{d=1}^{D} w'_{do} z_{do} - \left[\lambda^* e_o^{1-} + (1-\lambda^*) e_o^{1+}\right]\right)^2 + \left(e_o^{\text{Centralized}} / \sum_{d=1}^{D} w'_{do} z_{do} - \left[\lambda^* e_o^{2-} + (1-\lambda^*) e_o^{2+}\right]\right)^2$$

$$d'' = \left(e_o^{\text{Centralized}} / \sum_{r=1}^{s} u''_{ro} y_{ro} - \left[\lambda^* e_o^{1-} + (1-\lambda^*) e_o^{1+}\right]\right)^2 + \left(\sum_{r=1}^{s} u''_{ro} y_{ro} - \left[\lambda^* e_o^{2-} + (1-\lambda^*) e_o^{2+}\right]\right)^2$$

If $d' > d''$, then $e_o^1 = e_o^{\text{Centralized}} / \sum_{r=1}^{s} u''_{ro} y_{ro}$ and

$$e_o^2 = \sum_{r=1}^{s} u''_{ro} y_{ro};$$

If $d'' > d'$, the $e_o^1 = \sum_{d=1}^{D} w'_{do} z_{do}$ and

$$e_o^2 = e_o^{\text{Centralized}} / \sum_{d=1}^{D} w'_{do} z_{do};$$

If $d'' = d'$, then $e_o^1 = \sum_{d=1}^{D} w'_{do} z_{do}$ and

$$e_o^2 = e_o^{\text{Centralized}} / \sum_{d=1}^{D} w'_{do} z_{do}, \text{ or}$$

$$e_o^1 = e_o^{\text{Centralized}} / \sum_{r=1}^{s} u''_{ro} y_{ro} \text{ and } e_o^2 = \sum_{r=1}^{s} u''_{ro} y_{ro}.$$

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