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Context-dependent data envelopment analysis—Measuring attractiveness and progress

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Abstract

Data envelopment analysis (DEA) is a methodology for identifying the efficient frontier of decision making units (DMUs). Context-dependent DEA refers to a DEA approach where a set of DMUs are evaluated against a particular evaluation context. Each evaluation context represents an efficient frontier composed by DMUs in a specific performance level. The context-dependent DEA measures (i) the attractiveness when DMUs exhibiting poorer performance are chosen as the evaluation context, and (ii) the progress when DMUs exhibiting better performance are chosen as the evaluation context. The current paper extends the context-dependent DEA by incorporating value judgment into the attractiveness and progress measures. The method is applied to measuring the attractiveness of 32 computer printers. It is shown that the attractive measure helps (i) customers to select the best option, and (ii) printer manufacturers to identify the potential competitors.

Keywords: Data envelopment analysis (DEA); Attractiveness; Progress; Efficient; Product; Value judgment

1. Introduction

Data envelopment analysis (DEA), introduced by Charnes, Cooper and Rhodes (CCR) [1], is a mathematical programming method for measuring the relative efficiency of decision making units (DMUs) with multiple outputs and multiple inputs. DEA identifies efficient DMUs from a given set of DMUs. It is well known that adding or deleting an inefficient DMU or a set of inefficient DMUs does not alter the efficiencies of the existing DMUs and the efficient frontier. The inefficiency scores change only if the efficient frontier is altered. i.e., the performance of DMUs depends only on the identified efficient frontier. In contrast, researchers of the consumer choice theory point out that consumer choice is often influenced by the context, e.g., a circle appears large when surrounded by small circles and small when surrounded by larger ones. Similarly, a product may appear attractive against a background of less attractive alternatives and unattractive when compared to more attractive alternatives [2].

Considering this influence within the framework of DEA, one could ask "what is the relative attractiveness of a particular DMU when compared to others?" As in [3], one agrees that the relative attractiveness of DMU_x compared to DMU_y depends on the presence or absence of a third option, say DMU_z (or a group of DMUs). Relative attractiveness depends on the evaluation context constructed from alternative options (or DMUs).

In fact, a set of DMUs can be divided into different levels of efficient frontiers. If we remove the (original) efficient frontier, then the remaining (inefficient) DMUs will form a new second-level efficient frontier. If we remove this new second-level efficient frontier, a third-level efficient frontier is formed, and so on, until no DMU is left. Each such efficient frontier provides an evaluation context for measuring the relative attractiveness, e.g., the second-level efficient frontier serves as the evaluation context for measuring the

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relative attractiveness of the DMUs located on the first-level (original) efficient frontier. On the other hand, we can measure the performance of DMUs on the third-level efficient frontier with respect to the first- or second-level efficient frontier.

In this way, we obtain a context-dependent DEA where the relative attractiveness is obtained when DMUs having worse performance are chosen as the evaluation context, and the relative progress is obtained when DMUs having better performance are chosen as the evaluation context. The presence or absence (or the shape) of the evaluation context (efficient frontier) affects the relative attractiveness or progress of DMUs on a different level of efficient frontier. When DMUs in a specific level are viewed as having equal performance, the attractiveness measure or the progress measure allows us to differentiate the "equal performance" based upon the same specific evaluation context (or third option).

Note that different input/output measures play different roles in the evaluation of a DMU's performance. Customers may make trade-offs among different measures of a product. For example, suppose we want to buy a dot-matrix printer and we may, given the price, make trade-offs amongst the speed, print quality, and input buffer (memory) which are some of the most important features that distinguish 24-pin dot-matrix printers. We may not consider the printer memory feature to be very vital, because dot-matrix printers only use memory as a buffer space to download fonts. Thus, we give more consideration to speed and print quality. Perhaps, the printer is simply used to print long program codes or data-base listings, so that speed outweighs print quality.

Therefore, in measuring the relative attractiveness and progress, incorporation of value judgment is also very important. The current paper uses the result of Zhu [4] to develop a context-dependent DEA with value judgment. The method is applied to measure the relative attractiveness of a set of printers that is studied by Doyle and Green [5]. The application demonstrates that the context-dependent DEA helps practitioners to produce finer evaluation of efficiency in practical problems.

The rest of the paper is organized as follows. The next section presents the context-dependent DEA. We then incorporate the value judgment into the context-dependent DEA. The method is applied to a set of 32 printers. Conclusions are provided in the last section.

2. Context-dependent DEA

Our model formulation below uses a vector notion for inputs and outputs where DMU_j ($j = 1, 2, \ldots, n$) produces a vector of outputs $y_j = (y_{1j}, \ldots, y_{sj})$ by using a vector of inputs $x_j = (x_{1j}, \ldots, x_{mj})$.

Let $\mathbf{J}^1 = \{\mathrm{DMU}_j, j = 1, \ldots, n\}$ be the set of all n DMUs. We interactively define $\mathbf{J}^{l+1} = \mathbf{J}^l - \mathbf{E}^l$ where $\mathbf{E}^l = \{\mathrm{DMU}_k \in \mathbf{J}^l \mid \phi^*(l,k) = 1\}$, and $\phi^*(l,k)$ is the optimal value to the following linear programming problem:

$$\phi^*(l,k) = \max_{\lambda_j, \phi(l,k)} \phi(l,k)$$

$$\text{s.t.} \quad \sum_{j \in F(\mathbf{J}^l)} \lambda_j y_j \geqslant \phi(l,k) y_k;$$

$$\sum_{j \in F(\mathbf{J}^l)} \lambda_j x_j \leqslant x_k;$$

$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{J}^l). \tag{1}$$

where (x_k, y_k) represents the input and output vector of DMU_k, and $j \in F(\mathbf{J}^l)$ means $\mathrm{DMU}_j \in \mathbf{J}^l$, i.e., $F(\cdot)$ represents the correspondence from a DMU set to the corresponding subscript index set.

When $l = 1$, model (1) becomes the original output-oriented CCR model and DMUs in set $\mathbf{E}^1$ define the first-level efficient frontier. When $l = 2$, model (1) gives the second-level efficient frontier after the exclusion of the first-level efficient DMUs. And so on. In this manner, we identify several levels of efficient frontiers. We call $\mathbf{E}^l$ the lth-level efficient frontier. The following algorithm accomplishes the identification of these efficient frontiers by model (1). The efficient frontiers can be easily obtained by using the DEA Excel Solver provided in [6].

- *Step* 1: Set $l = 1$. Evaluate the entire set of DMUs, $\mathbf{J}^1$, by model (1) to obtain the first-level efficient DMUs, set $\mathbf{E}^1$ (the first-level efficient frontier).
- *Step* 2: Exclude the efficient DMUs from future DEA runs. $\mathbf{J}^{l+1} = \mathbf{J}^l - \mathbf{E}^l$. (If $\mathbf{J}^{l+1} = \emptyset$ then stop.)
- *Step* 3: Evaluate the new subset of "inefficient" DMUs, $\mathbf{J}^{l+1}$, by model (1) to obtain a new set of efficient DMUs $\mathbf{E}^{l+1}$ (the new efficient frontier).
- *Step* 4: Let $l = l + 1$. Go to step 2.
- *Stopping rule*: $\mathbf{J}^{l+1} = \emptyset$, the algorithm stops.

There exists an input-oriented version of model (1). However, the input-oriented version of model (1) yields the same stratification of the whole set of DMUs. Fig. 1 plots the three levels of efficient frontiers of 10 DMUs with two outputs and one single input of one (see Table 1).

Now, based upon these evaluation contexts $\mathbf{E}^l$ ($l = 1, \ldots, L$), we can obtain the relative attractiveness measure by the following context-dependent DEA:

$$\Omega_q^*(d) = \max_{\lambda_j, \Omega_q(d)} \Omega_q(d), \quad d = 1, \ldots, L - l_0$$

$$\text{s.t.} \quad \sum_{j \in F(\mathbf{E}^{l_0+d})} \lambda_j y_j \geqslant \Omega_q(d) y_q;$$

$$\sum_{j \in F(\mathbf{E}^{l_0+d})} \lambda_j x_j \leqslant x_q;$$

$$\lambda_j \geqslant 0 \quad j \in F(\mathbf{E}^{l_0+d}). \tag{2}$$

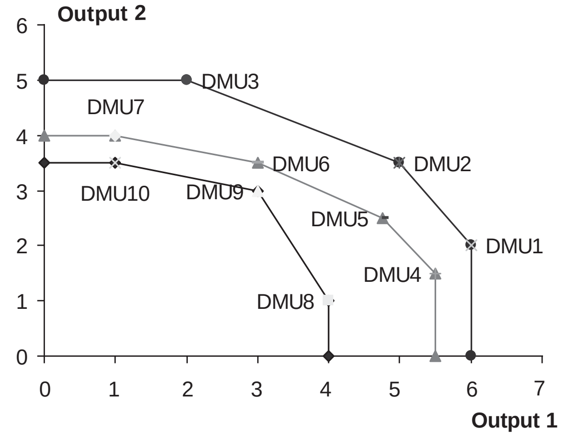

Fig. 1. Efficient frontiers in different levels.

Table 1
Sample DMUs

DMU	1	2	3	4	5	6	7	8	9	10
Output 1	6	5	2	5.5	4.75	3	1	4	3	1
Output 2	2	3.5	5	1.5	2.5	3.5	4	1	3	3.5

where $\text{DMU}_q = (x_q, y_q)$ is from a specific level $\mathbf{E}^{l_0}$, $l_0 \in \{1,\ldots,L-1\}$. We have (i) $\Omega_q^*(d) < 1$ for each $d=1,\ldots,L-l_0$, and (ii) $\Omega_q^*(d+1) < \Omega_q^*(d)$.

Definition 1. $\text{A}_q^*(d) \equiv 1/\Omega_q^*(d)$ is called the (output-oriented) *d-degree* attractiveness of DMU_q from a specific level $\mathbf{E}^{l_0}$.

Suppose, e.g., each DMU in the first-level efficient frontier represents an option, or product. Customers usually compare a specific DMU in $\mathbf{E}^{l_0}$ with other alternatives that are currently in the same level as well as with relevant alternatives that serve as evaluation contexts. The relevant alternatives are those DMUs, say, in the second- or third-level efficient frontier, etc. Given the alternatives (evaluation contexts), model (2) enables us to select the best option—the most attractive one.

In model (2), each efficient frontier of $\mathbf{E}^{l_0+d}$ represents an evaluation context for measuring the relative attractiveness of DMUs in $\mathbf{E}^{l_0}$. Note that $\text{A}_q^*(d)$ is the reciprocal of the optimal value to (2), therefore $\text{A}_q^*(d) > 1$. The larger the value of $\text{A}_q^*(d)$, the more attractive the DMU_q is, because this DMU_q makes itself more distinctive from the evaluation context $\mathbf{E}^{l_0+d}$. We are able to rank the DMUs in $\mathbf{E}^{l_0}$ based upon their attractiveness scores and identify the best one.

To obtain the progress measure for a specific $\text{DMU}_q \in \mathbf{E}^{l_0}$, $l_0 \in \{2,\ldots,L\}$, we use the following context-dependent DEA:

$$P_q^*(g) = \max_{\lambda_j, P_q(g)} P_q(g), \qquad g = 1,\ldots,l_0-1$$

$$\text{s.t.} \quad \sum_{j\in F(\mathbf{E}^{l_0-g})} \lambda_j y_j \geqslant P_q(g) y_q;$$

$$\sum_{j\in F(\mathbf{E}^{l_0-g})} \lambda_j x_j \leqslant x_q;$$

$$\lambda_j \geqslant 0 \quad j \in F(\mathbf{E}^{l_0-g}). \tag{3}$$

We have (i) $P_q^*(g) > 1$ for each $g = 1,\ldots,l_0-1$, and (ii) $P_q^*(g+1) > P_q^*(g)$.

Definition 2. The optimal value to (7), i.e., $P_q^*(g)$, is called the (output-oriented) *g-degree* progress of DMU_q from a specific level $\mathbf{E}^{l_0}$.

Each efficient frontier, $\mathbf{E}^{l_0-g}$, contains a possible target for a specific DMU in $\mathbf{E}^{l_0}$ to improve its performance. The progress here is a level-by-level improvement. For a larger $P_q^*(g)$, more progress is expected for DMU_q. Thus, a smaller value of $P_q^*(g)$ is preferred.

3. Context-dependent DEA with value judgment

In the previous section, both attractiveness and progress are measured radially with respect to different *levels* of efficient frontiers. The measurement does not require a priori information on the importance of the attributes (input/output) that feature the performance of DMUs. However, different attributes play different roles in the evaluation of a DMU's overall performance. Therefore, we introduce value judgment into the context-dependent DEA.

3.1. Incorporating value judgment into attractiveness measure

In order to incorporate such a priori information into our measures of attractiveness and progress, we first specify a set of weights related to the s outputs, u_r $(r = 1,\ldots,s)$ such that $\sum_{r=1}^{s} u_r + 1$. Based upon [4], we develop the following linear programming problem for $\text{DMU}_q = (x_q, y_q) = (x_{1q},\ldots,x_{mq}, y_{1q},\ldots,y_{sq})$ in $\mathbf{E}^{l_0}$, $l_0 \in \{1,\ldots,L-1\}$:

$$\Phi_q^*(d) = \max_{\lambda_j, \Phi_q^r(d)} \sum_{r=1}^{s} u_r \Phi_q^r(d), \qquad d = 1,\ldots,L-l_0$$

$$\text{s.t.} \quad \sum_{j\in F(\mathbf{E}^{l_0+d})} \lambda_j y_{rj} \geqslant \Phi_q^r(d) y_{rq}, \qquad r = 1,\ldots,s;$$

$$\sum_{j\in F(\mathbf{E}^{l_0+d})} \lambda_j x_{ij} \leqslant x_{iq}, \qquad i = 1,\ldots,m;$$

$$\Phi_q^r(d) \leqslant 1, \quad r = 1,\ldots,s;$$

$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0-d}). \tag{4}$$

Definition 4. $\bar{\bar{A}}_q^*(d) \equiv 1/\Phi_q^*(d)$ is called the (output-oriented) value judgment (VJ) *d-degree* attractiveness of DMU_q from a specific level $\mathbf{E}^{l_0}$.

Obviously, $\bar{\bar{A}}_q^*(d) > 1$. The larger the $\bar{\bar{A}}_q^*(d)$ is, the more attractive the DMU_q appears under the weights u_r ($r = 1,\ldots,s$). We now can rank DMUs in the same level by their VJ attractiveness scores incorporated with the preferences over outputs.

If one wishes to prioritize the options (DMUs) with higher values of the r_0th output, then one can increase the value of the corresponding weight u_{r_0}. These user-specified weights reflect the relative degree of desirability of the corresponding outputs. For example, if one prefers a printer with faster printing speed to one with higher print quality, then one may specify a larger weight for the speed (output). The constraints of $\Phi_q^r(d) \leqslant 1$ ($r = 1,\ldots,s$) ensure that in an attempt to make itself as distinctive as possible, DMU_q is not allowed to decrease some of its outputs to achieve higher levels of other preferred outputs.

Consider DMUs, 1, 2 and 3 in Table 1 and select the second-level efficient frontier as the evaluation background, i.e., we consider the VJ first-degree attractiveness.

Case I: If let $u_1 = u_2 = 0.5$, i.e., the preference over the two outputs is equal, then we have $\bar{\bar{A}}_1^*(1) = 1.0787$, $\bar{\bar{A}}_2^*(1) = 1.2019$ and $\bar{\bar{A}}_3^*(1) = 1.1429$. Thus, DMU2 is the most attractive one;

Case II: If let $u_1 = 0.98$ and $u_2 = 0.02$, i.e., we prefer the first output, then we have $\bar{\bar{A}}_1^*(1) = 1.0949$, $\bar{\bar{A}}_2^*(1) = 1.0077$ and $\bar{\bar{A}}_3^*(1) = 1.0050$. Thus, DMU1 is the most attractive one;

Case III: If $u_1 = 0.02$ and $u_2 = 0.98$, i.e., we prefer the second output, then we have $\bar{\bar{A}}_1^*(1) = 1.0030$, $\bar{\bar{A}}_2^*(1) = 1.0081$ and $\bar{\bar{A}}_3^*(1) = 1.2595$. Thus, DMU3 is the most attractive one.

It can be seen that different weight combinations lead to different attractiveness scores.

Note that $\bar{\bar{A}}_q^*(d)$ (or $\Phi_q^*(d)$) is an overall attractiveness of DMU_q in terms of outputs while keeping the inputs at their current levels. On the other hand, each individual optimal value of $1/\Phi_q^{r^*}(d), (r = 1,\ldots,s)$ measures the attractiveness of DMU_q in terms of each output dimension. Note that $\bar{\bar{A}}_q^*(d)$ is not equal to $\sum_{r=1}^s u_r A_q^r(d)$, where $\mathrm{A}_q^r(d) = 1/\Phi_q^{r*}(d)$.

Definition 5. For $\mathrm{DMU}_q \in \mathbf{E}^{l_0}$, $l_0 \in \{2,\ldots,L\}$, the optimal value $\bar{\bar{A}}_q^{r*}(d) \equiv 1/\Phi_q^{r*}(d)$ is called the (output-oriented) VJ *d-degree* output-specific attractiveness measure.

Consider case I of VJ first-degree attractiveness. When $u_1 = u_2 = 0.5$, we have (i) $A_1^1(1) = 1.1710$, $A_1^2(1) = 1$ for DMU1; (ii) $A_2^1(1) = 1.0526$, $A_2^2(1) = 1.4006$ for DMU2; and (iii) $A_3^1(1) = 1$, $A_3^2(1) = 1.3333$ for DMU3. Thus, DMU1 is the most attractive one in terms of the first output, whereas DMU2 is the most attractive one in terms of the second output.

Let $\Phi_q^r(d)y_{rq} = y_{rq} - s_q^r(d)$ ($r = 1,\ldots,s$) in (4). Since $\Phi_q^r(d) \leqslant 1$, $s_q^r(d) \geqslant 0$, model (4) is equivalent to the following linear programming problem:

$$\min_{\lambda_j, s_q^r(d)} \sum_{r=1}^s D_r s_q^r(d), \quad d = 1,\ldots,L-l_0$$

$$\text{s.t.} \quad y_{rq} - \sum_{j \in F(\mathbf{E}^{l_0+d})} \lambda_j y_{rj} = s_q^r(d), \quad r = 1,\ldots,s;$$

$$\sum_{j \in F(\mathbf{E}^{l_0+d})} \lambda_j x_{ij} \leqslant x_{iq}, \quad i = 1,\ldots,m;$$

$$s_q^r(d) \geqslant 0, \quad r = 1,\ldots,s;$$

$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0+d}). \tag{5}$$

where $D_r = u_r/y_{rq}$, i.e., u_r is normalized by the corresponding output quantity. $s_q^r(d)$ in (5) can be regarded as the maximum possible output reduction to a specific efficient frontier $\mathbf{E}^{l_0+d}$. Therefore, the output-specific attractiveness measure characterizes the difference between $\mathrm{DMU}_q \in \mathbf{E}^{l_0}$ and $\mathbf{E}^{l_0+d}$ in terms of a specific output.

With the output-specific (or input-specific) attractiveness measures, one can further identify which outputs (inputs) play important roles in distinguishing a DMU's performance. On the other hand, if $\Phi_q^{r_0^*}(d) = 1$, then other DMUs in $\mathbf{E}^{l_0+d}$ or their combinations can also produce the amount of the r_0th output of DMU_q, i.e., DMU_q does not exhibit better performance with respect to this specific output dimension. Therefore, DMU_q should improve its performance on the r_0th output to distinguish itself in the future.

3.2. Incorporating value judgment into progress measure

Similar to the development in the previous section, we can define the output-oriented VJ progress measure:

$$\bar{\bar{P}}_q^*(g) = \max_{\lambda_j, P_q^r(g)} \sum_{r=1}^s u_r P_q^r(g), \quad g = 1,\ldots,l_0-1$$

$$\text{s.t.} \quad \sum_{j \in F(\mathbf{E}^{l_0-g})} \lambda_j y_{rj} \geqslant P_q^r(g) y_{rq}, \quad r = 1,\ldots,s;$$

$$\sum_{j \in F(\mathbf{E}^{l_0-g})} \lambda_j x_{ij} \leqslant x_{iq}, \quad i = 1,\ldots,m;$$

$$P_q^r(g) \geqslant 1, \quad r = 1,\ldots,s;$$

$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0-g}). \tag{6}$$

Definition 6. The optimal value $\bar{\bar{P}}_q^*(g)$ is called the (output-oriented) VJ *g-degree* progress of DMU_q in a specific level $\mathbf{E}^{l_0}$.

The larger the $\bar{\bar{P}}_q^*(g)$ is, the greater the amount of progress is expected for DMU_q. Here the user-specified weights reflect the relative degree of desirability of improvement on the individual output levels.

Let $P_q^{r*}(g)$ represent the optimal value of (6) for a specific $g \in \{1, \ldots, l_0 - 1\}$. By Zhu [4], we know that $\sum_{j\in F(\mathbf{E}^{l_0-g})} \lambda_j^* y_{rj} = P_q^{r*}(g) y_{rq}$ holds at optimality for each $r = 1, \ldots, s$. Consider the following linear programming problem:

$$\max \sum_{i=1}^{m} s_i^-(g), \quad g = 1, \ldots, l_0 - 1$$

$$\text{s.t.} \quad \sum_{j\in F(\mathbf{E}^{l_0-g})} \lambda_j y_{rj} = P_q^{r*}(g) y_{rq}, \quad r = 1, \ldots, s;$$

$$\sum_{j\in F(\mathbf{E}^{l_0-g})} \lambda_j x_{ij} + s_i^-(g) = x_{iq}, \quad i = 1, \ldots, m;$$

$$s_i^-(g) \geqslant 0, \quad r = 1, \ldots, s;$$

$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0-g}). \tag{7}$$

Definition 7. (*Preferred global efficient target and preferred local efficient target*) The following point:

$$\begin{cases} \hat{y}_{rq} = P_q^{r*}(g) y_{rq}, & r = 1, \ldots, s, \\ \hat{x}_{iq} = x_{iq} - s_i^{-*}(g), & i = 1, \ldots, m \end{cases}$$

is a *preferred global efficient target* for $\text{DMU}_q \in \mathbf{E}^{l_0}$, $l_0 \in \{2, \ldots, L\}$ if $g = l_0 - 1$; otherwise, if $g < l_0 - 1$, it represents a *preferred local efficient target*, where $P_q^{r*}(g)$ is the optimal value in (6), and $s_i^{-*}(g)$ represent the optimal values in (7).

3.3. More discussion

In order to further investigate the property of models (4) and (6), we consider the dual program to (4):

$$\min \sum_{i=1}^{m} w_i x_{iq} + \sum_{r=1}^{s} e_r$$

$$\text{s.t.} \quad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} \leqslant 0, \quad j \in F(\mathbf{E}^{l_0+d});$$

$$\mu_r y_{rq} + e_r \geqslant u_r, \; r = 1, \ldots, s;$$

$$w_i, e_r, \mu_r \geqslant 0 \tag{8}$$

in which w_i and μ_r are the input and the output multipliers, respectively, and e_r are the dual variables associated with $\Phi_q^r(d) \leqslant 1$.

We rewrite the constraint of $\mu_r y_{rq} + e_r \geqslant u_r$ in (8) as $\mu_r \geqslant u_r - e_r/y_{rq} \geqslant u_r/y_{rq}$. It can be seen that the weights in (4) are the lower multiplier bounds in (8). Therefore, the value judgment here can be expressed by the assurance region concept [7]. The weights in (4) can be obtained by multiplying each lower output multiplier bound by its corresponding output quantity. In fact, the lower output multiplier bounds can be used as D_r in (5).

Moreover, substituting $\mu_r = u_r/y_{rq}$ into (8) yields $\sum_{i=1}^m w_i x_{ij} \geqslant R_j, j \in F(\mathbf{E}^{l_0+d})$, where $R_j = \sum_{r=1}^s u_r y_{rj}/y_{rq}$. Thus, R_j can be interpreted as the available resource level for each DMU_j, $j \in F(\mathbf{E}^{l_0+d})$. The range of R_j can be obtained through additional information, such as, price/cost data on inputs. If the number of DMUs in $\mathbf{E}^{l_0+d}$ is greater than the number of outputs, then we may use $\sum_{r=1}^s Y_j u_r = R_j$ to determine u_r, where $Y_j = y_{rj}/y_{rq}$, $j \in F(\mathbf{E}^{l_0+d})$.

Finally, the current discussion is based upon the output-oriented DEA models. Similar context-dependent DEA models can be obtained if we use input-oriented DEA models. See Appendix for the models.

4. An application

Doyle and Green [5] benchmarked 37 computer printers using DEA. We revisit their data set by using the newly developed context-dependent DEA. In order to keep the results consistent and comparable with Doyle and Green [5], we choose price (in US dollars) as the single input. The following features/measures are chosen as outputs: (1) input buffer; (2) mean time between failure (MTBF); (3) 80-column throughput; (4) graphics throughput; (5) sound level and (6) print quality (see Table 2).

There are two kinds of input buffers: standard and optional. Because some printers have zero values for either the standard or optional input buffer, we combine the two scores to give a composite input buffer score so that all scores are positive. The larger the buffer, the more output a computer can transmit to the printer and the sooner the computer is freed for other uses. As stated in Stewart [8], MTBF (in hours) is a significant specification of a manufacturer's rating of the durability of a printer. The current study does not have access to the MTBF of the following five printers: Star Micronics NB24-15, Toshiba P341SL, IBM Proprinter XL24, Star Micronics NB-15 and Toshiba P351SX.

The third and fourth outputs are measures of printing speed in characters per second (cps) which is the document length in bytes divided by the number of seconds to print it. (Higher numbers signify faster performance.) The fifth output is a measure of the noise level (in dBA) where lower numbers are preferable. Based upon [9], because it is an output measure, we subtract each number from 100 to obtain an adjusted score for the DEA analysis. The last output is a combined quality score for text and graphics quality scores where larger numbers indicate a higher quality. Note

Table 2
Data for the 32 printers

Printer name	DMU no.	Price	Input buffer	MTBF	80-column throughput	Graphics throughput	Sound level	Print quality
Epson LQ-500	1	499	8	4000	101	850	72	5
NEC P2200	2	499	8	4000	85	830	72	5
Seikosha SL-80AI	3	549	16	3200	56	451	68	4
Copal WH 6700	4	795	50	4000	102	450	69	3
Epson LQ-850	5	799	38	4000	148	1350	71	7
Printronix P1013	6	895	2	4000	107	683	78	6
Panasonic KX-P1524	7	899	45	4000	107	850	75	7
Brother M-1724L	8	949	32	4000	107	931	72	5
Citizen Tribute 224	9	949	24	5000	122	917	73	6
ALPS ALQ324	10	995	71	5000	105	562	69	6
Fujitsu DL3400	11	995	24	8000	146	1440	63	7
NEC P7	12	995	50	5000	111	1255	65	6
Sanyo PR-241	13	999	10	8000	90	955	68	6
Dataproducts 9044	14	1099	32	5000	121	687	72	5
Epson LQ-1050	15	1099	48	6000	147	1367	71	7
Facit B3450	16	1245	16	4000	134	1090	72	5
C. Itoh C-715A	17	1295	32	7200	131	1186	74	7
Nissho NP-2405	18	1295	36	6000	139	650	72	7
ALPS P2400C	19	1395	256	6000	146	1000	70	7
Okidata Microline 393	20	1399	30	4000	184	2400	67	9
Epson LQ-2500	21	1449	40	6000	128	1459	70	6
Fujitsu DL2600	22	1495	80	6000	146	1588	69	8
NEC P5XL	23	1495	40	7000	132	1421	68	7
Radio Shack DMP-2120	24	1599	64	3000	150	465	68	7
AT&T 477	25	1695	80	6000	146	1301	69	7
Hewlett-Packard RW480	26	1695	36	20000	191	542	69	15
Nissho NP-2410	27	1745	54	6000	169	683	71	12
NEC P9XL	28	1795	48	7000	170	1928	68	8
Mannesmann Tally MT330	29	1799	32	4800	205	1069	63	7
C. Itoh C-815	30	1995	42	7200	182	2823	72	10
Fujitsu DL5600	31	2195	24	8000	236	3176	68	12
Japan Dgtl. Labs JDL-850	32	2495	128	4000	169	497	63	9

that the last four outputs are among the test criteria used by Stewart [8]. Also, based upon Stewart [8], printers 1–13 are in the low-price category ($499–$999), printers 14–23 are in the middle-price category ($1000–$1499), printers 24–30 are in the high-price category ($1500–$1999) and printers 31 and 32 are in the deluxe price category ($2000–$2499).

By using the DEA model (1), we obtain four levels of efficient frontiers. They are

$$\mathbf{E}^1 = \{\mathrm{DMU}_j \mid j = 1, 2, 3, 5, 19, 20, 26\},$$

$$\mathbf{E}^2 = \{\mathrm{DMU}_j \mid j = 4, 7, 10, 11, 12, 15, 31\},$$

$$\mathbf{E}^3 = \{\mathrm{DMU}_j \mid j = 6, 8, 9, 13, 22, 27, 30\},$$

$$\mathbf{E}^4 = \{\mathrm{DMU}_j \mid j = 14, 16, 17, 18, 21, 23, 24, 25, 28, 29, 32\}.$$

It can be seen from the original DEA (CCR) model, seven printers in $\mathbf{E}^1$ are efficient. This result is slightly different from that of Doyle and Green [5], partly because we treat one of the outputs, sound level, in a different way. Note that three of the six "outstanding buys" selected by Stewart [8], namely, DMU1 (Epson LQ-500), DMU20 (Okidata Microline 393) and DMU26 (Hewlett-Packard RW480) are in the first-level efficient frontier and the remaining three, namely, DMU4 (Copal WH6700), DMU11 (Fujitsu DL3400) and DMU31 (Fujitsu DL5600) are in the second-level efficient frontier. We next discuss the 14 printers in $\mathbf{E}^1$ and $\mathbf{E}^2$ in detail.

First, by using (2) we consider the attractiveness and progress of the 14 printers when different efficient frontiers are chosen as evaluation contexts. Table 3 gives the results.

The number to the right of each score indicates the ranking position by the attractiveness measure. (① represents the top-rank position) Note that DMU19 (ALPS P2400C) and DMU4 (Copal WH 6700) are the most attractive printers in the first and second levels, respectively, no matter which

Table 3
Attractive and progress scores for the 14 printers in $\mathbf{E}^1$ and $\mathbf{E}^2$

Printer name	DMU no.	Background (efficient frontier)		
		Second-level First-degree[a]	Third-level Second-degree	Fourth-level Third-degree
Epson LQ-500	1	1.50092③	1.85446③	2.32072②
NEC P2200	2	1.50092③	1.78060④	2.30622③
Seikosha SL-80AI	3	1.51699②	1.85487②	2.28781④
Epson LQ-850	5	1.33046⑤	1.59208⑤	1.83955⑥
ALPS P2400C	19	2.57175①	3.42936①	3.57769①
Okidata Microline 393	20	1.18545⑥	1.31406⑦	1.59716⑦
Hewlett-Packard RW480	26	1.46755④	1.54022⑥	2.12224⑤
		First-degree[b]	First-degree	Second-degree
Copal WH 6700	4	1.16312③	1.50432①	1.72718①
Panasonic KX-P1524	7	1.13117②	1.28282⑤	1.53605⑤
ALPS ALQ324	10	1.27868⑦	1.41235②	1.60648③
Fujitsu DL3400	11	1.03020①	1.37561③	1.67563②
NEC P7	12	1.19557⑤	1.29241④	1.57736④
Epson LQ-1050	15	1.22295⑥	1.18991⑥	1.38623⑥
Fujitsu DL5600	31	1.18327④	1.09066⑦	1.34711⑦

[a]The number to the right of each score indicates the ranking position.
[b]This represents progress.

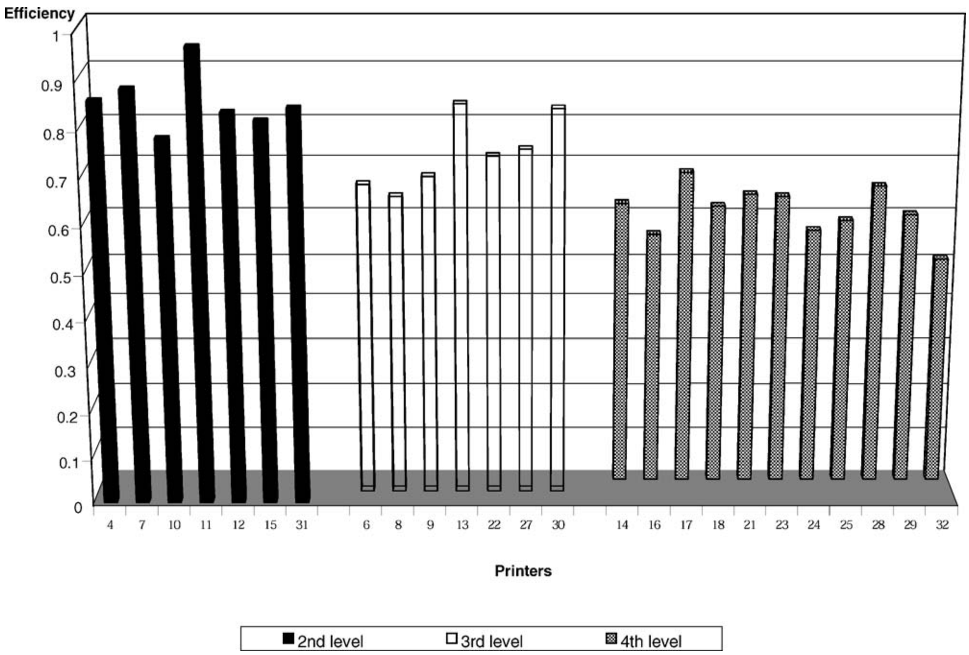

Fig. 2. Ranking of the inefficient printers by the original DEA.

evaluation context is chosen. Also, DMU1 (Epson LQ-500) and DMU11 (Fujitsu DL3400) have the second and third ranking positions, respectively.

Fig. 2 gives the ranking of the 25 inefficient DMUs in sets $\mathbf{E}^2$, $\mathbf{E}^3$ and $\mathbf{E}^4$ by the original output-oriented CCR model. The ranking scores are reciprocal of the output-oriented CCR efficiency scores which are equal to one if DMUs are in $\mathbf{E}^1$, and otherwise are greater than one if DMUs are in set $\mathbf{E}^2$ or $\mathbf{E}^3$ or $\mathbf{E}^4$. Table 3 also reports the progress scores for the printers in $\mathbf{E}^2$. The scores are actually the output-oriented

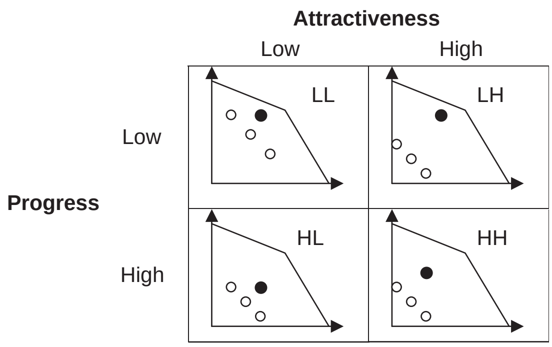

Fig. 3. Attractiveness—progress.

CCR scores. It can be seen that DMU10 is the worst printer in $\mathbf{E}^2$. However, it has a better performance in terms of the attractiveness score. DMU10 is ranked as second and third by the first-degree and the second-degree attractiveness scores, respectively.

In fact, for DMUs that are not located on the first or last level of efficient frontier, we can characterize their performance by their attractiveness and progress as shown in Fig. 3 where the solid circle represents the DMU being evaluated. The most desirable category is the low progress – high attractiveness (LH) and the least desirable category is the high progress – low attractiveness (HL). A high progress indicates that the DMU needs to improve its outputs substantially, and a high attractiveness indicates that the DMU does not have any close competitors. For example, for the printers in $\mathbf{E}^2$, we may categorize (i) Copal WH 6700 (DMU4) and Fujitsu DL3400 (DMU11) as LH, (ii) Panasonic KX-P1524 (DMU7) as LL, (iii) ALPS ALQ324 (DMU10) as HH, and (iv) NEC P7 (DMU12), Epson LQ-1050 (DMU15), and Fujitsu DL5600 (DMU31) as HL.

Next, we consider DMU19 (ALPS P2400C). Note that this printer has the largest input buffer, 256 k (the average value of the others is 40 k). Thus, the massive input buffer is likely to lead to the large attractiveness score for that printer, and consequently, the attractiveness measure for DMU19 may be biased. Therefore, we need to define some weights, $u_r (r = 1, \ldots, 6)$ to construct the output-oriented VJ attractiveness score by using model (4).

Stewart [8] writes:

> Among low-price units, the Epson LQ-500 ($499), the Copal Write Hand 6700 ($795), and the Fujitsu DL3400 ($995) each offer bargain hunters good combinations of speed and quality.

Thus, if we prefer speed and quality, we specify the following weights where more weight is put on 80-column throughout, graphics throughout and print quality which characterize speed and quality.

$$\textbf{Weight-1}: u_1 = 0.004,\ u_2 = 0.003,\ u_3 = 0.33,\ u_4 = 0.33,\ u_5 = 0.003,\ u_6 = 0.33,$$

Tables 4 and 5 report the VJ (first-degree) attractiveness scores for the printers in $\mathbf{E}^1$ and $\mathbf{E}^2$, respectively.

It can be seen that DMU1 (Epson LQ-500) and DMU11 (Fujitsu DL3400) are the top-ranked printers in $\mathbf{E}^1$ and $\mathbf{E}^2$, respectively. Note that DMU11 (Fujitsu DL3400) is the top-ranked unit among the inefficient DMUs by the CCR model (see Fig. 2). This observation strengthens the conclusion that these two printers are the best ones.

However, DMU4 (Copal WH6700) which is ranked highly by the CCR model does not have a large attractiveness score. When calculating the VJ attractiveness score for DMU4, model (4) identifies DMU8 and DMU9 as the referent DMUs. (The associated optimal lambda values are 0.013 and 0.824, respectively.) Thus, the unattractiveness of DMU4 is due to the presence of DMU8 and DMU9. Note that DMU4, DMU8 and DMU9 are all in the low-price category. Hence, DMU8 (Brother M-1724L) and DMU9 (Citizen Tribute 224) could be the potential competitors for DMU4 (Copal WH6700).

It can also be seen that DMU26 (Hewlett-Packard RW480) has a small attractiveness score of 1.04331 although it achieves a top rating in terms of text and graphics quality. Note that our VJ attractiveness measure is based on the situation where inputs are fixed at current levels. Model (4) identifies DMU7 (Panasonic KX-P1524) as the referent

Table 4
VJ attractiveness scores for the seven printers in $\mathbf{E}^1$ when $\mathbf{E}^2$ is chosen as the evaluation context[a]

Printer name	DMU no.	No weight	Weight-1	Weight-3
Epson LQ-500	1	1.50092③	1.42580①	1.39677
NEC P2200	2	1.50092③	1.33025②	1.27733
Seikosha SL-80AI	3	1.51699②	1.00125⑦	1.25382
Epson LQ-850	5	1.33046⑤	1.31890③	1.25382
ALPS P2400C	19	2.57175①	1.00255⑥	1.00319
Okidata Microline 393	20	1.18545⑥	1.09626④	1.00133
Hewlett-Packard RW480	26	1.46755④	1.04331⑤	1.06608

[a]The number to the right of each score indicates the ranking position.

Table 5
VJ attractiveness scores for the seven printers in $\mathbf{E}^2$ when $\mathbf{E}^3$ is chosen as the evaluation context[a]

Printer name	DMU no.	No weight	Weight-1	Weight-2
Copal WH 6700	4	1.50432①	1.00320	1.00431
Panasonic KX-P1524	7	1.28282⑤	1.06673	1.14002
ALPS ALQ324	10	1.41235②	1.00205	1.00347
Fujitsu DL3400	11	1.37561③	1.14645①	1.03367
NEC P7	12	1.29241④	1.04595	1.00436
Epson LQ-1050	15	1.18991⑥	1.09975	1.00398
Fujitsu DL5600	31	1.09066⑦	1.05477	1.00079

[a]The number to the right of each score indicates the ranking position.

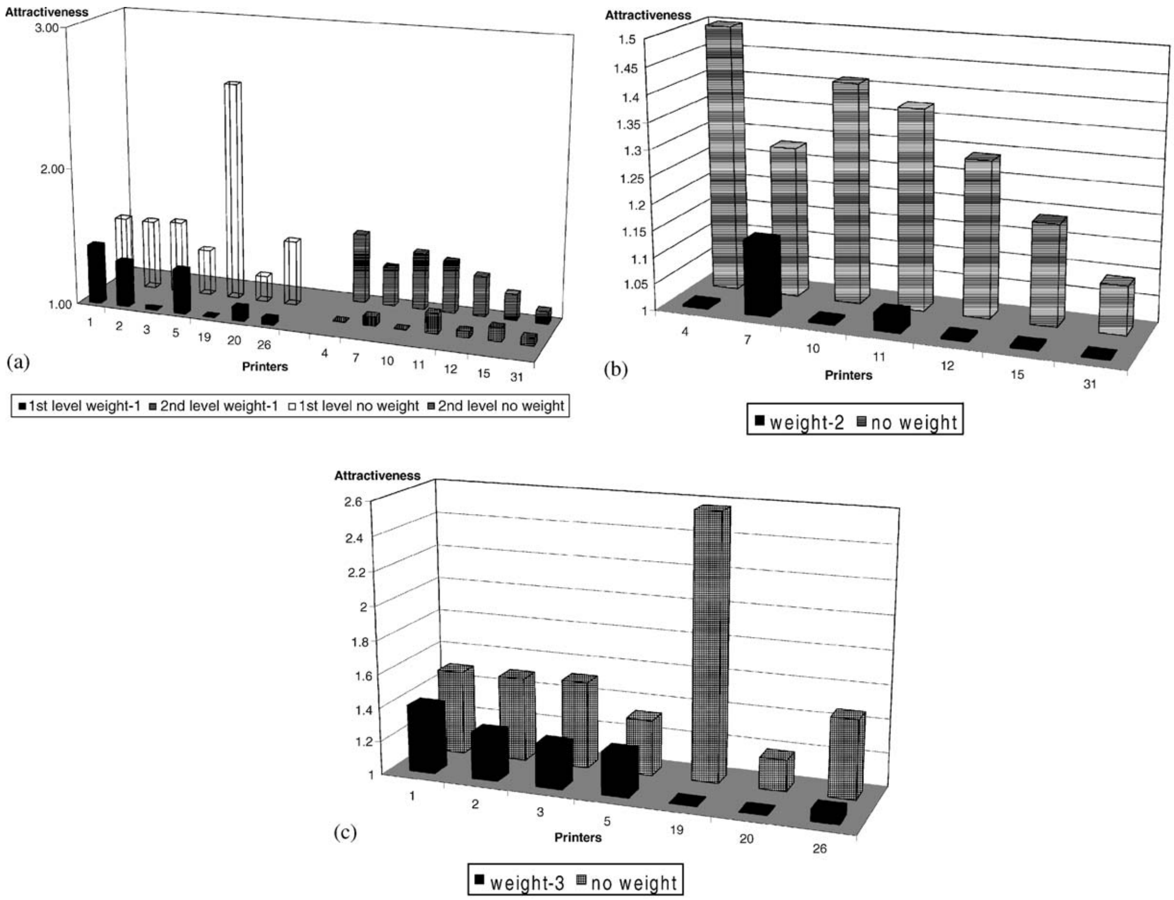

Fig. 4. (a) First-degree attractiveness under Weight-1; (b) attractiveness for second-level printers under Weight-2; and (c) attractiveness for first-level printers under Weight-3.

printer. If we examine the original data for the two printers

Printer name	DMU no.	Price	Input buffer	MTBF	80-column throughput	Graphics throughput	Sound level	Print quality
Panasonic KX-P1524	7	899	45	4000	107	850	75	7
Hewlett-Packard RW480	26	1695	36	20000	191	542	69	15

we observe that the price of DMU26 almost doubles that of DMU7. Note that DMU7 is in the low-price category and DMU26 is in the high-price category. However, DMU26 does not have a higher value of graphics throughput, and consequently, the presence of DMU7 makes DMU26 less attractive. DMU7 may be a better alternative for DMU26 if one's budget is restricted. In other words, in terms of the price and the printers in $\mathbf{E}^2$, DMU26 (Hewlett-Packard RW480) is not attractive among the seven printers in $\mathbf{E}^1$. This result is consistent with the statement in Stewart [8, p. 124],: "If you are willing to pay the price, you can definitely find speed and quality in one unit (Hewlett-Packard RW480)". Finally, note that DMU19 dropped to the sixth position in terms of attractiveness ranking.

If quality alone is the consideration, then we choose the following weights:

Weight-2 : $u_r = 0.005 (r = 1, \ldots, 5)$ and $u_6 = 0.975$.

From the last column of Table 5, we see that the most attractive printer is DMU7 (Panasonic KX-P1524), followed by the DMU11 (Fujitsu DL3400) which were suggested by Stewart [8] for quality consideration.

If we prefer 80-column throughout and quality, we specify the following weights:

Weight-3 : $u_1 = 0.005$, $u_2 = 0.005$, $u_3 = 0.49$,

$u_4 = 0.005$, $u_5 = 0.005$, $u_6 = 0.49$.

In this case, DMU20 (Okidata Microline 393) is the most unattractive printer among the seven printers in $\mathbf{E}^1$ (see last column in Table 4). Stewart [8] stated "The Okidata Microline 393 ($1399) looks more like a high-price unit in terms of 80-column throughout and quality". In fact, DMUs 11, 15 and 31 are in the reference set under model (4), i.e., these three DMUs serve as the evaluation context when measuring the VJ attractiveness of DMU20. From the optimal lambda values, we see that DMU11 plays a substantial role with $\lambda_{11}^* = 0.790$ compared to DMU15 ($\lambda_{15}^* = 0.121$) and DMU31 ($\lambda_{31}^* = 0.219$). In terms of the price, DMU20 obviously does not have the advantage in 80-column throughout and quality.

Different results are observed from Figs. 4a–c, when value judgment is incorporated into the attractiveness measure. In particular, since model (2) considers a radial reduction of all outputs which is restricted by the lower output levels, it cannot reflect the attractiveness on each output dimension. However, model (4) gives the maximum reduction on each output level for a particular DMU under consideration. The weights specified in model (4) prescribe priority for each reduction.

Finally, we illustrate how to identify which of the six features (outputs) of each printer in $\mathbf{E}^1$ exhibits the leading performance with respect to the printers in $\mathbf{E}^2$. That is, based upon $\mathbf{E}^2$ and the first-degree attractiveness, we determine, for a printer in $\mathbf{E}^2$, (a) the "superior" features that other printers may have difficulties to catch up, and (b) the "noninferior" features for which other printers or their combinations also achieve the same performance level. This analysis provides the manufacturers with information on (i) which features of a printer should be improved to gain a competitive edge, and (ii) the referent printers in $\mathbf{E}^2$ may be viewed as potential competitors.

Let us assume equal weights in model (4), i.e., $u_r = \frac{1}{6}, r = 1, \ldots, 6$. Table 6 reports the six output-specific attractiveness measures along with the referent printers. It can be seen that four printers in $\mathbf{E}^2$ appear in the reference set, of which three are outstanding buys, and in particular, Fujitsu DL3400 (DMU11) appears in every reference set. The two outstanding buys in $\mathbf{E}^2$, namely Okidata Microline 393 (DMU20) and Hewlett-Packard RW480 (DMU26), which are in the high/deluxe price category, do not exhibit good performance in terms of output-specific attractiveness measures. For instance, DMU20, which is the winner (middle price) in graphics tests [8], only has 1.36310 on its graphics throughput, and 1.0 on all other features. DMU26 exhibits good performance only on MTBF and print quality. However, Epson LQ-850 (DMU5) and ALPS P2400C (DMU19) exhibit a good performance based upon most of the output-specific attractive measures. This indicates that if no preference is given to specific output features, these two printers may be a good choice in the presence of the outstanding buy, DMU11 (Fujitsu DL3400).

Finally, if we remove DMU11 from $\mathbf{E}^2$, then the output-specific attractiveness for DMU1 is improved for each feature except for input buffer and graphics throughput (1, 1.46827, 1.51322, 1.36945, 2.12646, 1.57312 versus 1, 1, 1.37941, 1.48675, 1.50896, 1.42428). The new referent printer is DMU15. This indicates that removing an inefficient DMU affects the attractiveness of efficient DMUs.

5. Conclusions

Context-dependent DEA is developed to measure the attractiveness and progress of DMUs with respect to a given evaluation context. Different strata of efficient frontiers rather than the traditional first-level efficient frontier are used as evaluation contexts. In the original DEA, adding or deleting inefficient DMUs does not alter the efficiencies of the existing DMUs and the efficient frontier whereas under the context-dependent DEA, such action changes the performance of both efficient and inefficient DMUs. i.e., the context-dependent DEA performance depends on not only the efficient frontier, but also the inefficient DMUs. This change makes DEA more versatile and allows DEA to locally and globally identify better options. Value judgment is incorporated into the context-dependent DEA through a specific set of weights reflecting the preferences over various output (or input) measures. In particular, the attractiveness measure can be used to (i) identify DMUs that have outstanding performance and (ii) differentiate the performance of DEA efficient DMUs.

Table 6
Output-specific attractiveness scores for the printers in $\mathbf{E}^1$

Printer name	DMU no.	Input buffer	MTBF	80-column throughput	Graphics throughput	Sound level	Print quality
Epson LQ-500	1	1	1	1.37941	1.48675	1.50896	1.42428
Referent printer		Fujitsu DL3400 (DMU11)					
NEC P2200	2	1	1	1.16088	1.45176	1.50896	1.42428
Referent printer		Fujitsu DL3400 (DMU11)					
Seikosha SL-80AI	3	1	1	1	1	1.24542	1.36564
Referent printers		Copal WH 6700 (DMU4) and Fujitsu DL3400 (DMU11)					
Epson LQ-850	5	3.12997	1	2.02519	2.35939	1.56353	1.99627
Referent printer		Fujitsu DL3400 (DMU11)					
ALPS P2400C	19	21.26936	1.49549	1.99399	1.74911	1.61675	1.99399
Referent printer		Fujitsu DL3400 (DMU11), Epson LQ-1050 (DMU15), and Fujitsu DL5600 (DMU31)					
Okidata Microline 393	20	1	1	1	1.36310	1	1
Referent printers		Fujitsu DL3400 (DMU11)					
Hewlett-Packard RW480	26	1	1.46755	1	1	1	1.25790
Referent printer		Fujitsu DL3400 (DMU11)					

The application of comparing computer printers illustrates that in-depth information can be obtained by the context-dependent DEA when compared to the results obtained from the original DEA method. Context-dependent DEA identifies the most attractive printer among the outstanding buys located at two different levels of efficient frontiers. It also identifies the most attractive printer in terms of individual features, e.g., speed and quality. The method uncovers better options and prescribes possible improvement when a specific printer is rated as inefficient by the original DEA model. With a restricted budget, the DEA-efficient printers may not necessarily be the best choice. In our application, we are able to identify better alternatives. In addition, with a sensitivity analysis of weights, one could determine allowable weight ranges to be specified by users or experts. However, this type of study is beyond the scope of the current paper and is therefore classified as future research.

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Appendix A. Input-oriented context-dependent DEA

Here, we provide the input-oriented context-dependent DEA.

Consider the following linear programming problem for $\mathrm{DMU}_q = (x_q, y_q)$ in a specific level $\mathbf{E}^{l_0}$, $l_0 \in \{1, \ldots, L-1\}$:

$$H_q^*(d) = \min H_q(d), \quad d = 1, \ldots, L - l_0$$

$$\text{s.t.} \quad \sum_{j \in F(\mathbf{E}^{l_0+d})} \lambda_j x_j \leqslant H_q(d) x_q;$$

$$\sum_{j \in F(\mathbf{E}^{l_0+d})} \lambda_j y_j \geqslant y_q;$$

$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0+d}). \tag{A.1}$$

Note that dividing each side of the constraint of (A.1) by $H_q(d)$ gives

$$\sum_{j \in F(\mathbf{E}^{l_0+d})} \tilde{\lambda}_j x_j \leqslant x_q,$$

$$\sum_{j \in F(\mathbf{E}^{l_0+d})} \tilde{\lambda}_j y_j \geqslant \frac{1}{H_q(d)} y_q,$$

$$\tilde{\lambda}_j = \frac{\lambda_j}{H_q(d)} \geqslant 0, \quad j \in F(\mathbf{E}^{l_0+d}).$$

Therefore, (A.1) is equivalent to (2), and we have (i) $H_q^*(d) = 1/\Omega_q^*(d)$ for $\mathrm{DMU}_q \in \mathbf{E}^{l_0}$, $l_0 \in \{1, \ldots, L-1\}$, (ii) $H_q^*(d) > 1$ for each $d = 1, \ldots, L - l_0$, and $H_q^*(d+1) > H_q^*(d)$.

Definition A.1. $H_q^*(d)$ is called (input-oriented) *d-degree* attractiveness of DMU_q from a specific level $\mathbf{E}^{l_0}$.

The bigger the $H_q^*(d)$ is, the more attractive the DMU_q is. Model (A.1) determines the relative attractiveness score for DMU_q when outputs are fixed at their current levels. To measure the progress of $\mathrm{DMU}_q \in \mathbf{E}^{l_0}$, $l_0 \in \{2,\ldots,L\}$, we develop

$$G_q^*(g) = \min G_q(g), \quad g = 1,\ldots,l_0-1$$
$$\text{s.t.} \quad \sum_{j\in F(\mathbf{E}^{l_o-g})} \lambda_j x_j \leqslant G_q(\beta)x_q;$$
$$\sum_{j\in F(\mathbf{E}^{l_o-g})} \lambda_j y_j \geqslant y_q;$$
$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0-s}). \tag{A.2}$$

We have (i) $G_q^*(g) = 1/P_q^*(g)$ for $\mathrm{DMU}_q \in \mathbf{E}^{l_0}$, $l_0 \in \{2,\ldots,L\}$, (ii) $G_q^*(g) < 1$ for each $g = 1,\ldots,l_0-1$, and (iii) $G_q^*(g+1) < G_q^*(g)$.

Definition A.2. $M_q^*(g) \equiv 1/G_q^*(g)$ is called (input-oriented) *g-degree* progress of DMU_q from a specific level $\mathbf{E}^{l_0}$.

Obviously, $M_q^*(g) > 1$. For a larger $M_q^*(g)$, more progress is expected. Next, we develop the following linear programming problem for $\mathrm{DMU}_q = (x_q, y_q) = (x_{1q},\ldots,x_{mq},y_{1q},\ldots,y_{sq})$ in $\mathbf{E}^{l_0}$, $l_0 \in \{1,\ldots,\mathbf{L}-1\}$:

$$\bar{\bar{H}}_q^*(d) = \min \sum_{i=1}^{m} w_i H_q^i(d), \quad d = 1,\ldots,L-l_0$$
$$\text{s.t.} \quad \sum_{j\in F(\mathbf{E}^{l_0+d})} \lambda_j x_{ij} \leqslant H_q^i(d)x_{iq}, \quad i = 1,\ldots,m;$$
$$\sum_{j\in F(\mathbf{E}^{l_0+d})} \lambda_j y_{rj} \geqslant y_{rq}, \quad r = 1,\ldots,s;$$
$$H_q^i(d) \geqslant 1, \quad i = 1,\ldots,m;$$
$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0+d}),$$

where w_i $(i = 1,\ldots,m)$ such that $\sum_{i=1}^m w_i = 1$ are user-specified weights reflecting the preference over the input improvements.

Definition A.3. The optimal value $\bar{\bar{H}}_q^*(d)$ is called (input-oriented) VJ *d-degree* attractiveness of DMU_q in a specific level $\mathbf{E}^{l_0}$.

To measure the (input-oriented) VJ progress, we have

$$\bar{\bar{\mathrm{G}}}_q^*(g) = \min \sum_{i=1}^{m} w_i G_q^i(g), \quad g = 1,\ldots,l_0-1$$
$$\text{s.t.} \quad \sum_{j\in F(\mathbf{E}^{l_0-g})} \lambda_j x_{ij} \leqslant G_q^i(g)x_{iq}; \quad i = 1,\ldots,m;$$
$$\sum_{j\in F(\mathbf{E}^{l_0-g})} \lambda_j y_{rj} \geqslant y_{rq}; \quad r = 1,\ldots,s;$$
$$G_q^i(g) \leqslant 1, \quad i = 1,\ldots,m;$$
$$\lambda_j \geqslant 0, \quad j \in F(\mathbf{E}^{l_0-g}).$$

Definition A.4. The optimal value $\bar{\bar{M}}_q^*(g) \equiv 1/\bar{\bar{G}}_q^*(g)(\underline{\underline{\Delta}})$, is the (input-oriented) VJ *g-degree* progress of DMU_q from a specific level $\mathbf{E}^{l_0}$.

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